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Axiomatic Interpretation in the History of Economics

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Abstract

A theory must be interpreted as a consistent axiom system, and therefore an interpretation of a theory is a consistent axiomatic extension of the theory. A historical interpretation of a past theory is possible if and only if there is a stream of theories developed from the theory or a Scientific Research Program to which it belongs. We take three examples. First, an interpretation of the theory of utility and scarcity shows that an axiomatic interpretation makes historian's vision clearer. The second example about the marginal productivity theory shows that a mathematical interpretation is useful to solve a conflict of opinions in a past controversy. The third example shows that there can be plural interpretation of a past consistent theory. We must review each alternative interpretation and minimize parts of interpretation alien to the original text.

Keywords: Axiomatic Interpretation, Marginal Utility Theory, Marginal Productivity Theory, Natural value

JEL: B00, B2

1 Interpretation in the History of Economics

There are at least three reasons why interpretation of theories from the past is indispensable for research into the history of economics. First, in earlier stages in the progression of economic theory, most theories were inadequate from several points of view. Early economics first developed merely as economic thought, and until the 1940s, most economic theories were literary. However, since the marginal revolution in the 1870s, economic theories often came to be described in terms of mathematics, but most were inadequate as an axiom system. . Second, even if every theory is described as a consistent axiom system, some past theories are possibly ambiguous, at least from the viewpoint of the most advanced theories. Third, when the intellectual background of economics changes, it becomes necessary to rewrite or reconstruct past economic theories. Since the formalist revolution of the 1950s (Blaug 2003), the way of expressing economic theory has changed under the influence of Bourbakism (Weintraub, 2002). A typical example is Debreu's *Theory of Value* published in 1959. Modern economic theories come to be expressed as consistent axiom systems or as mathematical models. For present-day economists to understand a past economic theory, it is necessary to express that theory as a consistent axiom system or a mathematical model in which the main theorems are proven.

Interpretation is another logical way of understanding economic ideas and for developing economic theories. Any interpretation is in essence the theoretical contribution of the interpreter. For example, several interpretations of Keynes's *General Theory of Employment, Interest and Money* put forward Scientific Research Programs (SRPs) of macroeconomics like Keynesian, Orthodox Keynesian, New Keynesian, Post-Keynesian economics etc.(Snowdon and Vane, 2005). These are not historical interpretations, because they make certain theoretical contributions that are quite apart from the original theory. A historical interpretation must be that showing not only what is achieved by a past economic theory but also what is not achieved by it.

This paper has 6 sections. In section 2, we confirm that a historical interpretation is one in a theory developed from the original one in the same SRP, as well as being a consistent extension of the original theory. We show three examples of interpretation of the history of the general equilibrium theory. The first example, in Section 3, is an interpretation of the theory of UTILITY and scarcity in the marginal utility theory. This example shows that mathematical models support the intuitive understandings of historians of economics. The second one, in Section 4, applies an old debate, a quarrel, about the marginal productivity theory. A mathematical model is useful to organize the debate about the Exhaustion Theorem. The third one, in Section 5, shows that there can be two consistent interpretations of Wieser's theory of natural value. In Section 6, we summarize our conclusions.

2 Historical Interpretation of a Past Theory

Suppose a past theory T is developed into a certain theory of the present. If the theory T is significant in the history of economics, then it can be thought to have been developed to a certain theory of the present. If the theory T is insufficient, that is to say, it is not axiomatizable, includes incompatible propositions, or is impossible to be derived from its axioms, then we cannot understand it as it is posited. To understand a past insufficient theory T, we have to interpret it as a consistent theory. Therefore, any interpretation of an insufficient theory must be different from the original theory. From the logical definition of interpretation (Shoenfield 1967: 61-64), we can consider an interpretation to be an extension of the original theory. If there is an interpretation of T in T' and T' is consistent, then T is consistent (Shoenfield 1967: 64-65).

When we interpret a historical theory in the field of economics, we need to pay attention to the idea of the original theory. This idea must be shared by the theories in the SRP developed from the theory. Therefore, we must interpret a past insufficient theory in a particular present theory developed from a theory in the same SRP.

Suppose that archaeologists discovered a mountain of bones in a stratum of the Jurassic period. In the stratum, a variety of bones are mixed together, constituting distinct sets of a dinosaur's skeleton. The archaeologists classify them into a particular set of bones that forms the skeleton of the dinosaur. They must construct a skeleton from the set of bones. However, until the skeleton formed by the set of bones is completely restored, they cannot know which set of bones a particular bone is classified into. The skeleton of an existing animal can be applied to understand the skeleton structure of the dinosaur. When archaeologists try to identify a bone from a skeleton of some dinosaur, they can utilize as a guide another skeleton structure of an existing animal that is close to the dinosaur. However, any restoration needs to be done by the analogy to a species evolved from the dinosaur, because it should be the closest form to the dinosaur.

Similarly, historical interpretation of a theory must be an interpretation of it within a theory developed from the original one. Moreover, it is necessary to verify which part of skeleton/theory is lacking in comparison with the complete one.

