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Kyoto University
d-primitive words and D(1)-concatenated words

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Abstract
In this paper, we study d-primitive words and D(1)-concatenated words. It is shown that neither $D(1)$, the set of all primitive words, nor $D(1)D(1)$, the set of all D(1)-concatenated words, is regular. We also show that every d-primitive word, with the length of two or more, is D(1)-concatenated.

1 Introduction

The notion of primitivity of words plays a central role in algebraic coding theory and combinatorial theory of words (See [3] [4], and [6]).

Recently attention has been drawn to $D(1)$, the set of all d-primitive words, which is a proper subset of $Q$, the set of all primitive words. ([1], [7])
In this paper, we study languages $D(1)$ and $D(1)D(1)$, the set of all $D(1)$-concatenated words. We consider regularity of $D(1)$ and $D(1)D(1)$, and a relation between $D(1)$ and $D(1)D(1)$.

In section 2 some basic definitions and results are presented. In section 3, the following results are proved. (1) Neither $D(1)$ nor $D(1)D(1)$ is regular. (2) For $u,v,w \in \Sigma^+$ with $|u| = |v|$, $uvw \in D(1)$ if and only if $uv^+w \subseteq D(1)$. In section 4, we consider a relation between $D(1)$ and $D(1)D(1)$. It is proved that for a word $w$ in $D(1)$, with the length of two or more, $w$ is in $D(1)D(1)$.

2 Preliminaries

Let $\Sigma$ be an alphabet. $\Sigma^*$ denotes the free moniod generated by $\Sigma$, that is, the set of all finite words over $\Sigma$, including the empty word $\epsilon$, and $\Sigma^+ = \Sigma^* - \epsilon$. For $w$ in $\Sigma^*$, $|w|$ denotes the length of $w$. Any subset of $\Sigma^*$ is called a language over $\Sigma$.

For a word $u \in \Sigma^+$, if $u = vw$ for some $v,w \in \Sigma^*$, then $v$ ($w$) is called a prefix (suffix) of $u$, denoted by $v \leq_p u$ ($w \leq_s u$, resp.). If $v \leq_p u$ ($w \leq_s u$) and $u \neq v(w \neq u)$, then $v$ ($w$) is called a proper prefix (proper suffix) of $u$, denoted by $v <_p u$ ($w <_s u$, resp.). For a word $w$, let $Pref(w)$ ($Suff(w)$) be the set of all prefixes (suffixes, resp.) of $w$.

A nonempty word $u$ is called a primitive word if $u = f^n$, for some $f \in \Sigma^+$, and some $n \geq 1$ always implies that $n = 1$. Let $Q$ be the set of all primitive words over $\Sigma$. A nonempty word $u$ is a non-overlapping word if $u = vx = yv$ for some $x,y \in \Sigma^+$.
always implies that $v = \epsilon$. Let $D(1)$ be the set of all non-overlapping words over $\Sigma$. A words in $D(1)$ is also called a $d$-primitive word. For $u \in \Sigma^+$, $u$ is said to be $D(1)$-concatenated if there exist $x, y \in D(1)$ such that $xy = u$, i.e., $u \in D(1)D(1)$. (See [1] and [5]).

For $w \in \Sigma^+$ with $|w| \geq 2$, $Hlv\s(w)$ is defined as follows. If $w = xy$ for $x, y \in \Sigma^*$, with $|x| = |y|$, then $Hlv\s(w) = (x, y)$. If $w = xcy$, for $x, y \in \Sigma^*$, $c \in \Sigma$, with $|x| = |y|$, then $Hlv\s(w) = (x, y)$. For $x, y \in \Sigma^+$, if $(Pref(x) - \{\epsilon\}) \cap (Suff(y) - \{\epsilon\}) = \phi$, then $(x, y)$ is said to be a non-overlapping pair (n-o. pair).

**Lemma 1** ([2]) Let $u \in \Sigma^+$. Then $u \notin D(1)$ iff there exists a unique word $v \in D(1)$ with $|v| \leq (1/2)|u|$ such that $u = vv$ for some $w \in \Sigma^*$.

**Remark 1** Let $u, v \in \Sigma^+$. Obviously $uv \in D(1)$ implies that $(u, v)$ is a n-o. pair. The converse does not hold; for $u = abbbbba$, and $v = bb$, $(u, v)$ is a n-o. pair but $uv$ is not in $D(1)$. However, in the next Proposition, we show the above two are equivalent on the condition that $u$ and $v$ are in $D(1)$.

**Proposition 2** For $u \in \Sigma^+$, the following two are equivalent.

(1) $u, v, uv$, and $vu$ are in $D(1)$.

(2) $u, v$ are in $D(1)$, and $(u, v), (v, u)$ are n-o. pairs.

The next lemma is immediate by Lemma 1

**Lemma 3** (1) For a n-o. pair $(x, y)$ and $c \in \Sigma$, with $|x| = |y|$, both $xy$ and $xcy$ are in $D(1)$. (2) Let $w \in D(1)$. For every $x \in Pref(w) - \{\epsilon\}$ and $y \in Suff(w) - \{\epsilon\}$, $(x, y)$ is a n-o. pair.
3 Regularity of D(1) and D(1)-concatenated words

Proposition 4 $D(1)$ is not regular.

Proposition 5 $D(1)D(1)$ is not regular.

Proposition 6 Let $|u| = |w|$ for $u, v, w \in \Sigma^+$. Then $uvw \in D(1)$ if and only if $uv^+w \subseteq D(1)$.

Remark 2 Unfortunately, the previous proposition does not hold without the condition $|u| = |w|$. For example, let $u = babaa$, $v = ba$, and $w = a$. Then $uvw = bavaabaa \in D(1)$, but $uv^2w = (babaa)^2 \notin D(1)$.

4 d-primitive words and D(1)-concatenated words

In this section we consider a relation between primitive words and D(1)-concatenated words.

Lemma 7 Let $zxyx$ be in $D(1)$ for $z, x \in \Sigma^+$, $y \in \Sigma^*$. If $z$ is in $D(1)$, then $zx$ is also in $D(1)$.

Proposition 8 Let $|w| \geq 2$ for $w \in \Sigma^+$. If $w \in D(1)$, then $w$ is a $D(1)$-concatenated word. In other words, for a word $w$ in $D(1)$, with the length of two or more, $w$ is in $D(1)D(1)$. 
References


