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Global solvability of the Navier-Stokes equations in a rotating frame with spatially almost periodic data.

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1. INTRODUCTION

One of the most important unresolved questions concerning the Navier-Stokes equations is the global regularity and uniqueness of the solutions to the initial value problem. This question was posed in 1934 by Leray [30, 31] and is still left open for three dimensional flow. However if we pose some conditions on initial velocity, the smooth solution exists globally-in-time. More precisely, Kato [25], and Giga and Miyakawa [20] showed that if the initial velocity is small enough in L^n norm, then the unique smooth solution exists globally-in-time. This smallness condition is generalized by many authors (see [9, 21, 27, 35, 40]).

When an initial vector is close enough to a two-dimensional vector field, the unique smooth solution exists globally-in-time (see [12, 24]).

Babin, Mahalov and Nicolaenko [4] considered global solvability of the Navier-Stokes equations in a rotating frame with periodic initial data (see also [3, 5, 6, 7, 32]). They proved existence on infinite intervals of regular solutions to the 3D-Navier-Stokes equations with the Coriolis force. Chemin, Desjardins, Gallagher and Grenier [11] derived dispersion estimates on a linearized version of the 3D-Navier-Stokes equations with the Coriolis force. To construct such estimate, they handled eigenvalues and eigenfunctions of the Coriolis operator. Using the dispersive effect, they showed that there exists a global-in-time unique solution to the 3D-Navier-Stokes equations with a large Coriolis force with no smallness assumption on the initial data provided that the initial data decays at space infinity. Although these two results resemble each other, the mechanism is quite different. For periodic initial data there expects no dispersive effect for regularization of the flow, although the flow looks like two dimensional one for a large Coriolis force.

Problems concerning large-scale atmospheric and oceanic flows are known to be dominated by rotational effects. The Coriolis force appears in almost all of the models of oceanography and meteorology dealing with large-scale phenomena. For example, oceanic circulation featuring Tyhoon, Hurricane and Cyclone are caused by the large rotation. There is no doubt that other physical effects are of similar significance like salinity, natural boundary conditions and so on. However the first step in the study of more complex model is to understand the behavior of rotating fluids. This problem attracted many physicists and mathematicians. See [34] for references.

Let us mention almost periodic functions. Giga, Mahalov and Nicolaenko [19] proved existence of a local-in-time unique classical solution of the Navier-Stokes equations (with or without the Coriolis force) when the initial velocity is spatially almost periodic. They showed that the solutions is always spatially almost periodic any time provided that the solution exists. This fact follows from continuous dependence of the solution with respect to initial data in uniform topology. Giga, Inui, Mahalov and Matsui [15] established unique local existence for the Cauchy problem of the Navier-Stokes equations with the Coriolis force when initial data is in FM_0, Fourier preimage of the space of all finite Radon measures with no point mass at the origin. Some almost periodic functions are in FM_0. They also showed that the length of existence time-interval of mild solution is independent of the rotation speed. Giga, Jo, Mahalov and the author [18] considered properties of the solution to the Navier-Stokes equations with the Coriolis force in FM_0. They showed that when the initial data is almost periodic, the complex amplitude is analytic in time. In particular, a new mode cannot be created at any positive time.

In this paper we discuss existence on long time intervals of regular solutions to the 3D-Navier-Stokes equations in a rotating frame with spatially almost periodic data. (It is equivalent to 3D-Navier-Stokes equations for fully three dimensional initial data characterized by uniformly large vorticity. See [7, 23, 33] for example.) Since the initial data does not decay at space infinity, we are unable to use dispersion estimate by [11].