The purpose of historical interpretation is to understand a past theory, to confirm what is or is not shown by the theory, and to place the theory in its place in the history of economics. Any interpretation, including a theoretical contribution of an interpreter, is not really a historical but a theoretical one. There can be several interpretations of a past theory in a SRP. Suppose there is a past theory T, and a consistent extension of the past theory, T', in the SRP. If there is a consistent extension of the theory T', T'', in the same SRP, T''

is also an interpretation of T'. However, T' is a better interpretation of the original theory T than T'', because the former is closer to T and smaller than the latter. We must continue looking for an interpretation that is closer to the original theory. If an interpretation is shown to be a minimum and consistent extension of the original theory, it could be the best one. Of course, if the original theory is a consistent axiom system, then it stands as the best interpretation.

3 Theory of UTILITY and Scarcity as Marginal Utility Theory

Many economics historians have suggested that the theory of UTILITY and scarcity is very close to the marginal utility theory. For example, Schumpeter (1954: 297) states that "Even after 1776, that theory prevailed on the Continent, and there is an unbroken line of development between, Galiani and J. B. Say. Quesnay, Beccaria, Turgot, Verri, Condillac, and many other authors contributed to firmly establishing the theory. They all linked price and the pricing mechanism *directly* to what they conceived to be the fundamental purpose of economic activity, the satisfaction of wants. "

Most economics historians who research the theory of UTILITY and scarcity, agree that the theories of Galiani and others are close to the marginal utility theory, but that they lack the concept of marginal utility. [1] Schumpeter (1954: 297) states that "What separates Galiani from Jevons and Menger is, first, that he lacked the concept of marginal utility – though the concept of relative scarcity comes pretty near it – and, second, that he failed to apply his analysis to the problem of cost and of distribution. "

[1] See the references cited in Ekelund and Hebert (2002) and Kawamata (1995, 2009).

We will examine the theories of UTILITY and scarcity and show that Schumpeter's evaluation is correct, but not an exact exposition. We state that the theory of UTILITY and scarcity is certainly a marginal utility theory, but not defined as a result of the rational behavior of an individual economic agent.

3.1 Suppositions on the Concept of UTILITY

Things that were considered common knowledge when Galiani, Turgot, and Condillac were developing the theory of UTILITY and scarcity, are not explicitly described in their original texts. But from a modern point of view, they have to be explicitly formulated to understand the past theory. Theories of UTILITY and scarcity before the marginal revolution supposed that the utility function and the marginal utility function were measurable, separable, and additive, as Katzner(1970: 5–13) suggests. [2]

We cannot show that past economists made those assumptions, based on the original texts, because they are not described in the original texts. For us to understand past theories, however, it is necessary to complement some concepts and axioms thought to be lacking and reconstruct a complete theory that is an extension of that past theory.

3.2 UTILITY as Marginal Utility

We put together propositions for the theory of UTILITY and scarcity into an axiom system. Given resources of commodities, the value of a commodity is a subjective evaluation of that commodity by an individual. Thus, Galiani defines the value of a commodity as follows:

> Value, then, is a ratio; a compound of two ratios expressed as names UTILITY

[2] We denote the concept of UTILITY in the theory of UTILITY and scarcity by the SMALL CAPITAL description of UTILITY, because UTILITY is supposed not to be total utility, but to be marginal utility.

> and SCARCITY. I will explain what I mean by giving examples, so that there may be no dispute over the words. It is evident that air and water, though elements very useful for human life, have no value because they lack scarcity; whereas, a bag of sand from the shores of Japan would be a rare thing. However, assuming that it had no special utility, it would have no value. (Galiani 1927[1751]: 283) [3]

In the theory of UTILITY and scarcity, the value of a commodity is explained in terms of UTILITY and scarcity. Galiani states the following:

> A natural calf is nobler than a golden calf, but how much less its value is. I answer, that if a natural calf were as rare as one of gold, its price would be as much more than that of the golden calf as the utility and need of the one exceeds that of the other. These people imagine that value depends on a single factor, rather than on many which unite to form a compound ratio. Others, I believe, say that a pound of bread is more useful than a pound of gold. I answer that this is a shameful fallacy, due to not knowing that "more useful"and "less useful"are relative terms, which are measured according to the varying condition of individuals. If we are speaking of one who lacks bread and gold, the bread is certainly more useful; but the facts correspond to this and are not contrary, for no one will be found who would leave the bread and die of hunger, taking gold instead. Those who dig in the mines never forget to eat and sleep. But to a man who is satiated, is anything more useless than bread? Hence it is well if he satisfies other passions then. (Galiani 1927[1751]:

[3]While the items selected by Galiani might not be appropriate ones in today's World, the principal he seeks to highlight remains the same.

