数理解析研究所講究録
第 1659 巻 2009 年 204-215

On Siegel modular forms

(a report on joint work with S.Nagaoka)

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Introduction

Starting with Swinnerton-Dyer [11] and Serre [10], the mod p properties of elliptic modular forms and also their p-adic properties have been deeply studied. Some aspects of this theory were later generalized to other types of modular forms, e.g. Nagaoka and others considered sequences of Siegel Eisenstein series where the p-adic limit becomes a true modular form (e.g.[9]). Furthermore in our previous joint work [4] we constructed a level one Siegel modular form congruent 1 mod p. In our project in collaboration with Prof.Nagaoka we are concerned with generalizing some of Serre's results to the case of Siegel modular forms, especially his work on modular forms for congruence subgroups $\Gamma_0(p)$: He showed that modular forms for this group are always p-adic modular forms; a particularly interesting case is the weight 2, where he showed that modular forms for $\Gamma_0(p)$ are always congruent mod p to modular forms of level one of weight $p+1$; we mention that this result is very usefull also for other purposes [1] . To extend these results to Siegel modular forms, we follow Serre in the sense that we use a trace function from $\Gamma_0^n(p)$ to the full modular group. The main new problem is that certain modular forms of level p which are congruent 1 mod p and with divisibility by p in the other cusps are not available (there are $n+1$ cusps to be considered!). So the main point is to overcome or avoid this problem.

We only consider here characteristic zero classical modular forms and their reductions modulo p. For a more sophisticated (geometric) point of view we refer to work of Ichikawa [8].

§1 Preliminaries

1.1 Modular Forms

Let $\mathbb{H}_n$ denote the Siegel upper half space of degree n and Z a point of $\mathbb{H}_n$; then $\Gamma^n := Sp_n(\mathbb{Z}) = Sp_n(\mathbb{R}) \cap M_{2n}(\mathbb{Z})$ acts discontinously on $\mathbb{H}_n$.

Let $\Gamma \subset \Gamma^n$ be a congruence subgroup. We denote by $M_n^k(\Gamma)$ the space of Siegel modular forms of weight k for Γ. We will be only concerned with congruence subgroup of the form

$$\Gamma_0^n(N) := \left\{ \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp_n(\mathbb{Z}) \mid C \equiv O \pmod{N} \right\}.$$

For any function $f : \mathbb{H}_n \longrightarrow \mathbb{C}$ on the Siegel upper half space and any $k \in \mathbb{Z}_{\geq 0}$ we write

$$(f\mid_k M)(Z) = (\det M)^{k/2}\det(CZ+D)^{-k}f(MZ)$$

for any $M = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \in G^+Sp_n(\mathbb{R})$.
If F is an element of $M_n^k(\Gamma)$, then $F(Z)$ can be expressed as a Fourier series of the form

$$F(Z) = \sum_{0 \leq T \in \Lambda_n} a_F(T)e^{2\pi i \mathrm{tr}(TZ)},$$

where

$$\Lambda_n := \{T = (t_{ij}) \in Sym_n(\mathbb{Q}) \mid t_{ii}, 2t_{ij} \in \mathbb{Z}\}.$$

Taking $q_{ij} := e^{(2\pi i z_{ij})}$ with $Z = (z_{ij}) \in \mathbb{H}_n$, we write

$$\boldsymbol{q}^T := e^{2\pi i \mathrm{tr}(TZ)} = \prod_{i<j} q_{ij}^{2t_{ij}} \prod_{i=1}^{n} q_{ii}^{t_{ii}}.$$

Using this notation, we have the generalized q-expansion:

$$F = \sum_{0 \leq T \in \Lambda_n} a_F(T)\boldsymbol{q}^T = \sum \left(a_F(T)\prod_{i<j} q_{ij}^{2t_{ij}}\right)\prod_{i=1}^{n} q_{ii}^{t_{ii}}$$
$$\in \mathbb{C}[q_{ij}^{-1}, q_{ij}][\![q_{11}, \ldots, q_{nn}]\!].$$

