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<thead>
<tr>
<th>項目</th>
<th>内容</th>
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<tr>
<td>タイトル</td>
<td>SMITH PROBLEM FOR A FINITE OLIVER GROUP WITH NON-TRIVIAL CENTER (Geometry of Transformation Groups and Related Topics)</td>
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<td>著者</td>
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京都大学
SMITH PROBLEM FOR A FINITE OLIVER GROUP WITH NON-TRIVIAL CENTER

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1. INTRODUCTION

The Smith problem is that two tangential representations are isomorphic or not for a smooth action on a homotopy sphere with exactly two fixed points. Two real $G$-modules $U$ and $V$ are called Smith equivalent if there exists a smooth action of $G$ on a sphere $\Sigma$ such that $S^G = \{x, y\}$ for two points $x$ and $y$ at which $T_x(\Sigma) \cong U$ and $T_y(\Sigma) \cong V$ as real $G$-modules. We will consider a subset $Sm(G)$ of the real representation ring $RO(G)$ of $G$ consisting of the differences $U-V$ of real $G$-modules $U$ and $V$ which are Smith equivalent. We also define a subset $CSm(G)$ of $RO(G)$ consisting of the differences $U-V \in Sm(G)$ of real $G$-modules $U$ and $V$ such that for the sphere $\Sigma$ appearing in the notion of Smith equivalence of $U$ and $V$ satisfies that $\Sigma^p$ is connected for every $P \in \mathcal{P}(G)$. Moreover, we assume that $0 \in Csm(G)$ as definition.

In many groups, Smith equivalent modules are not isomorphic. In this paper we discuss the Smith problem for an Oliver group with non-trivial center. Throughout this paper we assume a group is finite.

2. TOPOLOGICAL VIEWPOINT

We denote by $\mathcal{P}(G)$ the family of subgroups of $G$ consisting of the trivial subgroup of $G$ and all subgroups of $G$ of prime power order, and by $\mathcal{L}(G)$ the family of large subgroups of $G$. Here, by a large subgroup of $G$ we mean a subgroup $H \leq G$ such that $O_p^0(G) \leq H$ for some prime $p$, where $O_p^0(G)$ is the smallest normal subgroup of $G$ such that $|G/O_p^0(G)| = p^k$ for some integer $k \geq 0$. A real $G$-module $V$ is called $\mathcal{L}(G)$-free if $\dim V^H = 0$ for each $H \in \mathcal{L}(G)$, which amounts to saying that $\dim V^{O_p^0(G)} = 0$ for each prime $p$ dividing $|G|$. Following [PS0], we denote by $LO(G)$ the subgroup of $RO(G)$ consisting of the differences $U-V$ of two real $\mathcal{L}(G)$-free $G$-modules $U$ and $V$ such that $\text{Res}_G^U(U) \cong \text{Res}_G^V(V)$ for every $P \in \mathcal{P}(G)$.

For two subgroups $P < H$ of $G$ with $P \in \mathcal{P}(G)$, and a smooth $G$-manifold $X$ or a real $G$-module $X$, we consider the number

$$d_X(P, H) = \dim X^P - 2 \dim X^H$$

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where $\dim$ means the dimension of the $G$-CW complex. Furthermore we define by $\dim Z = \dim X - \dim Y$ for a virtual real $G$-module $Z = X - Y$ of $RO(G)$. A smooth $G$-manifold $X$ satisfies the gap condition (GC) if $d_X(P, H) > 0$ for every pair $(P, H)$ of subgroups $P < H$ of $G$ with $P \in \mathcal{P}(G)$.

The following theorem goes back to [PSo], the Realization Theorem.

**Theorem 2.1 ([PSo]).** Let $G$ be a finite Oliver gap group. Then $LO(G) \subseteq CSm(G)$.

We impose a number of restrictions on a smooth $G$-manifold, in particular, a real $G$-module $X$. The restrictions are collected in the following conditions, where we consider series $P < H \leq G$ of subgroups $P$ and $H$ of $G$ always with $P \in \mathcal{P}(G)$. We say that a smooth $G$-manifold $X$ satisfies the weak gap condition (WGC) if the conditions (WGC1)–(WGC4) all hold (cf. [LM], [MP]), and we say that $X$ satisfies the semi-weak gap condition (SWGC) if the conditions (WGC1) and (WGC2) both hold.