287–288)

In this passage, Galiani supposes the UTILITY of a commodity h to be a UTILITY $u_h(\cdot)$ that an individual obtains by consuming a unit quantity c_h of the commodity in a situation where the individual has already consumed a quantity x_h of the commodity. That is, for every commodity h, the UTILITY means an additional utility of an unit c_h, when the quantity x_h has already been consumed. Denote total utility when the quantity x_h is consumed as $U_h(x_h)$ for every commodity h, then UTILITY of c_h is defined by the formula $U_h(x_h + c_h) - U_h(x_h)$. As we have seen in the passage, c_h is typically a unit of quantity, for example, a pound. For the level of UTILITY to be independent of a scale of unit quantity, the UTILITY has to be expressed by the following ratio.

$$u_h(x_h) = \frac{U_h(x_h + c_h) - U_h(x_h)}{c_h}.$$

Taking a limit, namely, $c_h \to 0$, then $u_h(x_h)$ comes to equal $\frac{\partial U_h}{\partial x_h}(x_h)$.

Then, Galiani states that the UTILITY $u_h(\cdot)$ decreases as the consumed quantity x_h increases. This means $u_h(x_h)$ is a decreasing function of x_h. In the theory of UTILITY and scarcity by Galiani(1927[1751]), therefore, $u_h(x_h)$ cannot be a total utility, because it is not an increasing function of x_h. It supports our interpretation of $u_h(x_h)$.

3.3 The Theory of UTILITY and Scarcity as an Axiom System

The group of propositions in the theory of UTILITY and scarcity is almost equivalent to the Kuhn-Tucker conditions for the rational behavior of an individual economic agent, such that:

(M) An economic agent chooses a consumption so as to maximize his/her utility under the constraint of resources.

Provided that the results of our consideration are valid, we proceed to interpret each proposition of Galiani. Since "value is a ratio compounded from two ratios expressed as utility and scarcity ", we can consider the value of commodity h , v_h , to be $u_h\left(x_h^*\right)$ where $x_h^* = \omega_h$. If a commodity is scarce, then $v_h = u_h\left(x_h^*\right) > 0$ where $x_h^* = \omega_h > 0$. "Air and water, which are elements very useful for human life, have no value, because they lack scarcity." This means that if a commodity is not scarce, $x_h^* < \omega_h$, then $v_h = 0$. These conditions can be coordinated with the following set of conditions, that is,

(Marginal Principle)		$x_h^* > 0,$	and	$\frac{\partial U_h}{\partial x_h}(x_h^*) = v_h^*$	for every h .
(Scarce Goods)	If	$\omega_h = x_h^*,$	then	$v_h^* > 0$	for every h .
(Free Goods)	If	$\omega_h > x_h^*,$	then	$v_h^* = 0$	for every h .

Comparing these conditions with the following ones,

$(KT1)$	$\frac{\partial U_h}{\partial x_h}(x_h^*) \leqq v_h^*$	$\left(\frac{\partial U_h}{\partial x_h}(x_h^*) - v_h^*\right)x_h^* = 0$	$x_h^* \geqq 0$
$(KT2)$	$\omega_h \geqq x_h^*$	$v_h^*(\omega_h - x_h^*) = 0$	$v_h^* \geqq 0$

we can see that the Corner solution case where if $\frac{\partial U_h}{\partial x_h}(x_h^*) < v_h^*$, then $x_h^* = 0$ for every h , is not referred to. This is understandable, because this condition makes sense in the theory of price, but does not make sense in the theory of value. The price system is given independent from the marginal condition, while the value system is defined by and equal to the marginal utility.

If we define the function $U(x)$, where $x = (x_1, x_2, \cdots, x_H)$, by

$$U(x) = \sum_{h=1}^{H} U_h(x_h) = \sum_{h=1}^{H} \int_0^{x_h} \frac{\partial U_h}{\partial x_h}(c_h)\,dc_h = \sum_{h=1}^{H} \int_0^{x_h} u_h(c_h)\,dc_h$$

then the UTILITY, $u_h\left(x_h^*\right)$, is defined as a derivative of the total utility function, $\frac{\partial U_h}{\partial x_h}(x_h^*)$.

[4] By Kuhn-Tucker's Equivalence Theorem (Intriligator 1981), Kuhn-Tucker conditions ($KT1$) and ($KT2$) are equivalent to the problem of a consumer's choice such that

$$\max \sum_{1}^{H} U_h \quad subject\ to \quad \sum_{1}^{H} (\omega_h - x_h) \quad or \quad \max \sum_{1}^{H} U_h + \sum_{1}^{H} \nu_h (\omega_h - x_h).$$

where ν_h is a Lagrangian multiplier to be a value of commodity h . [5] Therefore, under a certain set of implicit conditions, the theory of UTILITY and scarcity implies a group of conditions very close to the Kuhn-Tucker conditions, which are equivalent to the problem of a consumer's utility maximization under resource constraints. This means the theory of UTILITY and scarcity has almost the same structure as the marginal utility theory of Menger(1950[1871]).

