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# Tangential Representations at Fixed Points

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## 1. Basic problems

Let $G$ be a finite group throughout this paper. We mean by a (real) *G-module* a real $G$-representation (space) of finite dimension. Let $\mathcal{S}(G)$ denote the set of all subgroups of $G$ and let $\mathcal{P}(G)$ denote the subset of $\mathcal{S}(G)$ consisting of all subgroups of prime power order. Unless otherwise stated, $M$ will stand for a (smooth) $G$-manifold. S. Cappell-J. Shaneson referred the next problem to a basic problem on Algebraic and Differential Topology.

**Problem** (Basic Problem A). Let $x$, $y \in M^G$. How similar is a neighborhood of $x$ to that of $y$ as $G$-spaces?

If $x \in M^G$, then we can regard the tangent space $T_x(M)$ at $x$ in $M$ as a $G$-module. Thus the problem above is equivalent to ask

**Problem** (Basic Problem B). How similar is $T_x(M)$ to $T_y(M)$ as $G$-modules?

A specific case of the problem was posed by P. A. Smith.

**Problem** (Smith Problem). If $\Sigma$ is a homotopy sphere with exactly two fixed points $x$ and $y$, then is $T_x(\Sigma)$ isomorphic to $T_y(\Sigma)$ as $G$-modules?

We would like to study this problem in a slightly generalized form. Now let $\mathfrak{A}(2)$ denote the family of all (smooth) $G$-actions on manifolds with exactly 2 fixed points and let $\mathfrak{X} \subset \mathfrak{A}(2)$. We say that $G$-modules $V$ and $W$ are *$\mathfrak{X}$-related*, and write $V \sim_{\mathfrak{X}} W$, if there exists a smooth $G$-action on $M \in \mathfrak{X}$ such that $M^G = \{a, b\}$, $T_a(M) \cong_G V$ and $T_b(M) \cong_G W$. Let $\mathrm{RO}(G)$ denote the real representation ring of $G$. We define the *$\mathfrak{X}$-relation set* $\mathrm{RO}(G, \mathfrak{X})$ *of* $G$ by

$$\mathrm{RO}(G, \mathfrak{X}) = \{[V] - [W] \in \mathrm{RO}(G) \mid V \sim_{\mathfrak{X}} W\}$$

**Problem** (Basic Problem C). Describe $\mathrm{RO}(G, \mathfrak{X})$ in terms of Algebra (or Representation Theory)

We say that a $G$-action on a disk $D$ has a *linear boundary action* if the boundary $\partial D$ is $G$-diffeomorphic to the unit sphere $S(V)$ for some $G$-module $V$. A $G$-action on a homotopy sphere $\Sigma$ is called a *$G$-semilinear sphere* if $\Sigma^H$ is a homotopy sphere for each $H \leq G$. $G$-modules $V$ and $W$ are called *$\mathcal{P}$-matched* if $\mathrm{res}^G_P V \cong_P \mathrm{res}^G_P W$ for all $P \in \mathcal{P}(G)$.

We will discuss Basic Problem C for the following subfamilies of $\mathfrak{A}(2)$.

$$
\begin{aligned}
\mathfrak{E} &= \{G\text{-actions on Euclidean spaces} \in \mathfrak{A}(2)\} \\
\mathfrak{D} &= \{G\text{-actions on disks} \in \mathfrak{A}(2)\} \\
\mathfrak{D}_{\partial\text{-lin}} &= \{G\text{-actions on disks with linear boundary action} \in \mathfrak{A}(2)\} \\
\mathfrak{S} &= \{G\text{-actions on homotopy spheres} \in \mathfrak{A}(2)\} \\
\mathfrak{S}_{\text{s-free}} &= \{\text{semi free actions} \in \mathfrak{S}\} \\
\mathfrak{S}_{CS} &= \{\Sigma \in \mathfrak{S} \text{ such that } |\Sigma^H| = 2 \text{ or } \Sigma^H \text{ is connected } (\forall\ H \leq G)\ \} \\
\mathfrak{S}_{\text{s-lin}} &= \{G\text{-semilinear spheres} \in \mathfrak{A}(2)\} \\
\mathrm{p}\mathfrak{S} &= \{\Sigma \in \mathfrak{S}\ (\Sigma^G = \{x, y\}) \text{ such that } T_x(\Sigma) \text{ and } T_y(\Sigma) \text{ are } \mathcal{P}\text{-matched}\}
\end{aligned}
$$

With this notation, the Smith Problem is equivalent to ask whether $\mathrm{RO}(G, \mathfrak{S}) = 0$ or not.

