A singular perturbation problem for coupled nonlinear Schrödinger equations [1]

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1 Introduction and Main Result

In this note, we sketch the results and ideas of the forthcoming paper [16], and discuss locations of concentration points of the following singular perturbation problem for coupled nonlinear Schrödinger equations:

$$
\begin{cases}
-\varepsilon^2 \Delta u_1 + V_1(x)u_1 = \mu_1 u_1^3 + \beta u_1 u_2^2 & \text{in } \mathbf{R}^N, \\
-\varepsilon^2 \Delta u_2 + V_2(x)u_2 = \beta u_1^2 u_2 + \mu_2 u_2^3 & \text{in } \mathbf{R}^N, \\
\quad u_1(x), u_2(x) > 0 & \text{in } \mathbf{R}^N, \\
\quad u_1, u_2 \in H^1(\mathbf{R}^N).
\end{cases}
\tag{1}
$$

Here $N = 2, 3$, $\mu_1, \mu_2 > 0$, $\beta \in \mathbf{R}$ are constants, $V_1(x), V_2(x) : \mathbf{R}^N \to \mathbf{R}$ are given functions, and $\varepsilon > 0$ is a small parameter.

In [16], we will prove that (1) has a sequence of non-trivial positive solution $u_\varepsilon = (u_{\varepsilon,1}, u_{\varepsilon,2})$ such that after taking a subsequence, (u_{ε_n}) concentrates to a some point $P_0 \in \mathbf{R}$. Here we call $u = (u_1, u_2)$ a *non-trivial positive solution of (1)* if u satisfies the differential equations in (1) and $u_1(x), u_2(x) > 0$ in $\mathbf{R}^N$.

One of the difficulties to prove the existence of the above solutions of (1) is that (1) has semi-trivial positive solutions where we call $u = (u_1, u_2)$ a *semi-trivial positive solution of (1)* if u satisfies the differential equations in (1) and one of u_1 and u_2 is positive in $\mathbf{R}^N$ and the other is equal to zero identically. In fact, if $u_i(x) \in H^1(\mathbf{R}^N)$ is a positive solution of

$$
-\varepsilon^2 u_i + V_i(x)u_i = \mu_i u_i^3 \quad \text{in } \mathbf{R}^N,
\tag{2}
$$

then, $(u_1(x), 0)$ or $(0, u_2(x))$ solves the differential equations in (1).

The singular perturbation problem (1) is studied by Lin and Wei [19], Pomponio [25], Montefusco, Pellacci and Squassina [22] and Wei [27, 28]. In

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[19], they considered both of the existence and the behavior of non-trivial least energy solutions of (1). They treated (1) in both of attractive case ($\beta > 0$) and repulsive case ($\beta < 0$). In particular they showed the existence of a non-trivial least energy solution of (1) for a small $\varepsilon > 0$ under suitable conditions on the behavior of $V_1(x), V_2(x)$ as $|x| \to \infty$. Furthermore, they proved that if

$$\inf_{P \in \mathbf{R}^N} m(P) < \inf_{P \in \mathbf{R}^N} d_1(P_1) + \inf_{P \in \mathbf{R}^N} d_2(P_2)$$

(see (5), (6) and (8) for notation) holds, then after extracting a subsequence, the both components of non-trivial least energy solutions of (1) concentrate to the same point $P_0 \in \mathbf{R}^N$ where P_0 satisfies

$$m(P_0) = \inf_{P \in \mathbf{R}^N} m(P).$$

On the other hand, it also was proved in [12] that if the opposite inequality

$$\inf_{P \in \mathbf{R}^N} m(P) > \inf_{P \in \mathbf{R}^N} d_1(P_1) + \inf_{P \in \mathbf{R}^N} d_2(P_2)$$

holds, then each component of non-trivial least energy positive solutions of (1) concentrate to a different point $P_i \in \mathbf{R}^N$, where P_i satisfies

$$\inf_{P \in \mathbf{R}^N} d_1(P) = d_1(P_1), \quad \inf_{P \in \mathbf{R}^N} d_2(P) = d_2(P_2).$$