1. (WGC1) $d_X(P, H) \geq 0$ for every $P < H \leq G$, $P \in \mathcal{P}(G)$.
2. (WGC2) If $d_X(P, H) = 0$ for some $P < H \leq G$, $P \in \mathcal{P}(G)$, then $[H : P] = 2$, $\dim X^H > \dim X^K + 1$ for every $H < K \leq G$, and $X^K$ is connected.
3. (WGC3) If $d_X(P, H) = 0$ for some $P < H \leq G$, $P \in \mathcal{P}(G)$, and $[H : P] = 2$, then $X^K$ can be oriented in such a way that the map $g: X^K \to X^H$ is orientation preserving for any $g \in N_G(H)$.
4. (WGC4) If $d_X(P, H) = d_X(P, H') = 0$ for some $P < H$, $P < H'$, $P \in \mathcal{P}(G)$, then the smallest subgroup $\langle H, H' \rangle$ of $G$ containing $H$ and $H'$ is not a large subgroup of $G$.

Now, for a finite group $G$, we define subgroups $VLO(G)$, $WLO(G)$ and $MLO(G)$ of the free abelian group $LO(G)$ as follows.

- $VLO(G) = \{U - V \in LO(G) \mid U \oplus W$ and $V \oplus W$ both satisfy the gap condition for some real $\mathcal{L}(G)$-free $G$-module $W\}$
- $WLO(G) = \{U - V \in LO(G) \mid U \oplus W$ and $V \oplus W$ both satisfy the weak gap condition for some real $\mathcal{L}(G)$-free $G$-module $W\}$
- $MLO(G) = \{U - V \in LO(G) \mid U \oplus W$ and $V \oplus W$ both satisfy the semi-weak gap condition for some real $\mathcal{L}(G)$-free $G$-module $W\}$

Note that if $\mathcal{P}(G) \cap \mathcal{L}(G) = \emptyset$ then for an $\mathcal{L}(G)$-free real $G$-modules $U$ and $V$ there is a real $\mathcal{L}(G)$-free $G$-module $W$ such that both $U \oplus W$ and $V \oplus W$ satisfy (WGC2), and if $G$ is an Oliver group then for an $\mathcal{L}(G)$-free real $G$-modules $U$ and $V$ there is a real $\mathcal{L}(G)$-free $G$-module $W$ such that both $U \oplus W$ and $V \oplus W$ satisfy (WGC2) and (WGC4).

In general, $VLO(G) \subseteq WLO(G) \subseteq MLO(G) \subseteq LO(G)$ by definitions. But if $G$ is a gap group, then for every $U - V \in LO(G)$, there exists a real $\mathcal{L}(G)$-free $G$-module $W$ satisfying the gap condition, such that $U \oplus W$ and $V \oplus W$ also satisfy the gap condition, and thus $U - V \in VLO(G)$, and hence

$VLO(G) = WLO(G) = MLO(G) = LO(G)$. 


Therefore, the following theorem extends the result in Theorem 2.1 by using Theorem in [MP].

**Theorem 2.2.** Let $G$ be a finite Oliver group. Then $WLO(G) \subseteq CSm(G)$. 

### 3. Algebraic viewpoint

We denote by $PO(G)$ the subgroup of $RO(G)$ of $G$ consisting of the differences $U - V$ of representations $U$ and $V$ such that $\dim U^G = \dim V^G$ and $\text{Res}_P^G(U) \cong \text{Res}_P^G(V)$ for any subgroup $P$ of $G$ of prime power order. We note that in [PSo], $PO(G)$ is denoted by $IO(G, G)$. Similarly, we denote by $\overline{PO}(G)$ the subgroup of $RO(G)$ of $G$ consisting of the differences $U - V$ of representations $U$ and $V$ such that $\dim U^G = \dim V^G$ and $\text{Res}_P^G(U) \cong \text{Res}_P^G(V)$ for any subgroup $P$ of $G$ of odd prime power order and order 2.

