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BOREL DEFINABLE SUBGROUPS AND ALMOST INTERNALITY IN ROSY DEPENDENT GROUPS

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ABSTRACT. Let $G$ be a rosy dependent group with a countable language, weak canonical bases and elimination of hyperimaginaries. Let $p$ be a global $p$-generic type and $\Sigma$ be an $\emptyset$-invariant family of partial types. Under a certain assumption on independence relations of global types we show that if $p\frown \emptyset$ is not foreign to $\Sigma$, then there exists a Borel definable subgroup $H$ such that $G/H$ is almost $\Sigma$-internal.

1. Preliminaries

We review some definitions and facts. We only consider theories having elimination of hyperimaginaries and we will work in the eq-structures.

ROSY THEORIES (See [A]): Let $T$ be rosy and $\mathcal{M}$ be a big model. We work in $\mathcal{M}^{eq}$. We say that $T$ has weak canonical bases if for any type $p$ there exists the smallest algebraically closed subset $\text{wcb}_p(p)$ such that $p$ does not $p$-forks over $\text{wcb}(p)$. In [A] Adler showed that if $T$ has weak canonical bases then $\text{wcb}_p(\bar{a}/B) = \text{aker}((a_i)_{i<\omega}) := \{ d \in \text{acl}^{eq}((a_i)_{i<\omega}) : (a_i)_{i<\omega} \text{ is } d\text{-indiscernible} \}$, where $B = \text{acl}^{eq}(B)$ and $(a_i)_{i<\omega}$ is a Morley sequence of $\text{tp}(\bar{a}/B)$.

DEPENDENT THEORIES (See [HP]): Let $T$ be dependent. In [HP], it is shown that a global type $p \in S(\mathcal{M})$ does not fork over an algebraically closed set $A \subseteq \mathcal{M}$ if and only if $p$ is $\text{acl}^{eq}(A)$-invariant if and only if $p$ is strongly Borel definable over $\text{acl}^{eq}(A)$. (We say that $p$ is strongly Borel definable over $A$ if for any formula $\varphi(x, y)$, there exists a finite Boolean combination of partial types over $A$, say $D_{p,\varphi}(y)$, such that $\varphi(x, b) \in p$ if and only if $p \models D_{p,\varphi}(b)$ for any $b$. It is also mentioned that a global type $p$ is $A(\subseteq \mathcal{M})$-invariant, then any Morley sequence of $p$ over $A$ has the same type over $A$.

ROSY GROUPS (See [EKP]): Let $(G(x), \cdot)$ be a rosy group over $\emptyset$ and let $\downarrow^p$ be thorn-non-forking relation. There exists $p$-generic type $p \in S(A)$ for $G$ over $A$: $p(x) \vdash G(x)$ and if $a, b \in G$ with $a \models p$ and $a \downarrow^p A$, then $b \cdot a \downarrow^p A, b$ holds. And then $\text{tp}(b \cdot a/A)$ is also $p$-generic.

FOREIGNNESS and ALMOST INTERNALITY in ROSY THEORIES: Let

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Theorem 2.1. Let $G$ be a rosy group and let $H$ be a Borel definable subgroup over $B$. We say that $G/H$ is almost $\Sigma$-internal if for each $gH \in G/H$ there exist $C$ and $D \models \Sigma|B$ such that $g \downarrow B^p C$ and $gH$ is acl$^{eq}(B, C, D)$-invariant.

Here, $gH$ is an ultrimaginary element, we do not consider thorn-independence relation on ultraimaginaries, so we do not define $gH \downarrow B^p C$.

Proposition 2.2. Let $G$ be a rosy dependent group having weak canonical bases and a countable langage. Suppose that $G$ is sufficiently saturated and any global type (i.e. over $G$) does not $p$-fork over $A$ if and only if it does not fork over $A$. Let $p$ be a global $p$-generic type of $G$. If $p|\emptyset$ is not foreign to $\emptyset$-invariant family of partial types $\Sigma$, then there exists a Borel definable subgroup $H$ such that $|G/H| \geq \omega$ and $G/H$ is almost $\Sigma$-internal.

Proof. Note that $p$ is acl$^{eq}(\emptyset)$-invariance: By the definition of p-generics, $p$ does not $p$-fork over $\emptyset$. By our assumption $p$ does not fork over $\emptyset$, so we see acl$^{eq}(\emptyset)$-invariance of $p$ by dependence of $G$.

As $p|\emptyset$ is not foreign to $\Sigma$, there exist $A \subset G, a \models p|A$ and $b \models \Sigma$ such that $a \downarrow_A p b$. We may assume $a, \bar{b} \downarrow_A G$. By our assumption we have $a, \bar{b} \downarrow_A G$, where $\downarrow_A f$ denotes non-forking relation. Let $B := wc_{p}(a, \bar{b}/G)(= wc_{f}(a, \bar{b}/G))$.