1.2 p-adic modular forms

For any subring R of $\mathbb{C}$, we shall denote by $M_n^k(\Gamma)_R$ the R-module consisting of those F in $M_n^k(\Gamma)$ for which $a_F(T)$ is in R for every $T \in \Lambda_n$. From this, any element F in $M_n^k(\Gamma)_R$ may be regarded as an element of the formal power series ring $R[q_{ij}^{-1}, q_{ij}][\![q_{11}, \ldots, q_{nn}]\!]$.
For a prime number p, we denote by ν_p is the normalized additive valuation on $\mathbb{Q}$ (i.e. $\nu_p(p) = 1$), and the extension to a field $\boldsymbol{K}$. For a Siegel modular form $F = \sum a_F(T)\boldsymbol{q}^T \in M_n^k(\Gamma)(\boldsymbol{K})$ we put

$$\nu_p(F) := Inf\{\nu_p(a(T) \mid T \in \Lambda_n\}.$$

For two Siegel modular forms $F = \sum a_F(T)\boldsymbol{q}^T \in M_n^k(\Gamma)(\boldsymbol{K})$, $G = \sum a_G(T)\boldsymbol{q}^T \in M_n^l(\Gamma)(\boldsymbol{K})$, we write

$$F \equiv G \pmod{p^m}$$

if $\nu_p(F-G) \geq m + \nu_p(F)$.
A formal power series

$$F = \sum a_F(T)\boldsymbol{q}^T \in \mathbb{Q}_p[q_{ij}^{-1}, q_{ij}][\![q_{11}, \ldots, q_{nn}]\!]$$

is called a *p-adic* (*Siegel*) *modular form* (in the sense of Serre) if there exists a sequence of modular forms $\{F_m\}$ satisfying

$$F_m = \sum a_{F_m}(T)\boldsymbol{q}^T \in M_n^{k_m}(\Gamma^n)_{\mathbb{Q}} \quad \text{and} \quad \lim_{m\to\infty} F_m = F$$

where $\lim_{m\to\infty} F_m = F$ means that

$$\inf_{T\in\Lambda_n}(\nu_p(a_{F_m}(T) - a_F(T)) \longrightarrow +\infty \quad (m \to \infty).$$

1.3 $\Gamma_0^n(p)$, its cusps and the trace function to level one
We start from the double coset (Bruhat-) decomposition .

$$Sp_n(\mathbb{F}_p) = \cup_{i=0}^n P_n(\mathbb{F}_p) \cdot \omega_i \cdot P_n(\mathbb{F}_p),$$

where P_n denotes the Siegel parabolic subgroup of Sp_n and the ω_i parametrize the "inequivalent cusps" for $\Gamma_0^n(p)$:

$$\omega_i = \begin{pmatrix} 0_i & 0 & -1_i & 0 \\ 0 & 1_{n-i} & 0 & 0_{n-i} \\ 1_i & 0 & 0_i & 0 \\ 0 & 0_{n-i} & 0 & 1_{n-i} \end{pmatrix}$$

As a consequence of this decomopsition, we get

$$Sp_n(\mathbb{Z}) = \cup_{i=0}^n \cup_{\gamma_{ij}} \Gamma_0^n(p) \cdot \omega_i \cdot \gamma_{ij}$$

with certain $\gamma_{ij} \in P_n(\mathbb{Z})$.
The trace operator is defined by

$$tr : \begin{cases} M_n(\Gamma_0(p) & \longrightarrow & M_n^k(\Gamma^n) \\ f & \longmapsto & \sum_\gamma f \mid_k \gamma \end{cases},$$

where γ runs over $\Gamma_0^n(p)\backslash\Gamma^n$. Using the representatives from above, we can write the trace for $f \in M_n^k(\Gamma_0^n(p))$ as

$$tr(f) = \sum_{j=0}^n p^{\frac{j(j+1}{2}} (f \mid \omega_j) \mid \tilde{U}(j)$$

where the factor $p^{\frac{j(j+1)}{2}}$ comes from certain exponential sums and the operators $\tilde{U}(j)$ are certain operators acting on the Fourier expansion of $f \mid \omega_j$; we do not need to know here the explicit expression of the $\tilde{U}(j)$ in general, just the "extreme cases" should be made explict: $\tilde{U}(0)$ is the identity and $\tilde{U}(n)$ is a slight variant of the classical $U(p)$-operator, defined on Fourier series by

$$f = \sum_T a_f(T)e^{2\pi i tr(\frac{1}{p}TZ)} \longmapsto \sum_T a_f(pT)e^{2\pi i tr(TZ)}.$$