Here we may remark the following.

**Theorem** (G. E. Bredon [2]). *Let $G = C_n$ with $n = p^a$ and $\Sigma \in \mathfrak{S}$ with $\dim \Sigma = 2k$ and $x$, $y \in \Sigma^G$. Then $T_x(\Sigma) - T_y(\Sigma)$ is divisible by $p^h$ in* $\mathrm{RO}(G)$*, where* $h = \left[\dfrac{pk - n}{pn - n}\right]$.

By T. Petrie (e.g. [24]), the theorem above implies that if $\dim \Sigma \gg n$ then $T_x(\Sigma) \cong_G T_y(\Sigma)$. Thus, in the case $G = C_n$ with $n = 2^a \geq 8$, the set $\mathrm{RO}(G, \mathfrak{S})$ is not additively closed.

## 2. Preliminary

Let $\mathcal{H}$ be a set of subgroups of $G$. $G$-modules $V$ and $W$ are called *$\mathcal{H}$-matched* if $\mathrm{res}^G_H V \cong_H \mathrm{res}^G_H W$ for all $H \in \mathcal{H}$. A $G$-module $V$ is called *$\mathcal{H}$-free* if $V^H = 0$ holds for any $H \in \mathcal{H}$. For $M \subset \mathrm{RO}(G)$, and $\mathcal{H}$, $\mathcal{K} \subset \mathcal{S}(G)$, we define

$$M_{\mathcal{H}} = \{V - W \in M \mid V \text{ and } W \text{ are } \mathcal{H}\text{-matched}\}$$

$$M^{\mathcal{K}} = \{V - W \in M \mid V,\, W \text{ are } \mathcal{K}\text{-free}\}$$

$$M^{\mathcal{K}}_{\mathcal{H}} = M_{\mathcal{H}} \cap M^{\mathcal{K}}.$$

By Definition, we have $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = \mathrm{RO}(G, \mathfrak{S})_{\mathcal{P}(G)}$.

In some other papers, $V$ and $W$ are called *Smith equivalent* if $V \sim_{\mathfrak{S}} W$; $V$ and $W$ are called *s-Smith equivalent* if $V \sim_{\mathfrak{S}_{\text{s-lin}}} W$; $V$ and $W$ are called *primary Smith equivalent* if $V \sim_{\mathfrak{p}\mathfrak{S}} W$. The set $Sm(G) = \mathrm{RO}(G, \mathfrak{S})$ was usually called the *Smith set* and the set $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S})$ *primary Smith set*. By definition, $Sm(G)_{\mathcal{P}(G)} = \mathrm{RO}(G, \mathfrak{p}\mathfrak{S})$.

A finite group $G$ is called a *mod $\mathcal{P}$ cyclic group* if there exists a normal subgroup $P$ of $G$ such that $P$ is of prime power order and $G/P$ is cyclic. $G$ is called a *mod $\mathcal{P}$ hyperelementary group* if there exists a normal series $P \trianglelefteq H \trianglelefteq G$ such that $P$ and $G/H$ are of prime power order and $H/P$ is cyclic. $G$ is called an *Oliver group* if $G$ is not a mod $\mathcal{P}$ hyperelementary group. Thus $G$ is an Oliver group if and only if $G$ admits a $G$-action on a disk without fixed points.

Let $p$ be a prime. Let $G^{\{p\}}$ denote the smallest normal subgroup $H$ of $G$ such that $G/H$ has the order of a $p$-power. We refer $G^{\{p\}}$ to the *Dress subgroup of type $p$*. Let $G^{nil}$ denote the smallest normal subgroup $H$ of $G$ with nilpotent $G/H$. It follows that

$$G^{nil} = \bigcap_q G^{\{q\}}.$$

Let us adopt the following notation.

$$\mathcal{PC}(G) = \{\text{mod-}\mathcal{P} \text{ cyclic subgroups of } G\}$$

$$\mathcal{L}(G) = \{L \in \mathcal{S}(G) \mid L \supset G^{\{p\}} \text{ for some prime } p\}$$

$$\mathcal{M}(G) = \mathcal{S}(G) \smallsetminus \mathcal{L}(G)$$

## 3. Classical results (until 1996)

There are various affirmative answers to the Smith Problem. It is easy to see that if $V \sim_{\mathfrak{S}} W$ then $\mathrm{res}^G_P V \cong_P \mathrm{res}^G_P W$ for all $P \in \mathcal{P}(G)$ with $|P||4$. By Atiyah-Bott and Milnor, $V \sim_{\mathfrak{S}_{\text{s-free}}} W$ implies $V \cong_G W$. Sanchez showed that $V \sim_{\mathfrak{S}} W$ imples $\mathrm{Res}^G_P V \cong_P \mathrm{Res}^G_P W$ for any $P$ of odd-prime-power order.