Claim. Put $H = \{g \in G : g \cdot a, \bar{b} \equiv B (a, \bar{b})\}$ is a Borel definable (over $B$) subgroup of $G$.

$H \leq G$. Let $g \in K$. As $g^{-1} \in G$, it holds that $a, g^{-1} \cdot g \cdot a, \bar{b} \equiv G \cdot g^{-1} \cdot g \cdot a, \bar{b}$, we have $g^{-1} \in H$. Let $g, g' \in H$. As $g' \in G$, we have $g' \cdot g \cdot a, \bar{b} \equiv G \cdot g' \cdot a, \bar{b} \equiv G \cdot a, \bar{b}, g' \cdot g \in H$ follows.

The Borel definability of $H$ over $B$: Let $\varphi(u \cdot x, \bar{y}, \bar{z}) \in L$ and put $q = tp(a, \bar{b}/G)$. There exists a $D_{q, \varphi}(u, \bar{z})$ which defines a strongly Borel definable set over $B$ such that $\varphi(d \cdot x, \bar{y}, \bar{c}) \in q$ if and if only $D_{q, \varphi}(d, \bar{c})$ for any $d \in G, \bar{c} \subset G$. As $H = \bigcap_{\varphi \in L} \{g \in G : D_{q, \varphi}(d, \bar{c}) \rightarrow D_{q, \varphi}(d \cdot g^{-1}, \bar{c})\}$ for any $d \in G, \bar{c} \subset G$, the Borel definability of $H$ over $B$ follows.

Claim. $|G/H| \geq \omega$

As $p$ is acl$^{eq}(\emptyset)$-invariant, take $(g_i)_{i<\omega} \subset G$ be the Morley sequence of $p|B$. As $a, \bar{b} \downarrow_B (g_i)_{i<\omega}, g := g_j^{-1} \cdot g_i$ is p-generic over $B, a, \bar{b}$. So we have $g \cdot a \downarrow_B \bar{b}$.
As a $\bigwedge^p_B \bar{b}$, we see that $g \not\in H$ as desired.

Fix $g \in G$ and take a Morley sequence $(a_i)_{i<\omega} \subseteq G$ of $p|B, g$. Note that $g \bigwedge^p_B (a_i)_{i<\omega}$.

Again take a Morley sequence $(g \cdot a'_i, b_i)_{i<\omega}$ of $tp(a, \bar{b}/G)|B, g, (a_i)_{i<\omega}$. (tp$(a, \bar{b}/G)$ is $B$-invariant.) As $(a_i)_{i<\omega}$ and $(g \cdot a'_i)_{i<\omega}$ are Morley sequences of $p|B, g$, we have $(a_i)_{i<\omega} \equiv_B g (g \cdot a'_i)_{i<\omega}$. Let $(b'_i)_{i<\omega} \subseteq G$ be such that $(a_i, b'_i)_{i<\omega}$ is a Morley sequence of $tp(a, \bar{b}/G)|B, h$. So $wcb_p(a, \bar{b}/G) = wcb_p(a, \bar{b}/G) \subseteq acl^eq((a_i, b'_i)_{i<\omega})$. Therefore $tp(a, \bar{b}/G)$ is acl$^eq((a_i, b'_i)_{i<\omega})$-invariant and $b'_i \models \Sigma|B$.

**Claim.** Let $\sigma \in Aut(G/acl^eq((a_i, b'_i)_{i<\omega}, B))$. Then $\sigma(gH) = gH$.

As $tp(a, \bar{b}/G)$ is acl$^eq((a_i, b'_i)_{i<\omega})$-invariant, we have $g \cdot a, \bar{b} \equiv_G \sigma(g) \cdot a', \bar{b}'$. As $\sigma(g) \in G$ and $tp(a, \bar{b}/G)$ is $B$-invariant, we see $g \cdot a, \bar{b} \equiv_G \sigma(g) \cdot a', \bar{b}' \equiv_G \sigma(g) \cdot a, \bar{b}$, as desired.

We get $g \bigwedge^p_B (a_i)_{i<\omega}, b'_i \models \Sigma|B$ and $gH$ is acl$^eq((a_i, b'_i)_{i<\omega}, B)$-invariant.

**Question 2.3.**

1. Let $\mathcal{M}$ be a sufficiently saturated rosy model and any global type (i.e. over $\mathcal{M}$) does not $p$-fork over $A$ if and only if it does not fork over $A$. Then is $\mathcal{M}$ simple?

2. Can we find a Borel definable NORMAL subgroup $H$ as in the Proposition?

3. Can we find a $\Sigma$-connected component $G^\Sigma$ in a Borel definable way? ($G^\Sigma$ want to be foreign to $\Sigma$, connected and invariant under any definable automorphism as in [W].)

4. Does any superrosy field has monomial $U^p$-rank? If so, Nubling’s proof [N] that any supersimple field is n-ample for any $n < \omega$ works for any superrosy field.

**References**


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