In Montefusco et al. [22], they studied the case $\beta > 0$ and considered the existence of solutions which concentrates to a local minimum point of potential functions $V_1(x), V_2(x)$. More precisely, if $V_1(x), V_2(x)$ satisfy

$$\min_{|x-z| \le r} V_i(x) < \min_{|x-z|=r} V_i(x)$$

for some $z \in \mathbf{R}^N$ and $r > 0$, then for small $\varepsilon > 0$, there exists a solution $u_\varepsilon = (u_{\varepsilon,1}, u_{\varepsilon,2})$ of (1) such that $(u_{\varepsilon,1}, u_{\varepsilon,2}) \ne (0,0)$ and $u_{\varepsilon,1}(x) + u_{\varepsilon,2}(x)$ has exactly one global maximum point in $\{x \mid |x - z| < r\}$. However, in the case where $\beta > 0$ is small, one of $u_{\varepsilon,1}$ and $u_{\varepsilon,2}$ converges to 0 as $\varepsilon \to 0$. See also Pomponio [25] and Wei [27, 28] for another cases.

The aim in [16] is to prove that when $\beta > 0$ and β is relatively small (see Assumption 1), there exists a family of non-trivial positive solutions of (1) whose components concentrate to the same point as $\varepsilon \to 0$ without assuming the behavior of potentials $V_1(x), V_2(x)$ as $|x| \to \infty$.

To state our main result, we need some preparations. In the study of (1), the following constant coefficient system plays an important role:

$$\begin{cases} -\Delta u_1 + V_1 u_1 = \mu_1 u_1^3 + \beta u_1 u_2^2 & \text{in } \mathbf{R}^N, \\ -\Delta u_2 + V_2 u_2 = \beta u_1^2 u_2 + \mu_2 u_2^3 & \text{in } \mathbf{R}^N, \\ u_1(x), u_2(x) > 0 & \text{in } \mathbf{R}^N, \\ u_1, u_2 \in H^1(\mathbf{R}^N). \end{cases} \tag{3}$$

Here V_1, V_2 are positive constants. We remark that the system (3) appears as a limit problem of (1) after a suitable rescaling. The system (3) has been studied recently by many authors. See [2, 4, 5, 10, 12, 15, 18, 21, 26, 29, 30]. In the above articles, they considered both of cases $\beta > 0$ and $\beta < 0$, and proved the existence of non-trivial positive solutions of (3). Here the size of β plays an important role. For more precise statements, see the above articles and references therein.

In this note, we treat the case where β is positive and relatively small. Solutions of (3) is characterized as critical points of the following functional $J_{V_1,V_2} : H_r^1(\mathbf{R}^N) \times H_r^1(\mathbf{R}^N) = H_r \to \mathbf{R}$:

$$\begin{aligned} J_{V_1,V_2}(u_1, u_2) =& \frac{1}{2} \int_{\mathbf{R}^N} |\nabla u_1|^2 + V_1 u_1^2 + |\nabla u_2|^2 + V_2 u_2^2 dx \\ & - \frac{1}{4} \int_{\mathbf{R}^N} \mu_1 u_1^4 + 2\beta u_1^2 u_2^2 + \mu_2 u_2^4 dx, \end{aligned} \tag{4}$$

where

$$H_r^1(\mathbf{R}^N) = \{ \varphi_1 \in H(\mathbf{R}^N) \mid \varphi_1(x) = \varphi_1(|x|) \}.$$

To find a critical point of J_{V_1,V_2}, we consider the following minimizing problem: (See Lin–Wei [18] and Sirakov [26])

$$b(V_1, V_2) = \inf_{u \in \mathcal{M}_r(V_1,V_2)} J_{V_1,V_2}(u), \tag{5}$$

where

$$\begin{aligned} \mathcal{M}_r(V_1, V_2) = \{ & u = (u_1, u_2) \in H_r \mid u_1, u_2 \neq 0, \\ & J'_{V_1,V_2}(u)(u_1, 0) = J'_{V_1,V_2}(u)(0, u_2) = 0 \}. \end{aligned}$$