From an axiomatic point of view, the theory of UTILITY and scarcity is characterized by an axiom system consisting of Kuhn-Tucker conditions ($KT1$) and ($KT2$). While the marginal conditions are derived from the axiom of consumer's utility maximization, as are Kuhn-Tucker conditions, in the marginal utility theory, a group of conditions very close to the Kuhn-Tucker conditions are assumed in the theory of UTILITY and scarcity. This implies that in the theory of UTILITY and scarcity, the concept of marginal utility is a basic concept that is not defined in terms of other concepts. Furthermore, in the marginal utility theory, the concept of marginal utility is a derived concept defined in terms of other concepts. For this reason, historians have considered the concept of marginal utility to be missing the theory of UTILITY and scarcity.

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[4]Given the concept of the marginal utility function, Jevons(1965: 49–52) and Walras(1954[1874-77]: 76) derive the total utility function from the marginal utility function.

[5]The proposition such that a UTILITY is a decreasing function, with suppositions of cardinality, separability, and additivity, implies the concavity of a total utility function, which is a sufficient condition for a solution of the Kuhn-Tucker problem to exist.

3.4 The Significance of the Marginal Revolution

The results of the previous subsection show that the marginal principle was not an original achievement in the marginal revolution. The theory of UTILITY and scarcity just advances a group of propositions characterizing the value of a commodity utilizing the marginal principle. Those propositions are not organized on the basis of any set of more fundamental propositions. One of significances of the marginal revolution is that the group of propositions in the theory of UTILITY and scarcity is characterized by the rational behavior of an economic agent. This was noted by Menger(1950).

4 Marginal Productivity Theory and the Old Quarrel

The controversy on the Exhaustion Theorem in the marginal productivity theory is studied by Stigler(1941) and Jaffé(1964), who called the controversy *the old quarrel*; however, the theoretical development of the marginal productivity theory is not completely explained. Two difficulties should be pointed out. First, the Exhaustion Theorem can be established within the distinct specifications of an economic environment. This is the reason why the marginal productivity theory is controversial. Therefore, we must be consistent in our coordination of the theories. Second, the marginal productivity theory supposes a specified economic environment, and therefore the theory would vanish in the development of a general equilibrium theory. Without knowledge of the present microeconomic theory, it is impossible to understand the arguments, of the economists involved in the controversy, with any coherence.

We confirm what the economists involved in the controversy, namely, Wicksteed (1992[1894]), Walras (1954[1874-77]), Pareto (1964[1897], 1966[1909]), Barone (1965[1895]), and Wicksell (1958[1902]), say about the Exhaustion Theorem. Then, we interpret their theories in the general equilibrium theory and coordinate them.

4.1 The Old Quarrel over the Exhaustion Theorem

Wicksteed(1992[1894]) interpreted the classical theory of distribution as a producer theory and proved the Exhaustion Theorem within the framework of producer theory. Suppose a product denoted by y and n productive factors denoted by $z = (z_1, \cdots, z_n)$, where the price of the product is denoted by p and the prices of the productive factors by $q = (q_1, \cdots, q_n)$

A producer is characterized by a production function denoted by $y = f(z_1, \cdots, z_n)$. The producer chooses his production $\left(y^*, z_1^*, \cdots, z_n^*\right)$ so as to maximize his profit $py - q \cdot z$ under the constraint of production function. If there exists an equilibrium production (y^*, z^*) for a given price system (p^*, q^*), the following propositions are established.

(MPT1) $$q_1^* = p^* f_1, \cdots, q_n^* = p^* f_n.$$

(MPT2) $$y^* = f_1 z_1^* + \cdots + f_n z_n^* \text{ for any } (y^*, z^*).$$

where $f_h = \dfrac{\partial f}{\partial z_h}$ for every h. The producer equilibrium exists if f is homogeneous of degree one and satisfies the law of decreasing marginal productivity. Wicksteed states "under ordinary conditions of competitive industry, it is sensibly or approximately true that if every factor of production draws a remuneration determined by its marginal efficiency or significance, the whole product will be exactly distributed. "(Wicksteed 1992[1894]: 89)

Note that a producer theory assumes a price system is determined in a competitive equilibrium of a market economy; therefore, the Exhaustion Theorem is established for a given price system. If a production function is homogeneous of degree one, the output of product maximizing profit may be 0 or infinite for an arbitrarily determined price system. If so, the first theorem is not established, because an equilibrium production is a corner solution.

Walras constructed a general equilibrium theory with the support of his colleagues (Jaffé 1964, 1965). He consulted with Piccard about the problem of consumer's utility maximization with, and with Amstein about producer's cost minimization. Piccard answered Walras by explaining a solution in terms of elementary graphical presentation (Jaffé 1965: L. 211). Walras understood Piccard's explanation and utilized it in the first version of his *Éléments*. Amstein answered Walras by explaining a solution using a Lagrangian multiplier method (Jaffé 1965: L. 364). Amstein's explanation was, however, too technical for Walras to understand and therefore Walras did not utilize it in the first version of his *Éléments*; instead assuming a production technology of constant input-output coefficients. Then, in accord with a suggestion from Pareto (1955), he began to tackle the Amstein's explanation to construct a producer theory.