§2 Theta series attached to p-special lattices

We consider even integral lattices L in an m-dimensional euklidian space. Whenever convenient, we freely identify the lattice L with an even integral matrix S. We call L a p-special lattice, if it has an automorphism of order p with $\mathbf{0}$ as only fix point. It is our (somewhat naive) viewpoint that we consider such p-special lattices as principal source for constructing modular forms with desired congruence properties. Clearly, the degree n theta series attached to such p-special lattice will satisfy

$$\theta^n(L,Z) \equiv 1 \bmod p,$$

where (as usual) $\theta^n(L,Z) = \sum_{X\in\mathbb{Z}^{(m,n)}} exp(\pi i tr(S[X]Z)$.

It is a delicate problem to construct p-special lattices of level 1. The case of level p is somewhat simpler. Using two copies of the root lattice A_{p-1} we showed in [4]

Prop.2.1 *For all primes $p \geq n+3$ or $p \equiv 1 \bmod 4$ there exists a modular form F_{p-1} of degree n, level 1 and weight $p-1$ such that*

$$F_{p-1} \equiv 1 \bmod p$$

Remark: The lattice A_{p-1} is p-special, but it is also of level p; the desired level 1 modular forms is obtained from the theta series attached to the lattice $A_{p-1} \oplus A_{p-1}$by using the trace function, see [4].

Our main aim is to construct a modular form G of level p such that

$$\boxed{G \equiv 1 \bmod p}$$

and for all $i \geq 1$

$$\boxed{\text{the p-adic behaviour of } G \mid \omega_i \text{ should be “as good as possible”}}$$

By "as good as possible" we mean that we want to maximize successively the numbers $\nu_p(G \mid \omega_i)$ for all $i \geq 1$.
To construct such modular forms G we do not only need one p-special lattice of level p, but many of them (more precisely, we need p-special lattices with many different discriminants!)

Prop.2.2[1] *Let p be an odd prime, then there are p-special (positive definite, even) lattices of rank $p-1$, level p and determinant p^t for all $1 \leq t \leq p-2$.*

The *idea* is simple: We consider the field $K := \mathbb{Q}(\xi)$, where ξ is a primitive p-th root of unity. Let $\mathfrak{p}$ be the unique ramified prime ideal in K. As candidates for the lattices in question, we consider the powers $\mathfrak{p}^i$ with $i \in \mathbb{Z}$; the $\mathbb{Q}$-bilinear form to be considered will be given by $tr_{K/\mathbb{Q}}(x \cdot \bar{y})$. Then we have to investigate, which powers $\mathfrak{p}^i$ define integral lattices with level p. Clearly these lattices are p-special, because multiplication with ξ defines a special automorphism. We omit details.

By taking two copies of the lattices from above, we can construct many theta series of level p, which are congruent 1 mod p. However, these theta series will have high p-denominators in the other cusps ω_i, essentially the denominators in the cusps ω_i will be $d^{-\frac{i}{2}}$, where d is the discriminant of the lattice in question. The situation becomes better, if we consider appropriate linear combinations of such theta series.

Theorem 2.3: *Assume that $p \geq n$ if $p \equiv 1 \bmod 4$ or $p \geq n+3$ if $p \equiv 3 \bmod 4$. Then there exists a modular form G of level p, weight $p-1$ with the following properties:*

(A)

$$G \equiv 1 \mod p$$

(B) For $1 \leq j \leq n$

$$\nu_p(G \mid \omega_j) \geq -\frac{j(j-1)}{2} + 1$$

The Fourier expansion of $G \mid \omega_j$ has coefficients in $\mathbb{Z}$ for $j = 0, 1, 2$ and in $\mathbb{Z}[\frac{1}{p}]$ for $j \geq 3$.