Under the condition $\beta < \sqrt{\mu_1 \mu_2}$, we can prove that if a minimizer exists, then the minimizer is a critical point of J_{V_1,V_2}. Moreover, without loss of generality we can assume that the minimizer is a non-trivial positive solution of (3). In [18] and [26], they considered when $b(V_1, V_2)$ is attained and proved the existence of non-trivial positive solution of (3).

Remark 1.1. Under the condition $\beta > 0$, by the result of Busca and Sirakov [6], any non-trivial positive solution of (3) is a radially symmetric with respect to some point $P_0 \in \mathbf{R}^N$.

To state our main result, we suppose the following conditions.

Assumption 1 There exists a set $A = [\ell_{11}, \ell_{12}] \times [\ell_{21}, \ell_{22}] \subset \mathbf{R}^2$ such that
(i) For any $(\lambda_1, \lambda_2) \in A$, the following conditions are satisfied:

$$\int_{\mathbf{R}^N} |\nabla \varphi_1|^2 + (\lambda_1 - \beta \omega_{\lambda_2}^2)\varphi_1^2 dx, \quad \int_{\mathbf{R}^N} |\nabla \varphi_1|^2 + (\lambda_2 - \beta \omega_{\lambda_1}^2)\varphi_1^2 dx > 0$$

for all $\varphi_1 \in H_r^1(\mathbf{R}^N)\backslash\{0\}$. Here $\omega_{\lambda_i}(x) \in H_r^1(\mathbf{R}^N)$ is a unique positive radially symmetric solution of

$$-\Delta w_i + \lambda_i w_i = \mu_i w_i^3 \quad \text{in } \mathbf{R}^N.$$

(ii) $(V_1(P), V_2(P)) \in A$ for all $P \in \mathbf{R}^N$.

Remark 1.2. The condition (i) was introduced in Ambrosetti and Colorado [2]. They showed the existence of non-trivial positive solution of (3) under the condition (i) using the mountain pass theorem on the Nehari manifold.

We define the function $m(P) : \mathbf{R}^N \to \mathbf{R}$ as follows:

$$m(P) = b(V_1(P), V_2(P)). \tag{6}$$

As the second assumption, we assume

Assumption 2 There exists a bounded open set $\Lambda \subset \mathbf{R}^N$ such that

$$\inf_{P \in \Lambda} m(P) < \inf_{P \in \partial\Lambda} m(P). \tag{7}$$

We set

$$m_0 = \inf_{P \in \Lambda} m(P), \quad K = \{P \in \Lambda \mid m(P) = m_0\}.$$

Now we can state our main result.

Theorem 1.3 ([16]). *Suppose that Assumptions 1 and 2 hold, $0 < \beta < \sqrt{\mu_1\mu_2}$ and $V_1(x), V_2(x) \in C^1(\mathbf{R}^N)$. Then there exists an $\varepsilon_0 > 0$ such that (1) possesses a family of non-trivial positive solutions $((u_\varepsilon))_{\varepsilon\in(0,\varepsilon_0]} = ((u_{\varepsilon,1}, u_{\varepsilon,2}))_{\varepsilon\in(0,\varepsilon_0]}$ which satisfies the following properties:*
after taking a subsequence $\varepsilon_n \to 0$, there exists a sequence $(P_n) \subset \Lambda$, $P_0 \in K$ and $\omega(x) = (\omega_1(x), \omega_2(x))$ such that

$$P_{\varepsilon_n} \to P_0, \quad u_{\varepsilon,i}(\varepsilon_n x + P_{\varepsilon_n}) \to \omega_i(x) \quad \text{strongly in } H^1(\mathbf{R}^N).$$