To the contrary, there are negative answers to the Smith Problem. T. Petrie showed that if $G$ is an odd-order abelian group containing $C_{pqrs} \times C_{pqrs}$, where $p$, $q$, $r$, $s$ are distinct odd primes, then $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) \neq 0$. In addtion, Cappell-Shaneson showed that if $G = C_{4n}$ with $n \geq 2$ then $\mathrm{RO}(G, \mathfrak{S}_{CS}) \neq 0$.

Here we also recall classical results concerned with $\sim_{\mathfrak{E}}$ and $\sim_{\mathfrak{D}}$. By Petrie, if $G$ is an odd-order abelian group, then $\mathrm{RO}(G, \mathfrak{D})^{\mathcal{L}(G)} = \mathrm{RO}(G)^{\mathcal{L}(G)}_{\mathcal{P}(G)}$. R. Oliver showed that if $G$ is not of prime power order, then $\mathrm{RO}(G, \mathfrak{E}) = \mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)}$; if $G$ is an Oliver group, then $\mathrm{RO}(G, \mathfrak{D}) = \mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)}$.

## 4. Dimension Conditions on $G$-Modules

In order to apply an equivariant surgery theory to a $G$-manifold $M$, we require certain properties for $M^H$, where $H \in \mathcal{S}(G)$. If $V = T_x(M)$ with $x \in M^G$, then $\dim V^H$ is equal to the dimenison of the connected component of $M^H$ containing the point $x$.

Let $V$ be a $G$-module.

(1) We say that $V$ satisfies the *strong gap condition* if $\dim V^P > 2 \dim V^H + 2$ for all $P < H \leq G$ with $P \in \mathcal{P}(G)$.

(2) We say that $V$ satisfies the *gap condition* if $\dim V^P > 2 \dim V^H$ for all $P < H \leq G$ with $P \in \mathcal{P}(G)$.

(3) We say that $V$ satisfies the *weak gap condition* if the next dimension condition:

(Dim) $\dim V^P \geq 2 \dim V^H$ for all $P < H \leq G$ with $P \in \mathcal{P}(G)$

is satisfied and $V$ satisfies the orientation condition:

(Ori) $g : V^H \to V^H$ preserves orientation for any $g \in N_G(P) \cap N_G(H)$ such that $P \in \mathcal{P}(G)$, $P < H \leq G$ and $\dim V^P = 2 \dim V^H$.

A finite group $G$ is called a *gap group* if there exists a $G$-module $V$ such that $V$ is $\mathcal{L}(G)$-free and satisfies the gap condition.

## 5. Laitinen's Conjecture

E. Laitinen and K. Pawałowski were interested in determining the set $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S})$, namely $\mathrm{RO}(G, \mathfrak{S})_{\mathcal{P}(G)}$.

**Conjecture** (E. Laitinen). Let $G$ be an Oliver group. Then $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) \neq 0$ holds if and only if $\mathrm{RO}(G, \mathfrak{D}) \neq 0$.

For $g \in G$, let $(g)$ denote the conjugacy class $\{aga^{-1} \in G \mid a \in G\}$, and let $(g)^{\pm}$ denote the *real conjugacy class* $(g) \cup (g^{-1})$. Then $a_G$ stands for the number of all real conjugacy classes $(g)^{\pm}$ such that $g \in G$ is not of prime power order. If $G$ is an Oliver group, since $\mathrm{RO}(G, \mathfrak{D}) = \mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)}$, we obtain $\mathrm{rank}\mathrm{RO}(G, \mathfrak{D}) = a_G - 1$.

**Theorem** (E. Laitinen-K. Pawałowski, K. Pawałowski-R. Solomon, M. Morimoto). *Laitinen's Conjecture has been studied and is affirmative for Oliver gap groups $G$ satisfying one of the following conditions.*

(1) *$G$ is a perfect group* [9].