The Cauchy problem for the 3D-Navier-Stokes equations with the Coriolis force (NSC) are described as follows:

$$\begin{cases} \partial_t v^\Omega + (v^\Omega, \nabla) v^\Omega + \Omega e_3 \times v^\Omega - \Delta v^\Omega = -\nabla p^\Omega, \\ \nabla \cdot v^\Omega = 0, \ v^\Omega|_{t=0} = v_0, \end{cases} \tag{1.1}$$

where $v^\Omega = v^\Omega(x,t) = (v^{\Omega,1}(x,t), v^{\Omega,2}(x,t), v^{\Omega,3}(x,t))$ is the unknown velocity vector field and $p^\Omega = p^\Omega(x,t)$ is the unknown scalar pressure at the point $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ in space and time $t > 0$ while $v_0 = v_0(x)$ is the given initial velocity field. Here $\Omega \in \mathbb{R}$ is the Coriolis parameter, which is twice the angular velocity of the rotation around the vertical unit vector $e_3 = (0,0,1)$, the kinematic viscosity coefficient in normalized by one. By $\times$ we denote the exterior product, and hence, the Coriolis term is represented by $e_3 \times u = Ju$ with the corresponding skew-symmetric 3×3 matrix J.

We shall give the main ideas of the proof. First, we analyze the nonlinear term of NSC. We introduce operators

$$\begin{cases} F_{(0,0,0)} : \text{ operator for pure two dimensional interactions}, \\ F_{(1,0,1)} : \text{ skew-symmetric-catalytic operator}, \\ F_{(1,1,0)} : \text{ non-skew-symmetric-catalytic operator}, \\ F_{(1,1,1)} : \text{ operator for strict three dimensional interactions}, \\ F_c^{\Omega,t} : \text{ non-resonant operator}, \end{cases}$$

and write NSC in the form

$$\partial_t u = \Delta u + \sum_{\mu \in D} F_\mu(u,u) + F_c^{\Omega,t}(u,u),$$

where $D = \{((0,0,0),(1,0,1),(1,1,0),(1,1,1)\}$. If the term $F_c^{\Omega,t}$ is vanishing, then the equations are similar to the 2D-Navier-Stokes equations. We call such a system an extended 2D-Navier-Stokes equation (E2DNS). In fact, the solution to E2DNS is independent of the Coriolis force. The key is to prove global existence of a unique smooth solution to E2DNS. Babin, Mahalov and Nicolaenko [4] used energy inequality of E2DNS to show global unique existence of a solution. However, a straightforward application of energy inequality is impossible if the initial data is almost periodic function. What is worse, there is no good Hilbert space for almost periodic functions, so we cannot use eigenvalues and eigenfunctions of the Coriolis operator as Chemin, Desjardins, Gallagher and Grenier [11] did. To overcome these difficulties, we use FM_0 spaces (Fourier preimage of the space of all finite Radon measures with no point mass at the origin) proposed by Giga, Inui, Mahalov and Matsui (see [15]). We instead employ mild solutions of E2DNS in FM_0 so that this equation turns into a linear one if we choose an appropriate frequency set. Once the equation becomes linear, it is easy to show that the solution to E2DNS exists globally-in-time.

Babin, Mahalov and Nicolaenko [4] handled periodic L^2 Sobolev spaces, and Chemin, Desjardins, Gallagher and Grenier [11] handled L^2 Sobolev spaces in $\mathbb{R}^3$. Thus our result is not included in such results since we use almost periodic functions. Moreover we introduce useful decomposition to clarify the analysis of the nonlinear term of NS, which have never been used before.

2. Function spaces, Riesz transforms, the Helmholtz projection and local solution

In this section we shall give definition of function spaces suitable for almost periodic functions included in $BUC(\mathbb{R}^3)$ (Bounded uniformly continuous functions) . Note that almost periodic functions in the sense of Bohr belonging to BUC are already studied. See [8, 10] for example. To define such function spaces, we need the definition of frequency sets Λ and $\Lambda(\gamma)$. These sets are different from one defined in [16, Definition 1.1.]