Here $\omega(x) = (\omega_1(x), \omega_2(x))$ is a non-trivial positive solution of the limit problem

$$\begin{cases} -\Delta u_1 + V_1(P_0)u_1 = \mu_1 u_1^3 + \beta u_1 u_2^2 & \text{in } \mathbf{R}^N, \\ -\Delta u_2 + V_2(P_0)u_2 = \beta u_1^2 u_2 + \mu_2 u_2^3 & \text{in } \mathbf{R}^N, \end{cases}$$

and satisfies

$$J_{V_1(P_0),V_2(P_0)}(\omega) = m_0 = b(V_1(P_0), V_2(P_0)).$$

Remark 1.4. (i) In Theorem 1.3, we do not suppose conditions on $V_1(x), V_2(x)$ as $|x| \to \infty$.
(ii) We note that $b(V_1, V_2)$ and $m(P)$ depend on $\beta > 0$. As $\beta \to 0$, we can prove that

$$b(V_1, V_2) \to \left(\frac{V_1^{(4-N)/2}}{\mu_1} + \frac{V_2^{(4-N)/2}}{\mu_2} \right) d_0 \quad \text{uniformly w.r.t. } (V_1, V_2) \in A.$$

Here

$$d_0 = \int_{\mathbf{R}^N} \frac{1}{2}(|\nabla\omega_0|^2 + \omega_0^2) - \frac{1}{4}\omega_0^4 dx$$

and $\omega_0(x) \in H_r^1(\mathbf{R}^N)$ is a unique radially symmetric positive solution of $-\Delta w + w = w^3$ in $\mathbf{R}^N$. So, we can conclude that

$$m(P) \to d_1(P) + d_2(P) \quad \text{in } L_{\text{loc}}^\infty(\mathbf{R}^N),$$

where

$$d_1(P) = \frac{V_1^{(4-N)/2}}{\mu_1} d_0, \quad d_2(P) = \frac{V_2^{(4-N)/2}}{\mu_2} d_0. \tag{8}$$

Thus if there exists a bounded open set $\Lambda \subset \mathbf{R}^N$ such that

$$\inf_{P\in\Lambda}(d_1(P) + d_2(P)) < \inf_{P\in\partial\Lambda}(d_1(P) + d_2(P))$$

holds, then Assumption 2 is satisfied for sufficiently small $\beta > 0$. About another cases in which Assumption 2 is satisfied, see Remark 2.4.

An another aim of this note is to discuss locations of concentrating points of (1). The following result which has been proved by Wang [31] shows that the positivity of β is crucial in Theorem 1.3. (See also Wang and Zeng [32])

Proposition 1.5. *Suppose that $V_1, V_2 \in C^1(\mathbf{R}^N)$ satisfies Assumption 1 and there exist $C, \alpha > 0$ such that*

$$|\nabla V_1(x)| + |\nabla V_2(x)| \leq C(|x|^\alpha + 1) \quad \text{for all } x \in \mathbf{R}^N.$$

Let (u_ε) be a family of non-trivial positive solution of (1) with $\beta = 0$ and there exist $\varepsilon_n \to 0$ and $(P_{n,i}) \subset \mathbf{R}^N$ $(i = 1, 2)$ such that

$$P_{n,i} \to P_{0,i} \in \mathbf{R}^N, \quad u_{\varepsilon,i}(\varepsilon_n x + P_{n,i}) \to \omega_i(x) \not\equiv 0 \quad \text{stronly in } H^1(\mathbf{R}^N),$$

then

$$\nabla V_1(P_{0,1}) = 0, \quad \nabla V_2(P_{0,2}) = 0$$

holds.

By Proposition 1.5, if there is no point $P \in \mathbf{R}^N$ such that $\nabla V_1(P) = V_2(P) = 0$, then there does not exist a family of non-trivial positive solution which has a property in Theorem 1.3 even if Assumption 2 is satisfied. Thus we can conclude that Theorem 1.3 holds due to the positivity of β.

We discuss the differences of locations of concentrating points between the single nonlinear Schrödinger equations and our system (1) in section 3.