(2) *$G$ is a nonsolvable group:*

- *Case $G \not\cong P\Sigma L(2, 27)$:* [20].
- *Case $G = P\Sigma L(2, 27)$:* $\mathrm{RO}(G, \mathfrak{S}) = \mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)} \cong \mathbb{Z}$ [12].

(3) *$G$ has a normal subgroup $N$ such that $G/N \cong C_{pq}$ with distinct odd primes $p$, $q$* [20].

(4) *$G$ is of odd order* [20].

Let $SG(m, n)$ denote the $n$th small group of order $m$ given by the computer software GAP [5].

**Theorem** (A. Koto-M. Morimoto-Y. Qi, M. Morimoto, T. Sumi). *Laitinen's Conjecture fails and* $\mathrm{RO}(G, \mathfrak{S}) = 0$ *for Oliver groups $G$ satisfying one of the following conditions.*

(1) $G = \mathrm{Aut}(A_6)$ *(nongap group, $G/G^{nil} = C_2 \times C_2$)* [14].

(2) $G = SG(72, 44)$ (*gap group*, $G/G^{nil} = C_6$) [28].

(3) $G = SG(288, 1025)$ (*gap group*, $G/G^{nil} = C_6$) [28].

(4) $G = SG(432, 734)$ (*nongap group*, $G/G^{nil} = C_2$) [28].

(5) $G = SG(576, 8654)$ (*nongap group*, $G/G^{nil} = C_2 \times C_2$) [28].

(6) $G = SG(1176, 220)$ (*gap group*, $G/G^{nil} = C_3$) [7].

(7) $G = SG(1176, 221)$ (*gap group*, $G/G^{nil} = C_3$) [7].

## 6. Determination of $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S})$

Throughout this section, let $G$ be an Oliver group.

**Theorem** (K. Pawałowski-R. Solomon [20]). *Let $G$ be an Oliver group.*

(1) *If $G$ is a gap group, then* $\mathrm{RO}(G, \mathfrak{S})^{\mathcal{L}(G)}_{\mathcal{P}(G)} = \mathrm{RO}(G)^{\mathcal{L}(G)}_{\mathcal{P}(G)}$.

(2) *If $G$ is either an Oliver group of odd order or a nonsolvable group* $\not\cong \mathrm{Aut}(A_6)$, $P\Sigma L(2, 27)$ *and if $a_G \geq 2$, then* $\mathrm{RO}(G)^{\mathcal{L}(G)}_{\mathcal{P}(G)} \neq 0$.

Let us define the following subsets of $\mathrm{RO}(G)$.

$\mathrm{RO}[\mathcal{H}^{\mathcal{L}}](G) = \{V - W \in \mathrm{RO}(G) \mid V, W$ are $\mathcal{L}(G)$-free and satisfy (Dim)$\}$

$\mathrm{RO}[\mathcal{W}^{\mathcal{L}}](G) = \{V - W \in \mathrm{RO}(G) \mid V, W$ are $\mathcal{L}(G)$-free and satisfy (Dim), (Ori)$\}$

where (Dim) and (Ori) stand for the dimension condition and the orientation condition, respecively, appearing in the weak gap condition (see Section 4).

By definition,

$$2 \cdot \mathrm{RO}[\mathcal{H}^{\mathcal{L}}](G) \subset \mathrm{RO}[\mathcal{W}^{\mathcal{L}}](G) \subset \mathrm{RO}[\mathcal{H}^{\mathcal{L}}](G).$$

If $G$ is a gap group, then $\mathrm{RO}[\mathcal{W}^{\mathcal{L}}](G) = \mathrm{RO}(G)^{\mathcal{L}(G)}$.

By the Deleting-Inserting Theorem by M. Morimoto stated in [16, Appendix], we obtain the next basic theorem.

**Theorem 6.1.** *If $G$ is an Oliver group, then*

$$\mathrm{RO}[\mathcal{W}^{\mathcal{L}}](G)_{\mathcal{P}(G)} \subset \mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) \cap \mathrm{RO}(G, \mathfrak{D}_{\partial\text{-}lin}).$$

**Corollary 6.2.** *If $G$ is an Oliver group with* $\mathrm{RO}[\mathcal{H}^{\mathcal{L}}](G)_{\mathcal{P}(G)} \neq 0$, *then* $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) \neq 0$.