[1]this result is based on an email communication by E.Bayer-Fluckiger

Comments:

- The case $p \equiv 1 \bmod 4$ is somewhat better, because in addition we have a p-pecial unimodular lattice
- The p-denominators of $G \mid \omega_j$ seem to be quite bad for large j, e.g. taking just the theta series attached to a p-special lattice of discriminant p^2 as G would give p^{-j} as highest denominator in $G \mid \omega_j$; the construction above however is better for small j.
- In degree 1 Serre [10] takes

$$g := E_{p-1} - p^{p-1}E_{p-1}(p\tau)$$

where E is a level 1 form congruent to 1 mod p (more precisely he takes the level 1 Eisenstein series of weight $p-1$). This construction does not generalize to higher degree (in the sense that the p-denominators become too large in the other cusps for $n > 1$).

- We just give the formula for G in the case of $p \equiv 1 \bmod 4$: Let L_t be any p-special lattice of rank $2p-2$ with $\det(L_t) = p^{2t}$ with $0 \leq t \leq n$. Then we put

$$G := \sum_{t=0}^{n} (-1)^t p^{\frac{t(t-1)}{2}} \theta^n(L_i).$$

An inspection of the Fourier expansions in the cusps ω_j yields the result.

§3 Application I: From weight 2 to weight p+1

We put

$$M_n^k(\Gamma_0^n(p))^0 := \{f \in M_n^k(\Gamma_0(p) \mid \forall j : \nu_p(f \mid \omega_j) > -j - 1 + \nu_p(f)\}$$

Using a construction from §2 we can show

Prop.3.1: *Assume that* $p \geq n$ *if* $p \equiv 1 \bmod 4$ *or* $p \geq n+3$ *if* $p \equiv 3 \bmod 4$. *Then for all* $f \in M_n^k(\Gamma_0(p))^0$ *there is* $h \in M_n^{k+p-1}(\Gamma^n)$ *with*

$$f \equiv h \bmod p$$

Proof: Let $G \in M_{p-1}^n(\Gamma_0^n(p)$ be as in Theorem 2.3 of the previous section.

Assuming $\nu_p(f) = 0$ we consider the trace

$$tr(f \cdot G) = f \cdot G + \sum_{j=1}^{n} p^{\frac{j(j+1)}{2}} (f \cdot G) \mid \omega_j \mid \tilde{U}(j)$$

Then the contributions for $j \geq 1$ are all congruent zero mod p.

Remarks:

- The set of modular forms $M_n^k(\Gamma_0^n(p))^o$ satisfying the conditions of prop.3.1 is *not* a vector space in general.
- Clearly certain theta series $\theta^n(L)$ do satisfy the conditions above, namely if $\det(L) = p^2$; this does not imply that this remains true for linear combinations of such theta series.
- In [1] we treated a similar situation for degree 1. There is was easy to apply the theorem also for situations where the condition on $\nu_p(f \mid \omega_1)$ was not satisfied: We just enlarged the weight of the function G by taking an appropriate power of G. This does no longer work in our case because $\nu_p(G \mid \omega_j)$ is negative for $j \geq 2$.
- In degree one the theory of newforms implies, that

  $$M_1^2(\Gamma_0(p))^0 = M_1^2(\Gamma_0(p))$$

  and therefore *all* modular forms of level p and weight 2 are congruent mod p to level 1 modular forms of weight p+1. In higher degree such a theory of newforms is not (or not yet ?) available for $\Gamma_0^n(p)$ and it is even unclear whether such a theory would imply the equality $M_n^2(\Gamma_0(p))^0 = M_n^2(\Gamma_0(p))$ for any $n \geq 2$.