In section 2, we sketch the proof of Theorem 1.3. In order to obtain the solutions of (1), we use the ideas of Byeon and Jeanjean [7]. For more details, see the forthcoming paper [16]. In section 3, we prove the result which is related to Proposition 1.5 and discuss locations of concentration points.

2   Sketch of Proof of Theorem 1.3

In this section, we sketch a proof of Theorem 1.3. Before proving Theorem 1.3, we prepare some notation.

We set $H = H^1(\mathbf{R}^N) \times H^1(\mathbf{R}^N)$ and $H_r = H_r^1(\mathbf{R}^N) \times H_r^1(\mathbf{R}^N)$. Next, we define a norm on H as follows: for $\varphi = (\varphi_1, \varphi_2)$ and an open set $D \subset \mathbf{R}^N$,

$$\|\varphi_1\|_{H^1}^2 := \int_{\mathbf{R}^N} |\nabla \varphi_1|^2 + \varphi_1^2 dx, \quad \|\varphi_1\|_{H^1(D)}^2 := \int_D |\nabla \varphi_1|^2 + \varphi_1^2 dx,$$

$$\|\varphi\|_H^2 := \|\varphi_1\|_{H^1}^2 + \|\varphi_2\|_{H^1}^2, \quad \|\varphi\|_{H(D)}^2 := \|\varphi_1\|_{H^1(D)}^2 + \|\varphi_2\|_{H^1(D)}^2.$$

2.1 Properties of non-trivial positive solutions of (3)

In this subsection, we state the properties which non-trivial positive solutions of (3) satisfy. We begin with the following definition.

Definition 2.1. We define the sets $\mathcal{S}_{V_1,V_2} \subset H_r$, $\mathcal{S}_A \subset \mathbf{R}^2 \times H_r$ and $\mathcal{S}_K \subset \mathbf{R}^N \times H_r$ as follows:

$$\mathcal{S}_{V_1,V_2} = \{ \ \omega = (\omega_1, \omega_2) \in H_r \mid \omega_1, \omega_2 > 0, \ J'_{V_1,V_2}(\omega) = 0, \ J_{V_1,V_2}(\omega) = b(V_1, V_2) \},$$
$$\mathcal{S}_A = \{ \ (V_1, V_2, \omega) \in \mathbf{R}^2 \times H_r \mid (V_1, V_2) \in A, \ \omega \in \mathcal{S}_{V_1,V_2} \},$$
$$\mathcal{S}_K = \{ \ (P, \omega) \in \Lambda \times H_r \mid \omega_1, \omega_2 > 0, \ J'_{V_1(P),V_2(P)}(\omega) = 0,$$
$$J_{V_1(P),V_2(P)}(\omega) = m(P), \ P \in K \}.$$

$\mathcal{S}_A$ has nice properties.

Proposition 2.2. *(i) There exist* $C_1, C_2, C_3, C_4 > 0$ *such that for all* $(V_1, V_2, \omega) \in \mathcal{S}_A$, *it follows that*

$$C_1 \leq \|\omega_1\|_{H^1} \leq C_2, \quad C_1 \leq \|\omega_2\|_{H^1} \leq C_2,$$
$$|\omega(x)| + |\nabla \omega(x)| \leq C_3 \exp(-C_4|x|) \quad \text{for all } x \in \mathbf{R}^N.$$

(ii) $\mathcal{S}_A$ *is compact in* $\mathbf{R}^2 \times H_r$. *Namely, for any sequence* $(V_{n,1}, V_{n,2}, \omega_n) \subset \mathcal{S}_A$ *there exist subsequence* $(V_{n_j,1}, V_{n_j,2}, \omega_{n_j})$ *and* $(V_{0,1}, V_{0,2}, \omega_0) \in \mathcal{S}_A$ *such that*

$$(V_{n_j,1}, V_{n_j,2}) \to (V_{0,1}, V_{0,2}), \quad \omega_{n_j} \to \omega_0 \text{ strongly in } H_r.$$

Using the properties in Proposition 2.2, we can prove the following lemma.