Hence, Walras read Wicksteed's *Essay*. Walras immediately noticed that the Exhaustion Theorem could be derived from the conditions of producer equilibrium and free competition equilibrium. Walras supposed that the first proposition of the Exhaustion Theorem (MPT1) was derived from producer cost minimization under the constraint of the production technology. Then, the second proposition (MPT2) was derived from (MPT1) and the condition of free competition equilibrium such that $p^* y^* = q_1 z_1^* + \cdots + q_n z_n^*$. As Pareto (1955) suggested, Walras's demonstration of (MPT1) was not valid. (MPT1) is derived from producer's profit maximization. Walras supposed that his Exhaustion Theorem was more general than Wicksteed's, but he could not prove it because he was not familiar enough with mathematics to solve optimization problems.

Walras first consulted Pareto about his idea, but Pareto was not very interested in it. Pareto (1966[1909]) had denied the possibility of production function homogeneous of degree one that the Exhaustion Theorem assumes. He understood Euler's Theorem and the fact that the Exhaustion Theorem is established if the production function is homogeneous of degree one (Pareto 1964[1897]: 717, n.2). Pareto (1966[1909]: 631-39) supposes a general economic environment, where producers have variable productive factors and fixed ones, and thus the production function cannot be homogeneous of degree one.

Walras then consulted Barone. Barone (1965[1895) revised Walras's theory by taking account of Pareto's advice about a production theory (Pareto 1955), and proved the Exhaustion Theorem in the theory of a competitive market. He submitted a review of Wicksteed's *Essay* to *Economic Journal*, where Barone tried to show that Walras's Theorem is more general than Wicksteed's Exhaustion Theorem. Edgeworth, the editor of *Economic Journal*, rejected it. Walras was angry at this, and criticized the English theory of distribution in Walras(1954[1895]), which is rewritten as 36^{th} lesson of Walras(1954[1874-77]). Having explained Barone's reasoning, Walras states that "M. Barone deduced this proposition with logical rigour from my theory of economic equilibrium. Mr. Wicksteed, however, fell short of establishing it for the more general case and would have been better inspired if he had not made such efforts to appear ignorant of the works of his predecessors. "(Walras 1954[1895]: 495) [6]

In Walras's reasoning, the most important fact is that his claim is apparently valid, but it does not make sense if a free competition equilibrium does not exist. This is the problem of the existence of the competitive equilibrium of a market economy. Walras did not suggest any condition for the existence of producer's equilibrium and market equilibrium. Walras thought he had proven the existence of competitive equilibrium of a market economy by confirming that the number of unknowns is equal to that of demand-supply equilibrium equations. Among his contemporaries, his reasoning was common knowledge and the problem of the existence of equilibrium was an open question. However, the existence

[6] See Jaffé(1964) about the old quarrel in detail.

of market equilibrium is proven from the property of economic environment characterizing consumer's preference and producer's technology. It is necessary to assume not only decreasing marginal productivity for every productive factor, but also, for example, the homogeneity of degree one of the production function.

Later, Wicksell(1958[1902]) showed that the Exhaustion Theorem is valid if the production function has a U-shaped long-run average cost curve as well as it being homogeneous of the first degree. This supports Walras's and Barone's claim that they proved the Exhaustion Theorem within the more general framework of general equilibrium.

4.2 The Exhaustion Theorem in the General Equilibrium Theory

We coordinate the assertions on the Exhaustion Theorem of Wicksteed, Walras, Barone, Pareto, and Wicksell, and show that we can interpret them as specified theories of a general marginal productivity theory. It is necessary to interpret the marginal productivity theories in the controversy within the framework of the general equilibrium theory. We can define the marginal productivity theory as one of a competitive market where the following propositions are established: (1) the prices of productive factors are determined, (2) the prices of productive factors are proved to be equal to the marginal productivity of the respective productive factors, and (3) if (2) is valid, then the value of the total product is completely exhausted by the share of the productive factors. Proposition (1) is derived from the theory of price determination. The theory of a competitive market is one of alternatives. The existence of equilibrium is essential. Proposition (2) is derived from producer profit maximization under the constraint of production technology. In general, the solution is characterized by the Kuhn=Tucker Equivalence Theorem (Intriligator 1984). Proposition (3) is Euler's Theorem (Apostol 1967), which is a tautology because it is a mathematical theorem. Euler's Theorem implies that the Exhaustion Theorem is equivalent to the linear homogeneity of the total production function. [7]

The economic theories in the controversy assume certain specified economic environments. First, all the commodities are distinctly classified into products and productive factors in the marginal productivity theory. However, whether a commodity is a product or a productive factor is not *a priori* determined, but is determined depending on the property of the economic environment, such as a consumer's preference and a producer's technology. Second, there is no joint product in the marginal productivity theory. This implies that a production function is expressed by $y = f(z_1, \cdots, z_n)$. Third, since the marginal productivity theory is a theory of a competitive market, it is necessary to prove the existence of a competitive equilibrium of a market economy. The production technology is expressed by a differentiable production function satisfying the law of decreasing marginal productivity. However, to prove the existence of an equilibrium, it is sufficient to assume continuity, convexity, and several adequate conditions of an consumption set, preference ordering, and a production set; but not necessary to assume differentiability of utility and production functions (Debreu 1959). This is because the marginal principle of distribution is significant in the economic environment characterized by differentiable functions, but not in the economic environment characterized by convex analysis. Fourth, a production technology exhibiting constant returns to scale is specific. It may exhibit decreasing or increasing returns to scale depending on its properties.