X.M. Ju applied the theorem above and obtained the next result.

**Theorem** (X.M. Ju). *Let $X_2 = C_2 \times \cdots \times C_2$ be the n-fold cartesian product of $C_2$, where $n \geq 1$. Then $G = S_5 \times X_2$ is a nongap Oliver group,*

$$\mathrm{RO}(G, \mathfrak{S}) = \mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = \mathrm{RO}(G)^{\{A_5\}}_{\mathcal{P}(G)}$$

*and*

$$\mathrm{rank}_{\mathbb{Z}}\mathrm{RO}(G)^{\{A_5\}}_{\mathcal{P}(G)} = 2^n - 1.$$

**Lemma 6.3** ([7]). *Let $G$ be a finite group not of prime power order, $N$ a normal subgroup of $G$, $N_2$ a Sylow 2-subgroup of $N$.*

(1) *If $G/N \cong C_2$ and $V \sim_{\mathfrak{S}} W$, then $V^N = 0 = W^N$ or $\mathrm{res}^G_N V \cong_N \mathrm{res}^G_N W$.*

(2) *If $G/N \cong C_p$ with $p$ odd prime, $N_2$ is normal in $N$, and $V \sim_{\mathfrak{S}} W$ then $V^N = 0 = W^N$ or $\mathrm{res}^G_N V \cong_N \mathrm{res}^G_N W$.*

**Lemma 6.4** ([7]). *Let $G$ be a finite group not of prime power order and $G_2$ a Sylow 2-subgroup of $G$.*

(1) *If $G/G^{\{2\}} \cong C_2 \times \cdots \times C_2$, then $\mathrm{RO}(G, \mathfrak{S}) \subset \mathrm{RO}(G)^{\{G^{\{2\}}\}}$.*

(2) *If $G_2$ is normal in $G$ and $G/G^{\{3\}} \cong C_3 \times \cdots \times C_3$, then $\mathrm{RO}(G, \mathfrak{S}) \subset \mathrm{RO}(G)^{\{G^{\{3\}}\}}$.*

**Theorem 6.5** ([12]). *Let $G$ be either $SG(864, 2666)$ or $SG(864, 4666)$. Then $G$ is an Oliver group with $G/G^{nil} \cong C_3$ and*

$$\mathrm{RO}(G, \mathfrak{S}) = \mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = \mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)} \cong \mathbb{Z}.$$

Let $G$ be a finite Oliver group of order $\leq 2000$. T. Sumi (2006) tried to see whether $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = 0$ or not. Putting his computation together with our results, we can determine whether $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = 0$ or not for $G$ except ones in the next list:

| $G(m, n)$ | $a_G$ | gap? | $G/G^{nil}$ |
|---|---|---|---|
| $G(864, 4663)$ | 3 | No | $C_8$ |
| $G(864, 4672)$ | 5 | Yes | $Q_8 \times C_3$ |
| $G(1152, 155470)$ | 2 | Yes | $C_6$ |
| $G(1152, 157859)$ | 2 | Yes | $C_6$ |

List 1

## 7. Conjectures

We have several conjectures related to the Smith Problem which are not yet proved.

**Conjecture** (S. E. Cappell-J. L. Shaneson). If $V \sim_{\mathfrak{S}_{CS}} W$ and the actions on $V$ and $W$ are pseudofree, then $V \simeq_G W$ ($G$-homeomorphic).

**Conjecture 7.1.** If $G$ is an Oliver group with $\mathrm{RO}(G)^{\mathcal{L}(G)}_{\mathcal{P}(G)} \neq 0$, then $\mathrm{RO}(G, \mathfrak{S})^{\mathcal{L}(G)}_{\mathcal{P}(G)} \neq 0$.

Let $c_G$ denote the number of the conjugacy classes $(C)$ of cyclic subgroup $C$ of $G$ such that the order of $C$ is not of prime power order. Let $\Gamma$ denote the Galois group $Gal(\mathbb{Q}(\zeta))$, where $\zeta = \exp\left(\dfrac{2\pi\sqrt{-1}}{|G|}\right)$

**Conjecture 7.2.** If $G$ is an Oliver group with $c_G \geq 2$, then $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S})^{\Gamma} \neq 0$

**Conjecture 7.3.** If $G$ is an Oliver group, then $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) \subset \mathrm{RO}(G, \mathfrak{D}_{\partial\text{-lin}})$.

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