By a theorem of Sanchez [Sa], the difference of two Smith equivalent representations lies in $\overline{PO}(G)$ and the difference of two $\mathcal{P}$-matched Smith equivalent representations lies in $PO(G)$.

We define the Laitinen number $a_G$ as the number of real conjugacy classes in $G$ represented by elements of $G$ not of prime power order. The rank of $PO(G)$ is equal to the maximum of 0 and $a_G - 1$. Moreover the rank of $\overline{PO}(G)$ is equal to the rank of $PO(G)$ plus the number of all real conjugacy classes represented by 2-elements of order $\geq 8$. Now, let $H$ be a normal subgroup of $G$. We denote by $PO(G, H)$ the subgroup of $RO(G)$ consisting of the differences $U - V$ of representations $U$ and $V$ such that $U^H \cong V^H$ as representations over $G/H$, and $\text{Res}_P^G(U) \cong \text{Res}_P^G(V)$ for any subgroup $P$ of prime power order. Again, we note that in [PSo], $PO(G, H)$ is denoted by $IO(G, H)$. It holds that $PO(G) = PO(G, G)$.

Let $b_{G/H}$ be the number of all real conjugacy classes in $G/H$ which are images from real conjugacy classes of $G$ represented by elements not of prime power order by the surjection $G \to G/H$. Then the rank of $PO(G, H)$ is equal to $a_G - b_{G/H}$ (see [PSo]).

**Proposition 3.1** (cf. [PSo]). It holds that

$$PO(G, G^{nil}) \leq LO(G) \leq PO(G) \leq \overline{PO}(G) \leq RO(G).$$

Note that $G^{nil} = \cap_p O^p(G)$. Also it is known that

$$LO(G) \subseteq CSm(G) \subseteq Sm(G)$$

if $G$ is an Oliver gap group.

### 4. Upper restriction

Let $S$ be a set of primes dividing $|G|$ and 1, and let denote by $G^{\cap S}$ the normal subgroup of $G$ defined as

$$G^{\cap S} = \bigcap_{L \trianglelefteq G; [G:L] \in S} L.$$
Theorem 4.1 ([M07a, KMK]). Let $G$ be a finite Oliver group. We set $S = \{2, 3\}$ if a Sylow 2-subgroup of $G$ is normal and set $S = \{2\}$ otherwise. Then it holds that
\[
CSm(G) \subseteq PO(G, G^{\cap S}) \quad \text{and} \quad Sm(G) \subseteq PO(G, G^{\cap S}).
\]
In addition if $G$ is a gap group and $G^{nil} = G^{\cap S}$, then it holds that
\[
LO(G) = CSm(G) = PO(G, G^{nil}).
\]
Here $G^{nil}$ is the minimal subgroup among normal subgroups $N$ of $G$ such that $G/N$ is nilpotent.

In particular, $a_G = b_{G^{\cap S}}$ yields that $CSm(G) = 0$.

Proposition 4.2 (cf. [PSu07]). $G/G^{\cap S}$ is an elementary abelian group.

5. Known results

In this section we summarize several known results ([Ju, M07a, M07b, PSo, PSu07, Su]). First we treat a non-solvable group. Pawalowski and Solomon [PSo] showed that $0 \neq PO(G, G^{nil}) \subset CSm(G)$ if $G$ is a non-solvable gap group with $a_G \geq 2$, Pawałowski and Sumi [PSu07] showed that $0 \neq LO(G) \cap CSm(G)$ if $G$ is a non-solvable gap group with $a_G \geq 2$, except $Aut(A_6)$, $P\Sigma L(2, 27)$, and Morimoto [M07a, M07b] showed that $Sm(Aut(A_6)) = 0$ and $CSm(P\Sigma L(2, 27)) \neq 0$. Combining these results we can state that

Theorem 5.1. For a finite non-solvable group $G$, $Sm(G) = 0$ if and only if $a_G \leq 1$ or $G \equiv Aut(A_6)$.

We say that an element not of prime power order is an NPP element. Morimoto showed the following theorem to get $CSm(P\Sigma L(2, 27)) \neq 0$.