We define now a *subspace* of $M_n^k(\Gamma_0(p))$ by the condition

$$M_n^k(\Gamma_0(p))' := \{f \in M_n^k(\Gamma_0(p)) \mid \forall j : \ (*)_j \text{holds}\}$$

where $(*)_j$ denotes the following relation:
For $1 \leq j \leq n$ we decompose $Z \in \mathbb{H}_n$ as

$$Z = \begin{pmatrix} \tau & z \\ z^t & w \end{pmatrix} \qquad (\tau \in \mathbb{H}_j, w \in \mathbb{H}_{n-j})$$

Then by $(*)_j$ we mean the condition

$$f\mid_k \omega_j(\begin{pmatrix} p\tau_1 & z \\ z^t & \frac{1}{p}w \end{pmatrix}) = (-1)^j p^{-j} f(\frac{1}{p}Z) \mid \tilde{U}^j(p) \qquad (*)_j$$

Here $\tilde{U}^j(p)$ acts on periodic functions defined on $\mathbb{H}_n$ which are periodic for $p \cdot Sym_n(\mathbb{Z})$ by

$$f = \sum_T a(T)exptr(2\pi i tr(\frac{1}{p}TZ) \longmapsto f \mid \tilde{U}^j(p) = \sum_{T, t_1 \equiv 0(p)} a(T)exptr(\frac{1}{p}TZ)$$

and t_1 denotes the symmetric matrix of size j in the upper left corner of T. Clearly, the condition $(*)_j$ implies $\nu_p(f \mid \omega_j) \geq -j$ and therefore we have the inclusion

$$M_n^k(\Gamma_0(p))' \subseteq M_n^k(\Gamma_0(p))^0 \subseteq M_n^k(\Gamma_0(p)).$$

We remark, that for $n = 1$ this space plays an essential role in [1].
It is remarkable that the full space generated by quaternary theta series $\theta^n(L)$ with L of determinant p^2 and rank 4 satisfies the condition above; this is an easy consequence of the fact, that a quaternion algebra over $\mathbb{Q}$,ramified only in p is anisotropic mod p, when viewed as a quadratic space over $\mathbb{F}_p$.

Definition: For a prime p we put

$$Y^n(p) := \mathbb{C}(\theta^n(L) \mid L \text{ quaternary, level p, } det(L) = p^2)$$

This is precisely the vector space of "Yoshida liftings" of level p, see [12, 2]. Summarizing the considerations above, we get

Prop.3.2 For any prime p we have

$$Y^n(p) \subseteq M_n^k(\Gamma_0(p))' \subseteq M_n^2(\Gamma_0(p))^0 \subseteq M_n^2(\Gamma_0(p))$$

Combining all this we obtain as main result of this section:

Theorem 3.3: *Assume that* $p \geq n$ *if* $p \equiv 1 \bmod 4$ *or* $p \geq n+3$ *if* $p \equiv 3 \bmod 4$. *Then all elements of the space* $Y^n(p)$ *of Yoshida liftings are congruent* $\bmod\, p$ *to modular forms of level one of weight* $p+1$.

Remark: Assume that n is greater or equal to 5. Then the Yoshida lifts are singular modular forms [6]. The Corollary asserts that we have found modular forms of level one, weight $p+1$ degree n such that all their Fourier coefficients $a(T)$ with T of rank greater than 4 are congruent zero mod p.

§4 Application II: level p modular forms are p-adic

To generalize Serre's result about modular forms for $\Gamma_0(p)$ being p-adic modular forms we cannot follow his strategy directly. The problem of the (non-)existence of a modular form for $\Gamma_0^n(p)$ with the necessary properties ($F \equiv 1 \bmod p$ and $F \mid \omega_i \equiv 0 \bmod p$ for all $i > 0$) was discussed before. We need a variant of Serre's approach.
We use a modular form $\mathcal{K}_{p-1}$ on $\Gamma_0^n(p)$ with Fourier coefficients in $\mathbb{Z}$ satisfying

$$\begin{aligned} \mathcal{K} \mid \omega_i &\equiv 0 \bmod p \qquad (0 \leq i \leq n-1) \\ \mathcal{K} \mid \omega_n &\equiv 1 \bmod p \end{aligned}$$

The existence of such a modular form is not a problem at all: We may use

$$\mathcal{K}_{p-1} := p^n \theta^n(L),$$

where L is any p-special lattice of rank $2p-2$ and determinant p^2.