Definition. (Countable sum closed frequency set in $\mathbb{R}^3$.) We say that $\Lambda \subset \mathbb{R}^3$ is countable sum closed frequency set in $\mathbb{R}^3$ if Λ is countable set in $\mathbb{R}^3$ and it satisfies the following equality:

$$\Lambda = \{a + b : a, b \in \Lambda\}.$$

Remark. $\mathbb{Z}^3$, $\{e_1m_1+\sqrt{2}e_2m_2+e_2m_3+e_3m_4 : m_1, \cdots, m_4 \in \mathbb{Z}\}$ and $\{e_1m_1+(e_1+e_2\sqrt{2})m_2 + (e_2 + e_3\sqrt{3})m_3 : m_1, m_2, m_3 \in \mathbb{Z}\}$ are countable sum closed frequency sets, where $\{e_j\}_{j=1}^3$ is a standard orthogonal base in $\mathbb{R}^3$.

Definition. (Countable sum closed frequency set in $\mathbb{R}^3$ (depending on γ).) Let

$$\Lambda(\gamma) := \{(n_1, n_2, n_3) \in \mathbb{R}^3 : (n_1, n_2, n_3/\gamma) \in \Lambda\}$$

for $\gamma \in \mathbb{R} \setminus \{0\}$.

Remark. Let $\gamma \in \mathbb{R} \setminus \{0\}$. $\Lambda(\gamma)$ is a countable sum closed frequency set in $\mathbb{R}^3$ if and only if Λ is also countable sum closed frequency set in $\mathbb{R}^3$.

First, we define scalar valued function spaces $X^{s,\Lambda(\gamma)}$, $X_0^{s,\Lambda(\gamma)}$ and $\dot{X}^{s,\Lambda(\gamma)}$.

Definition. (3D-scalar valued function spaces.) For $s \geq 0$, let

$$X^{s,\Lambda(\gamma)} := \{g \in BUC(\mathbb{R}^3) : g(x) = \sum_{n\in\Lambda(\gamma)} a_n e^{in\cdot x}, \|g\|_s < \infty\},$$

where

$$\|g\|_s := \sum_{n\in\Lambda(\gamma)} (1 + |n|^2)^{s/2}|a_n|.$$

The infinite sum is understood in the sense of absolute uniform convergence. Let us define $X_0^{s,\Lambda(\gamma)}$ as follows:

$$X_0^{s,\Lambda(\gamma)} := \{g \in X^{s,\Lambda(\gamma)} : a_0 = 0\}.$$

Remark. $X_0^{s,\Lambda(\gamma)}$ is a closed subspace of $X^{s,\Lambda(\gamma)}$ with the norm $\|\cdot\|_s$.

Second, we define three-dimensional vector valued function spaces $\mathcal{X}^{s,\Lambda(\gamma)}$ and $\mathcal{X}_\sigma^{s,\Lambda(\gamma)}$.

Definition. (3D-vector valued function spaces.) Let

$$\mathcal{X}^{s,\Lambda(\gamma)} := \{v = (v^1, v^2, v^3) \in (X^{s,\Lambda(\gamma)}(\mathbb{R}^3))^3 :$$
$$\|v\|_s := \|v^1\|_s + \|v^2\|_s + \|v^3\|_s < \infty\}.$$

Let us define three-dimensional vector valued divergence free function spaces as follows:

$$\mathcal{X}_\sigma^{s,\Lambda(\gamma)} := \{v = (v^1, v^2, v^3) \in \mathcal{X}^{s,\Lambda(\gamma)} :$$
$$n^1 a_n^1 + n^2 a_n^2 + n^3 a_n^3 = 0 \quad \text{for} \quad n = (n^1, n^2, n^3) \in \Lambda(\gamma)\}.$$

We define $\mathcal{X}_0^{s,\Lambda(\gamma)}$, $\mathcal{X}_{0,\sigma}^{s,\Lambda(\gamma)}$ and $\dot{\mathcal{X}}^{s,\Lambda(\gamma)}$ in the same way since the definitions are similar to $X_0^{s,\Lambda(\gamma)}$, $X_{0,\sigma}^{s,\Lambda(\gamma)}$ and $\dot{X}^{s,\Lambda(\gamma)}$. Clearly, $\mathcal{X}^{s,\Lambda(\gamma)} = \mathcal{X}_0^{s,\Lambda(\gamma)} \oplus \mathbb{C}^3$ (topological direct sum).