Theorem 5.2 (Morimoto). Let $G$ be an Oliver gap group. Suppose that $O^2(G)$ has a dihedral subgroup $D_{2pq}$ of order $2pq$ with distinct primes $p$ and $q$ and $G$ has two real conjugacy classes of NPP elements contained in $O^2(G)$. Then $CSm(G) \neq 0$.

To show $LO(G) \cap CSm(G) \neq 0$ for a non-solvable group with $LO(G) \neq 0$, Pawalowski and Sumi introduced a basic pair (cf. [PSu07, Su]). Let $f: G \rightarrow G^{nil}$ be a natural homomorphism. For two NPP elements $x$ and $y$ of an finite Oliver group $G$, we call $(x, y)$ a basic pair, if $f(x) = f(y)$, $x$ is not real conjugate to $y$, and one of the following claims is satisfied:

1. $x$ and $y$ are elements of some gap subgroup of $G$.

2. $|x|$ is even and the involution of $\langle x \rangle$ is conjugate to the involution of $\langle y \rangle$ in $G$.

We denote by $\pi(G)$ the set of all primes dividing the order of $G$. Note that $\langle x \rangle G^{nil} = \langle y \rangle G^{nil}$ as $f(x) = f(y)$. Recall that if $|x|$ is even, then for the involution $c$ of $\langle x \rangle$, $c \in O^2(G)$ or $|\pi(O^2(G))| \geq 2$, then $\langle x \rangle O^2(G)$ is a gap group.

Theorem 5.3 ([PSu07]). If an Oliver group has a basic pair, it holds $LO(G) \cap CSm(G) \neq 0$. 

\end{quote}
Recall that $LO(G/G^{nil}) \subseteq LO(G)$. Furthermore we have

**Proposition 5.4.** $2LO(G/G^{nil}) \subseteq WLO(G)$ and in particular $LO(G/G^{nil}) \neq 0$ implies $CSm(G) \neq 0$.

Then $LO(G) \cap CSm(G) = 0$ implies $LO(G/G^{nil}) = 0$. Thus the following proposition is important.

**Proposition 5.5 ([PSu07]).** Let $H$ be a nilpotent group with $LO(H) = 0$. Then $H$ is isomorphic to one of the following groups:

1. a $p$-group for a prime $p$,
2. $C_2 \times P$ for an odd prime $p$ and a $p$-group $P$, or
3. $P \times C_3$ for a 2-group $P$ such that any element is self-conjugate.

**Lemma 5.6.** If $a_G \geq 2$ and $LO(G) = 0$ it holds $|\pi(G/G^{nil})| = 1, 2$.

**Proof.** If $|\pi(G/G^{nil})| \geq 3$, then $G/G^{nil}$ is a gap group with $LO(G/G^{nil}) \neq 0$, a contrary. If $|\pi(G/G^{nil})| = 0$, then $G$ is perfect and thus rank $LO(G) = a_G - 1 > 0$, a contrary. \hfill \square

**Theorem 5.7.** If $LO(G) \cap CSm(G) = 0$, then $G$ has no element $x$ with $|\pi(\langle x \rangle)| \geq 3$.

**Proof.** We assume that $x$ is an element of $G$ of order $pqr$ such that $p, q, r$ are distinct primes. It is clear that $a_G \geq 4$. We may assume that $x^{pq} \in G^{nil}$ by Lemma 5.6. Then $(x^{pq}x^{qr}x^{pr})$ is a basic pair, a contrary. \hfill \square

Thus $|\pi(\langle c \rangle)| \leq 2$ for each non-trivial element $c \in Z(G)$.

6. **Induced Modules and $PO(G)$**

Let $G$ be a finite group and $NPP(G)$ be the set of all elements of $G$ not of prime power order. Note that $NPP(G)$ does not contain the identity element. For the real representation ring $RO(G)$, the real vector space $RO(G) \otimes \mathbb{R}$ is identified with the vector space consisting of all maps from the set of real conjugacy classes of $G$ to the real number field $\mathbb{R}$. We denote by $1^G_{(g)_{\pm}^{G}}$, the map defined by $1^G_{(g)_{\pm}^{G}}((g)_{\pm}^{G}) = 1$ and $1^G_{(g)_{\pm}^{G}}((a)_{\pm}^{G}) = 0$ if $a$ is not real conjugate to $g$. Then

$$RO(G) \otimes \mathbb{R} \cong \langle 1^G_{(g)_{\pm}^{G}} \mid (g)_{\pm}^{G} \subseteq G \rangle$$

and

$$RO(G)_{P(G)} \otimes \mathbb{R} \cong \langle 1^G_{(g)_{\pm}^{G}} \mid g \in NPP(G) \rangle.$$