Theorem 4.1: *Let p be a prime with $p \geq 5$. Let f be an element of $M_n^k(\Gamma_0(p))$. Then for any $\alpha \in \mathbb{N}$ there exists $\beta \in \mathbb{N}$ (depending on α, f) and $H \in M_n^{k+\beta\cdot(p-1)}$ such that*

$$\nu_p(f-H) \geq \nu_p(f) + \alpha.$$

The dependence of β on α will be clarified below.

Proof: As usual, we assume $\nu_p(f) = 0$.
For the moment we consider (for an arbitrary modular form $g \in M_n^k(\Gamma_0^n(p))$ and arbitrary $\beta = \kappa p^\gamma$

$$Tr_\beta(g) := p^{-\frac{n(n+1)}{2}} \cdot tr(g \cdot \mathcal{K}_{p-1}^\beta)$$

The trace decomposes into $n+1$ pieces Y_j which we consider separately: For $0 \leq j \leq n$ we have to look at

$$Y_j := p^{\frac{j(j+1)}{2} - \frac{n(n+1)}{2}} \left(g \mid_k \omega_j \cdot (\mathcal{K}_{p-1} \mid \omega_j)^\beta\right) \mid \tilde{U(j)}.$$

Then for $j < n$ we have

$$\nu_p(Y_j) \geq \nu_p(g \mid_k \omega_j) + \nu_p(\mathcal{K}_{p-1}) \cdot \beta$$

Clearly this becomes large if β is large.
The contribution for $j = n$ needs a more detailed study:
We write $(\mathcal{K}_{p-1} \mid \omega_n)^\beta$ as $1 + p^{\gamma+1}X$ with a Fourier series X with integal Fourier coefficients. Then

$$\left(g \mid_k \omega_n \cdot (\mathcal{K}_{p-1}^\beta \mid \omega_n)\right) \mid \tilde{U}(n) = g \mid_k \omega_n \mid \tilde{U}(n) + p^{\gamma+1} \left(g \mid_k \omega_n \cdot X\right) \mid \tilde{U}(n).$$

Now we use that the $U(p)$ operator is invertible as a Hecke operator for $\Gamma_0(p)$ [3]. Therefore we may choose g such that

$$g \mid_k \omega_n \mid \tilde{U}(n) = f.$$

With this choice of g the contribution for $j = n$ to the trace of $g \cdot \mathcal{K}_{p-1}^\beta$, which we call Y_n satisfies

$$\nu_p(Y_n - f) \geq \gamma + 1 + \nu_p(g \mid_k \omega_n).$$

Summarizing this, we see that $H := Tr_\beta(g \cdot \mathcal{K}_{p-1})$ is congruent to f if we choose γ to be large enough.

Remark: We wrote $\beta = \kappa \cdot p^\gamma$ in the proof in order to emphasize different roles played by β and γ. We have to choose γ large enough to assure the congruence for Y_n, but to make the other Y_j divisible by a high power of p it is sufficient that β becomes large.
Remark: If we compare our result with Serre's in the degree one case, our result is slightly weaker: It is possible that the application of $\tilde{U}(n)^{-1}$ introduces additional powers of p in the denominator (which weakens our congruences somewhat).

§5 Further aspects

Here we shortly mention extensions of our results

- We can more generally show that modular forms for $\Gamma_0^n(p^m)$, $m \geq 1$ are p-adic.
- We can extend all results to modular forms for $\Gamma_0^n(p^m)$ with real nebentypus.

- We can also treat vector-valued modular forms (we must first modify the notion of p-adic modular form properly).

- For a modular form f we can consider the $n \times n$ matrix Df of its holomorphic derivatives. We can show that this is a (vector-valued) p-adic modular forms; this is also true for matrices of minors of derivatives. Here our proof is completely different from Serre's: We use modular forms congruent 1 mod p^m and holomorphic bilinear operators (generalized Rankin-Cohen operators as considered by Ibukiyama [7])

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