Remark. $X^{s,\Lambda(\gamma)}$, $X_0^{s,\Lambda(\gamma)}$ $\mathcal{X}^{s,\Lambda(\gamma)}$ $\mathcal{X}_\sigma^{s,\Lambda(\gamma)}$ $\mathcal{X}_0^{s,\Lambda(\gamma)}$, $\mathcal{X}_{0,\sigma}^{s,\Lambda(\gamma)}$ are Banach spaces.

Let us consider the function space $X_0^{0,\Lambda(\gamma)}$ more precisely. It is easy to see that this function space is a closed subspace of FM_0 (the Fourier preimage of the space of all finite Radon measures with no point mass at the origin) which is introduced in [15]. The space FM_0 is strictly smaller than $\dot{B}^0_{\infty,1}$ as is proved in [15, Appendix A]. Thus the space $X_0^{0,\Lambda(\gamma)}$ is strictly smaller than BUC.

Third, we define two-dimensional vector valued function spaces. To treat the two-dimensional Navier-Stokes equations, it is convenient to set the following operators $\mathcal{Q}_0$, $\mathcal{Q}_1$ $\mathcal{Q}_0^h$, $\mathcal{Q}_0^3$ and function spaces $\mathcal{Q}_0^h\mathcal{X}_0^{s,\Lambda}$, $\mathcal{Q}_0^h\mathcal{X}_{0,\sigma}^{s,\Lambda}$.

Definition. (Splitting vertically oscillating and non-oscillating parts.)

For $u = (u^1, u^2, u^3) \in \mathcal{X}^{s,\Lambda(\gamma)}$,

$$u^j(x) = \sum_{n\in\Lambda(\gamma)} c_n^j e^{in\cdot x} \quad (j = 1, 2, 3),$$

let $\mathcal{Q}_\ell u := (\mathcal{Q}_\ell u^1, \mathcal{Q}_\ell u^2, \mathcal{Q}_\ell u^3)$ $(\ell = 0, 1)$ with

$$\mathcal{Q}_0 u^j(x_1, x_2) := \sum_{n\in\Lambda(\gamma), n_3=0} c_n^j e^{in\cdot x}, \ \mathcal{Q}_1 u^j(x) := \sum_{n\in\Lambda(\gamma), n_3\neq 0} c_n^j e^{in\cdot x},$$

for $j = 1, 2, 3$.

Remark. A direct calculation yields

$$\mathcal{Q}_0 u^j(x_1, x_2) = \lim_{r\to\infty} \frac{1}{2r} \int_{-r}^{r} u^j(x_1, x_2, x_3) dx_3$$

and

$$\mathcal{Q}_1 u^j(x) = u^j(x) - \mathcal{Q}_0 u^j(x_1, x_2).$$

See [10] for example.

Definition. (Splitting 2D-two vector and 2D-one vector parts.)

Let $\mathcal{Q}_0^h w := (\mathcal{Q}_0 w^1, \mathcal{Q}_0 w^2, 0)$ and $\mathcal{Q}_0^3 w := (\mathcal{Q}_0 w^3, 0, 0)$.

Remark. It is easy to see that $u = (\mathcal{Q}_0 + \mathcal{Q}_1)u = (\mathcal{Q}_0^h + \mathcal{Q}_0^3 + \mathcal{Q}_1)u$ and that $\|w\|_s = \|\mathcal{Q}_0 w\|_s + \|\mathcal{Q}_1 w\|_s = \|\mathcal{Q}_0^h w\|_s + \|\mathcal{Q}_0^3 w\|_s + \|\mathcal{Q}_1 w\|_s$.

Now we define two-dimensional vector valued function spaces $\mathcal{Q}_0^h \mathcal{X}_0^{s,\Lambda}$, $\mathcal{Q}_0^h \mathcal{X}_{0,\sigma}^{s,\Lambda}$ as follows.