Let $K$ be a subgroup of $G$. The induced map $\text{Ind}_K^G 1^K_{(k)_{\pm}^{K}}$ has a non-zero value at $(g)_{\pm}^{G}$ only if $g$ is real conjugate to $k$ in $G$, i.e. $(g)_{\pm}^{G} = (k)_{\pm}^{G}$, since

$$\text{Ind}_K^G 1^K_{(k)_{\pm}^{K}}((a)_{\pm}^{G}) = \sum_{b \in G/K, b^{-1}ab \in K} 1^K_{(k)_{\pm}^{K}}(((b^{-1}ab)_{\pm}^{K}).$$
We denote by $RO(G)_{P(G)}$ the subset of $RO(G)$ consisting the differences $U - V$ of real representations $U$ and $V$ such that $\text{Res}^G_P(U) \cong \text{Res}^G_P(V)$ for $P \in P(G)$. It is clear that

$$PO(G) = \text{Ker}(\text{Fix}^G : RO(G)_{P(G)} \to \mathbb{R}).$$

We have the following commutative diagram.

$$\begin{array}{ccc}
RO(K)_{P(K)} \otimes \mathbb{R} & \longrightarrow & (\text{Ind}_K^G RO(K)_{P(K)}) \otimes \mathbb{R} \\
\downarrow & & \downarrow \\
\langle 1_{(k)^{K}}^{K} | k \in \text{NPP}(K) \rangle & \longrightarrow & \langle 1_{(g)^{G}}^{G} | g \in \text{NPP}(G) \rangle
\end{array}$$

It holds that

$$(\text{Ind}_K^G RO(K)_{P(K)}) \otimes \mathbb{R} = (\text{Ind}_K^G RO(K))_{P(G)} \otimes \mathbb{R}$$

and then that

$$(\text{Ind}_K^G RO(K)_{P(K)}) \otimes \mathbb{Q} = (\text{Ind}_K^G RO(K))_{P(G)} \otimes \mathbb{Q}.$$
(2) If two involutions $x$ and $y$ of $G$ outside of $O^2(G)$ are not conjugate then $C_G(x)$ or $C_G(y)$ is a 2-group.

The author does not know a group $G$ with $MLO(G) \neq LO(G)$.

7. NON-TRIVIAL CENTRAL

In this section we consider whether $CSm(G) = 0$ or not for an Oliver group $G$ with $a_G \geq 2$. In the section 5 we know completely it for a non-solvable group $G$. From now on we assume that $G$ is an Oliver solvable group with $LO(G) \cap CSm(G) = 0$ and $a_G \geq 2$. Recall that $PO(G, Gn) \neq 0$ implies $a_G \geq 2$.

Lemma 7.1. If $Z(G) \neq \{1\}$ then $|\pi(G^{nil})| = 2$.

Proof. Since $LO(G/G^{nil}) = 0$, $G/G^{nil}$ is isomorphic to $P$, $C_2 \times P$, or $C_3 \times P_2$, where $P$ is a $p$-group and $P_2$ is a 2-group. Then for some subgroup $K$ of $G$, the sequence $G^{nil} \leq K \leq G$ such that $|\pi(G/K)| = 1$ and $K/G^{nil}$ is cyclic. Thus $|\pi(G^{nil})| \geq 2$. We assume that $|\pi(G^{nil})| \geq 3$. Take distinct primes $p$, $q$, $r$ in $\pi(G^{nil})$. Let $c \in Z(G)$ be an element of prime order. We may assume that $|c| \neq q, r$. Take elements $x_q$ and $x_r$ of $G^{nil}$ of order $q$ and $r$ respectively. Then $cx_q$ and $cx_r$ are NPP elements of distinct order. Therefore $(cx_q, cx_r)$ is a basic pair. 

Lemma 7.2. $Z(G)$ has no NPP element.