Lemma 2.3. *(i)* $b(V_1, V_2) : A \to \mathbf{R}$ *is continuous.*
(ii) $b(V_1, V_2)$ *is strictly increasing in* V_1 *(resp.* V_2*) for fixed* V_2 *(resp.* V_1*). In particular, if* $V_{11} < V_{12}$ *or* $V_{21} < V_{22}$ *holds, then it follows that*

$$b(V_{11}, V_{21}) < b(V_{12}, V_{22}).$$

Remark 2.4. From Lemma 2.3, the function

$$m(P) = b(V_1(P), V_2(P))$$

is continuous on $\mathbf{R}^N$ and $\mathcal{S}_K$ is compact. Thus we see that if there exist a bounded open set $\tilde{\Lambda} \subset \mathbf{R}^N$ and a point $P_0 \in \tilde{\Lambda}$ such that

$$V_1(P_0) < \inf_{P \in \partial\tilde{\Lambda}} V_1(P), \quad V_2(P_0) < \inf_{P \in \partial\tilde{\Lambda}} V_2(P),$$

then, $m(P)$ satisfies Assumption 2 with $\Lambda = \tilde{\Lambda}$ for all $\beta > 0$ which satisfies Assumption 1.

Finally, we give some remarks about the minimizing problem (5). In order to obtain non-trivial positive solution of (3), we consider the minimizing problem (5). On the other hand, there is an another minimizing problem:

$$\tilde{b}(V_1, V_2) = \inf_{u \in \mathcal{M}(V_1, V_2)} J_{V_1, V_2}(u) \tag{9}$$

where

$$\mathcal{M}(V_1, V_2) = \{ \ u = (u_1, u_2) \in H \mid u_1, u_2 \not\equiv 0,$$
$$J'_{V_1, V_2}(u)(u_1, 0) = J'_{V_1, V_2}(u)(0, u_2) = 0\}.$$

Under the condition $0 < \beta < \sqrt{\mu_1 \mu_2}$, if there exists a minimizer of the problem (9), the minimizer is a non-trivial positive solution of (3). It is easy to see that the inequality $\tilde{b}(V_1, V_2) \leq b(V_1, V_2)$ holds. Furthermore, we can prove that the opposite inequality holds.

Lemma 2.5. *Under the assumptions in Theorem 1.3, the equality* $\tilde{b}(V_1, V_2) = b(V_1, V_2)$ *holds.*

2.2 Nehari type manifold and the Palais–Smale condition

In this subsection, we consider the following problem:

$$
\begin{cases}
-\Delta u_1 + V_1(\varepsilon x)u_1 = \mu_1 u_1^3 + \beta u_1 u_2^2 & \text{in } \mathbf{R}^N, \\
-\Delta u_2 + V_2(\varepsilon x)u_2 = \beta u_1^2 u_2 + \mu_2 u_2^3 & \text{in } \mathbf{R}^N, \\
u_1(x), u_2(x) > 0 & \text{in } \mathbf{R}^N, \\
u_1, u_2 \in H^1(\mathbf{R}^N).
\end{cases}
\tag{10}
$$

The system (10) appears when we consider a rescale function $u(x) = v(\varepsilon x)$. To treat the existence of non-trivial positive solution of (10), we define the following functional $I_\varepsilon : H \to \mathbf{R}$,

$$I_\varepsilon(u) = \frac{1}{2} \int_{\mathbf{R}^N} |\nabla u_1|^2 + V_1(\varepsilon x)u_1^2 + |\nabla u_2|^2 + V_2(\varepsilon x)u_2^2 dx$$
$$- \frac{1}{4} \int_{\mathbf{R}^N} \mu_1 u_1^4 + 2\beta u_1^2 u_2^2 + \mu_2 u_2^4 dx.$$