Definition. (2D-vector valued function spaces.) For $s \geq 0$, let

$$\mathcal{Q}_0^h \mathcal{X}_0^{s,\Lambda} := \{v(x) = (v^1, v^2) \in (BUC(\mathbb{R}^2))^2 : \\ v^j = \sum_{n \in \Lambda \setminus \{0\}, n_3 = 0} a_n^j e^{in\cdot x}, \quad \text{for} \quad j = 1, 2, \ \|v\|_s := \|v^1\|_s + \|v^2\|_s < \infty\},$$

where

$$\|v^j\|_s := \sum_{n \in \Lambda \setminus \{0\}, n_3 = 0} (1 + |n|^2)^{s/2} |a_n^j|.$$

Let

$$\mathcal{Q}_0^h \mathcal{X}_{0,\sigma}^{s,\Lambda} := \{v(x) = (v^1, v^2) \in \mathcal{Q}_0^h \mathcal{X}_0^{s,\Lambda} : \\ n_1 a_n^1 + n_2 a_n^2 = 0 \quad \text{for} \quad n \in \Lambda \quad \text{with} \quad n_3 = 0\}.$$

Theorem. (Local solution.) Assume that $v_0 = \sum_{n \in \Lambda(\gamma) \setminus \{0\}} a_n e^{in\cdot x} \in \mathcal{X}_{0,\sigma}^{0,\Lambda(\gamma)}$. Then there is a local-in-time unique mild solution v^Ω satisfying

$$v^\Omega \in C([0, T_{v_0}], \mathcal{X}_{0,\sigma}^{0,\Lambda(\gamma)}), \quad T_{v_0} \geq \frac{C}{\|v_0\|_0}, \quad \sup_{0<t<T_{v_0}} \|v^\Omega\|_0 \leq 10\|v_0\|_0,$$

where C is a positive constant independent of Ω.

Moreover $v^\Omega = v^\Omega(x,t)$ is expressed as

$$v^\Omega(x,t) = \sum_{n \in \Lambda(\gamma) \setminus \{0\}} a_n^\Omega(t) e^{in\cdot x}, \quad a_n^\Omega(t) \to a_n \quad (t \to 0).$$

3. Global solvability of the 3D Navier-Stokes equations in a rotating frame with spatially almost periodic data

In this section we state a result of global solvability of the 3D-Navier-Stokes equations in a rotating frame with spatially almost periodic data. This is our main result.

Theorem. Let Λ be a countable sum closed frequency set. There exists a set $\Gamma \subset \mathbb{R} \setminus \{0\}$ (depending on Λ) whose complement set is countable.

Let us impose the following two assumptions.

(1) Take $\gamma \in \Gamma$.

(2) Take $v_0 \in \mathcal{X}_{0,\sigma}^{0,\Lambda(\gamma)}$ such that the initial value problem for the 2D Navier-Stokes equations admits a global-in-time unique solution in $C([0,\infty)\mathcal{Q}_0^h\mathcal{X}_{0,\sigma}^{0,\Lambda(\gamma)})$ with a initial data $\mathcal{Q}_0^h v_0 \in \mathcal{Q}_0^h\mathcal{X}_{0,\sigma}^{0,\Lambda(\gamma)}$.

Then for any $T > 0$ there exists Ω_0 depending only on v_0 and T such that if $|\Omega| > \Omega_0$, then there exists a mild solution $v^\Omega \in C([0,T] : \mathcal{X}_{0,\sigma}^{0,\Lambda(\gamma)})$ of equation (1.1) with an initial data $v_0 \in \mathcal{X}_{0,\sigma}^{0,\Lambda(\gamma)}$.

Remark If $\mathcal{Q}_0^h v_0$ is a periodic function, there exists a global-in-time unique solution to the 2D Navier-Stokes equations in $C([0,\infty)\mathcal{Q}_0^h\mathcal{X}_{0,\sigma}^{0,\Lambda(\gamma)})$.

Remark If $\|(-\Delta)^{-\frac{1}{2}}\mathcal{Q}_0^h v_0\|_0$ is small enough, there exists a global-in-time unique solution to the 2D Navier-Stokes equations in $C([0,\infty)\mathcal{Q}_0^h\mathcal{X}_{0,\sigma}^{0,\Lambda(\gamma)})$. See [16, 17].

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