Proof. We suppose that $Z(G)$ has an NPP element $c$ of order $pq$ where $p$ and $q$ are primes. Then $|\pi(G)| = 2$ and $\pi(G) = \pi(\langle c \rangle) = \{p, q\}$ by Theorem 5.7. First we show that $G^{nil}$ is not a subgroup of $\langle c \rangle$. Suppose $G^{nil} \leq \langle c \rangle$. Let $f: G \to G/\langle c \rangle$ be a canonical epimorphism. Note that $\pi(G/\langle c \rangle) = \{p, q\}$. Since $f(G)$ is nilpotent, $O^p(f(G))$ is a Sylow $p$-subgroup of $f(G)$ and a Sylow $p$-subgroup $O^p(G)_p$ of $O^p(G)$ is normal and its quotient $O^p(G)/O^p(G)_p$ is cyclic. This is a contrary against $G$ is Oliver. 

Thus we can take an element $x$ of $G^{nil}$ which is not in $\langle c \rangle$. Since $f$ sends two NPP elements $xc$ and $c$ to elements of distinct order, $xc$ and $c$ are not real conjugate. It is clear that they are sent to the same element by $G \to G/G^{nil}$. Then $(xc, c)$ is a basic pair, which is a contrary. Thus $Z(G)$ has no NPP element. 

The following can be straightforward checked.
Lemma 7.3. Let \( c \in Z(G) \) be an element of order a prime \( p \). If \( G^{nil} \) has an element \( x \) of order \( q^2 \) for some prime \( q \neq p \), then \( G \) has a basic pair \( (cx, cx^q) \).

We define the DressLength\((G)\) as the minimal length \( n \) of sequences

\[
G = G_0 > G_1 > G_2 > \cdots > G_n = \{1\}
\]

such that \( O^{p_j}(G_{j-1}) = G_j \) with some prime \( p_j \) for each \( j \). In convenient, we assume DressLength\((G)\) = \( \infty \) if there is no sequence as above. For example, DressLength\((G)\) = \( \infty \) for a non-solvable group. It is easy to see that DressLength\((G)\) \( \geq 3 \) if \( G \) is an Oliver group and that DressLength\((G)\) \( \geq 3 \) if \( G \) is a gap group.

Now we recall classical results. A finite group is called a CP group if it has no NPP elements.

Lemma 7.4 (Higman, cf. [PSO, Lemma 2.5]). Let \( H \) be a finite solvable CP group. Then one of the following conclusions holds:

1. \( H \) is a \( p \)-group for some prime \( p \); or
2. \( H = K \rtimes C \) is a Frobenius group with kernel \( K \) and complement \( C \), where \( K \) is a \( p \)-group and \( C \) is a \( q \)-group of \( q \)-rank 1 for two distinct primes \( p \) and \( q \); or
3. \( H = K \rtimes C \rtimes A \) is a 3-step group, in the sense that \( K \rtimes C \) is a Frobenius group as in the conclusion (2) with \( C \) cyclic, and \( C \rtimes A \) is a Frobenius group with kernel \( C \) and complement \( A \), a cyclic \( p \)-group.

Proposition 7.5 ([Hu, Proposition 22.3 and Remark on p.193]). \( \text{Aut}(C_{2^a}) = C_2 \times C_{2^{a-2}} \)

where \( x \mapsto x^5 \) is a generator of \( C_{2^{a-2}} \) and \( x \mapsto x^{-1} \) is a generator of \( C_2 \). \( \text{Aut}(C_{p^n}) = C_{p^{a-1}(p-1)} \) for an odd prime \( p \).

With these results we use a Frattini subgroup and a Fitting subgroup and then we obtain the following results.

Theorem 7.6. Let \( G \) be an Oliver solvable group with \( a_G \geq 2 \) and \( Z(G) \neq \{1\} \). If \( CSm(G) = 0 \), then it holds the following.

1. \( Z(G) \) has no NPP element.
2. If \( Z(G) \) is a \( p \)-group, an element of \( G^{nil} \) not of \( p \) power order has prime order.
3. \( |\pi(G)| = 2 \).
4. \( \text{DressLength}(G) = 3, 4 \).

References


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