These then are the reasons why the modern theory of a producer does not refer to the Exhaustion Theorem. The theorem assumes a much too specific economic environment for modern economic theories.

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[7]EULER'S THEOREM: Let $f(x) = 0$ be continuous, and also differentiable. Then f is homogeneous of degree k if and only if $kf(x) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(x)\, x_i$.

4.3 Reconstruction of Walras's Exhaustion Theorem

We show that in the competitive equilibrium of a market economy, Walras's condition that every producer makes neither profit nor loss is equivalent to the condition that the total production function is homogeneous of degree zero. Let $y_j = \left(y_{j1}, \cdots, y_{jn}\right)$ be a production of producer j, and $f_j\left(y_j\right) \leqq 0$ be a production function of producer j, which is homogeneous of degree zero, for every $j \in \{1, \cdots, n\}$. Then, the total production function is defined as $f(y) \leqq 0$ such that $f(y) = f_1(y_1) + \cdots + f_n(y_n)$. The total production function is homogeneous of degree zero. The equilibrium condition of producer j is described by:

$$y_j^* \text{ maximizes } p^* \cdot y_j \text{ subject to } f_j\left(y_j\right) \leqq 0, \text{ for every } j.$$

Then, the price of productive factor q_h relative to the price of product p is equal to the marginal productivity of the productive factor f_h', for very productive factor h. Therefore, the value of the product is equal to the value of all the productive factors.

In a competitive situation, the following theorem is valid (Debreu 1959: 3.4 (1)).

DECENTRALIZATION THEOREM: Given a price system p^*,

$$y^* \text{ maximizes } p^* \cdot y \text{ subject to } f(y) \leqq 0, \text{ if and only if}$$

$$\left(y_j^*\right) \text{ maximizes } p^* \cdot y_j \text{ subject to } f_j\left(y_j\right) \leqq 0, \text{ for every } j.$$

The Decentralization Theorem states that every individual producer maximizes his profit subject to his production function if and only if the total production maximizes the total profit subject to the total production function. By producer profit maximization, the price of a productive factor is equal to the price by the marginal productivity of the productive factor for every productive factor for every individual equilibrium production. Under the Decentralization Theorem, this is also valid for the total equilibrium production.

In the general equilibrium theory, the Exhaustion Theorem is established for the total equilibrium production. Therefore, applying Euler's Theorem, the total production function is homogeneous of degree zero. Inversely, if the total production function is homogeneous of degree zero, the Exhaustion Theorem is established, and it is not necessary for an individual production function to be homogeneous of degree zero (Debreu 1959: 88, n.1). This is the implication of Walrasian Exhaustion Theorem. If the marginal productivity theory were completed, then it would be the theory of McKenzie (1959) where differentiability is satisfied.

4.4 Pareto's consideration

Pareto extended and generalized Walrasian general equilibrium theory. He generalized the concept of utility from the separable and additional utility function to the ordinal one. He also generalized the concept of production technology from the production function in which all the productive factors are fixed or variable, to that in which some productive factors are variable and others fixed (Pareto 1966[1909]: 636). Therefore Pareto's production function $y = f(z_1, \cdots, z_n)$ cannot be homogeneous of degree one, because it has both variable productive factors and fixed ones. Thus, Pareto had a negative view about the Exhaustion Theorem. However, as long as he adopts Walras's concept of free competition equilibrium, Pareto's theory implies the existence of a competitive equilibrium in a market economy in which the Exhaustion Theorem is established.

Consider the problem of cost minimization subject to a production function with a fixed productive factor $\bar{y} = f^S(z_1, \cdots, \bar{z_h}, \cdots, z_n)$, and the problem of cost minimization subject to a production function without a fixed productive factor $\bar{y} = f(z_1, \cdots, z_n)$. Then, we have the cost function with a fixed factor h, $c^S(q_1, \cdots, q_n, \bar{y}, \bar{z_h})$, and that without a fixed factor, $c(q_1, \cdots, q_n, \bar{y})$. We can consider f as the production function of the industry, and

f^S as that of the individual producer. We can also consider c as the cost function of the industry, and c^S as that of the individual producer. The Envelop Theorem characterizes the relationship between these cost functions. [8] In Walras's free competition equilibrium, the Exhaustion Theorem is established and the production function of the industry f is homogeneous of degree one. Then, the average cost curve of the industry AC is horizontal, and the average cost curve of an individual producer AC^S is tangential to that, as is shown in the figure 1. If a producer has a fixed productive factor, the producer can choose the input of all the variable productive factors to minimize cost. Thus, even if an individual production function f^S has a fixed factor and is not homogeneous of degree one, the Exhaustion theorem can be established if the production function f is homogeneous of degree one. Pareto did not realize that a free competition equilibrium implies the production function of the industry is homogeneous of degree one.

Figure1 Cost structure in a free competition equilibrium

In modern microeconomics, the Envelop Theorem is considered to characterize the relationship between short-run and long-run cost functions. The production function without fixed productive factor should be interpreted as the long-run one. Therefore, the Exhaustion Theorem should be considered to be a long-run equilibrium of free competition (Osana, 1987).

[8] ENVELOP THEOREM: For $y = \bar{y} > 0$, $c^S(q_1, \cdots, q_n, \bar{y}, \bar{z_h}) = c(q_1, \cdots, q_n, \bar{y})$, then

$$\mu^S = \frac{\partial c^S(q_1, \cdots, q_n, \bar{y}, \bar{z_h})}{\partial y} = \frac{\partial c(q_1, \cdots, q_n, \bar{y})}{\partial y} = \mu.$$

where μ^S is the Lagrangian multiplier of the cost minimization problem with a fixed productive factor h, and μ is the Lagrangian multiplier of the cost minimization problem without a fixed productive factor.

4.5 The Old Quarrel in Retrospect

As long as the marginal productivity theory is a distribution theory, it must be one that determines the prices and inputs of all the productive factors. Therefore, we can say that Wicksteed's Exhaustion Theorem is not sufficient without a theory of a competitive market. Within the framework of general equilibrium theory, a price system is determined at market equilibrium, and the Exhaustion Theorem is equivalent to the homogeneity of degree zero of the total production function. It is not necessary to assume the homogeneity of degree zero of an individual production function.

However, microeconomic theories should be described in terms of an economy consisting of many consumers, many producers, and many commodities, where each consumer i must be characterized by i's preference and each producer j must be characterized by j's production technology. Therefore Wicksell (1958[1902]) assumed an individual production function first exhibiting increasing and then decreasing returns to scale.

Jaffé (1964) states that the old quarrel was not over the distribution theory but over the producer theory. Stigler (1941) focuses his attention on the controversy over the Exhaustion Theorem. But we have showed that the old quarrel was also over the theory of a competitive market. This point of view supports an understanding of Walras's assertion; that his Exhaustion Theorem is more general than Wicksteed's.

5 Historical Interpretation of an Ambiguous Theory

There can be plural interpretations of the same text because of the definition of interpretation. Waterman (2003) takes examples of Morishima's and Samuelson's interpretations of Ricardo (Morishima, 1989; Samuelson, 1978) and Marx (Morishima, 1973, ch.4; Samuelson, 1971). We take an example of Negishi's (1989) and Kawamata's (1995, 2009) interpretation of Wieser's theory of natural value (Wieser, 1893[1889]). Wieser defines natural value as:

> The relation of natural value to exchange value is clear. Natural value is one element in the formation of exchange value. It does not, however, enter simply and thoroughly into exchange value. On the one side, it is disturbed by human imperfection, by error, fraud, force, and chance; and on the other side, by the present order of society, by the existence of private property, and by differences between rich and poor, as a consequence of which a second element, namely, purchasing power, mingles itself in the formation of exchange value. For natural value, goods are estimated simply according to their marginal utility; for exchange value, it is according to a combination of their marginal utility and purchasing power. (Wieser 1893[1889]: 61-2)

This statement implies that Wieser sees the natural value of a commodity as an exchange value when the incomes (purchasing power) of all individual agents are equal to one another. Two interpretations are possible. One is based on interpreting the income as a real income; the other has it as a nominal income. The former interpretation is that of Negishi (1989). He characterizes the natural value as a shadow price where the utilitarian social welfare function is maximized. The natural value is the marginal utilitarian social welfare of a commodity when the utilitarian social welfare is maximized. The latter interpretation is that of Kawamata (1995, 2009). He defines the natural value based on the distribution economy of Malinvaud (1977: 107-110), where the resources of commodities are not privately owned and, given a price system and a nominal income, a consumer chooses his/her consumption so as to maximize his/her utility subject to the income constraint. Therefore, the natural value of a commodity is the competitive price of the commodity when the nominal income is the same for every consumer. In this case, the allocation at the natural value is a fair allocation.

Note that the idea of real income was used by Hicks (1946[1939]) in the context of the Slutsky equation in the 1930s. It is not reasonable to suppose that before Hicks, Wieser considered a real income as the utility level without any necessity of defining it in his theory. Wieser defines the natural value not in terms of social welfare function, but in terms of exchange value.

When a past theory contains incompatible sets of propositions, that is to say, the theory is inconsistent, it is impossible to interpret the theory as a whole. To interpret an inconsistent theory, we have only to focus our attention on a consistent set of propositions. Then, we interpret the consistent parts of the original theory. There can be several interpretations depending on which part of the original theory is interpreted. In such cases, we must review whether or not an individual interpretation is accurate enough to be a historical interpretation. If we interpret the theory as a historical fact, we can review each alternative interpretation and minimize parts of the interpretation alien to the original text.

6 Concluding Remarks

There is a historical interpretation of a past theory if and only if there is a stream of theories developed from that past theory, or a Scientific Research Program to which the theory belongs. A theory must be interpreted as a consistent axiom system; therefore, an interpretation of a theory will be a consistent axiomatic extension of the theory. We showed three examples of historical interpretations in the history of the general equilibrium theory. The first example of interpretation of the theory of utility and scarcity shows that axiomatic interpretation is effective in interpreting an insufficient theory of the past from which a

SRP developed. The second shows that a past theory T may include invalid propositions in a present one that is thought to have been developed from the theory T . The history of economic theory is formed by theories dominating their contemporary ones, which have arisen from a SRP. The set of abandoned theories includes those which are invalid or are incompatible with the present established theories. The third example shows that there may be plural historical interpretations of a consistent theory that is decisive relative to the core set of theorems. This is because a past theory that has been regarded as consistent and complete may be ambiguous from the viewpoint of a present theory, even if the past theory was regarded as complete by its contemporaries.

Our concept of historical interpretation is the same as that of Rational Reconstruction (RR) and Mathematical Modeling (MM) of Waterman (2003). RR is a set of interpretations of economic thought or of economic theory in the SRP to which they belong, where these are not always expressed in terms of mathematics. RR but not MM is a set of economic thoughts and literary economic theories among which a certain concept of development is defined. MM is a set of mathematical interpretations of economic thoughts or economic theories, where these do not always belong to a stream of theories or SRP developed from them. There are sometimes significant open questions in past theories. Resolving such open questions from the history of economics makes a theoretical contribution to present day economics. Morishima's and Samuelson's interpretations of the theories of Ricardo and Marx are such theoretical interpretations that contribute to modern economics. They are examples of MM but not RR. If a researcher resolves open questions posited in a certain text from the history of economics, it is the same as resolving open questions posited at the forefront of present day economic research.

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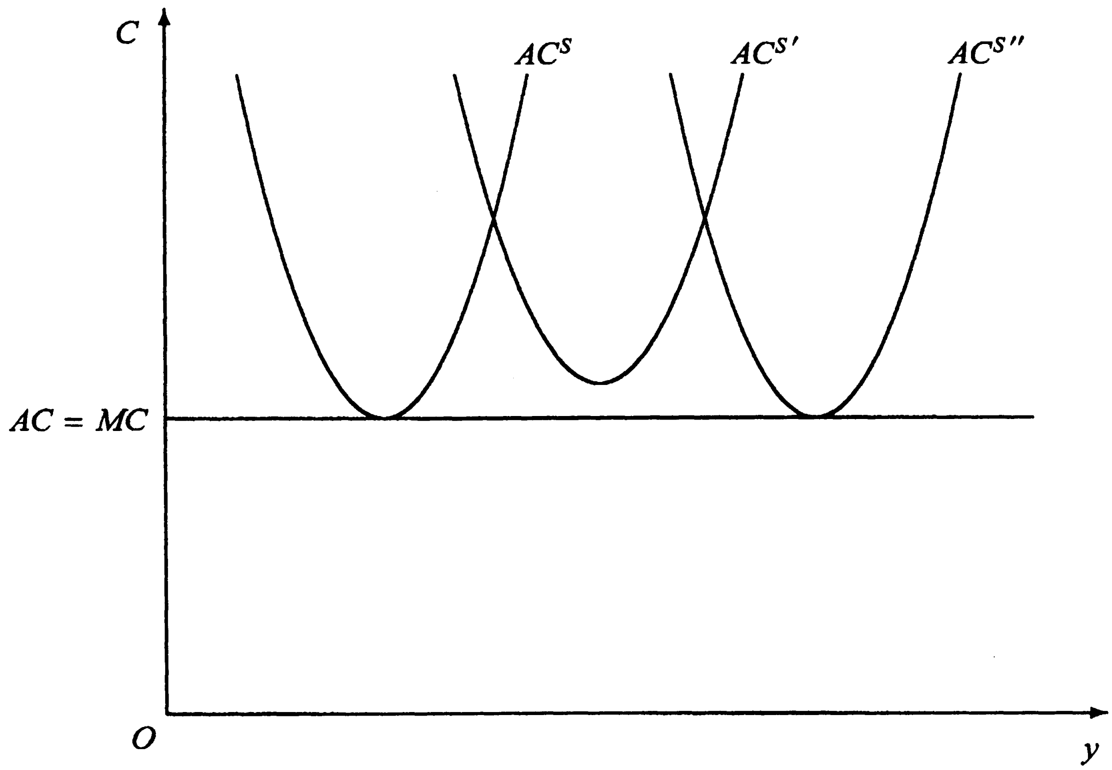

Figure 1: Cost structure in a free competition equilibrium