A solution of (10) is characterized as a critical point of I_ε. In order to obtain non-trivia positive solutions of (10), we consider the following Nehari type manifold $\mathcal{M}_\varepsilon$:

$$\mathcal{M}_\varepsilon = \{ \ u \in H \mid u_1, u_2 \not\equiv 0, \ I'_\varepsilon(u)(u_1, 0) = I'_\varepsilon(u)(0, u_2) = 0\}.$$

Under the condition $0 < \beta < \sqrt{\mu_1\mu_2}$, a critical point of I_ε is equivalent to a critical point of $I_\varepsilon|_{\mathcal{M}_\varepsilon}$. So, we will find a critical point of $I_\varepsilon|_{\mathcal{M}_\varepsilon}$.

One of difficulties in finding a critical point of $I_\varepsilon|_{\mathcal{M}_\varepsilon}$ is that the functional I_ε does not satisfy the Palais–Smale condition. Here we say the functional I_ε satisfies the Palais–Smale condition if for any sequence $(u_n) \subset \mathcal{M}_\varepsilon$ which satisfies

$$I_\varepsilon(u_n) \to c, \quad I_\varepsilon'(u_n) \to 0 \text{ in } H^*,$$

then (u_n) has a strongly convergent subsequence.

Following the ideas in Byeon and Jeanjean [7], we consider the following sets. For $\varepsilon, \delta, d > 0$, we set

$$\mathcal{S}_{\varepsilon,\delta} = \{ \omega_P(x - \tilde{P}/\varepsilon) \mid P \in K, |P - \tilde{P}| \leq \delta, \omega_P \in \mathcal{S}_K \},$$

$$\mathcal{S}_{\varepsilon,\delta}^d = \{\omega_P(x - \tilde{P}/\varepsilon) + \varphi(x) \mid \omega_P(x - \tilde{P}/\varepsilon) \in \mathcal{S}_{\varepsilon,\delta}, \; \varphi \in H, \; \|\varphi\|_H < d\}.$$

Remark 2.6. The set type of $\mathcal{S}_{\varepsilon,\delta}^d$ is introduced in Byeon and Jeanjean [7] to study the singular perturbation problem for single nonlinear Schrödinger equations with a general nonlinearity. They proved the existence of solutions which concentrate to some point without using the uniqueness and nondegeneracy of the limit problem.

To prove Theorem 1.3, we consider the existence of local minimum point of I_ε on $\mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta}^d$, i.e., we consider the following minimizing problem:

$$m_\varepsilon = \inf_{u \in \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta}^d} I_\varepsilon(u). \tag{11}$$

The set $\mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta}^d$ and the functional I_ε have the following properties.

Proposition 2.7. *There exist $d_1, \delta_1, \varepsilon_1 > 0$ such that for all $\varepsilon \in (0, \varepsilon_1)$, if $(u_j) \subset \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta_1}^{d_1}$ satisfies*

$$I_\varepsilon'(u_j) \to c, \quad I_\varepsilon'(u_j) \to 0 \text{ in } H^*,$$

then (u_j) has a strongly convergent subsequence in H and its limit u_0 is a critical point of I_ε and $I_\varepsilon(u_0) = c$.

Proposition 2.8. *There exist $d_2, \delta_2 > 0$ such that if a sequence $(\varepsilon_j), \varepsilon_j \to 0$ and $(u_j) \subset H$ which satisfy*

$$u_j \in \mathcal{M}_{\varepsilon_j} \cap \mathcal{S}_{\varepsilon_j,\delta_2}^{d_2}, \quad I_{\varepsilon_j}(u_j) \to c_0 \leq m_0, \quad I_{\varepsilon_j}'(u_j) \to 0 \quad \text{in } H^*,$$

then $c_0 = m_0$ holds and there exist a subsequence (we still denote (j)), $(P_j) \subset \mathbf{R}^N$, $P_0 \in K$ and $\omega_0 \in \mathcal{S}_{V_1(P_0),V_2(P_0)}$ such that

$$P_j \to P_0, \quad u_j(x + P_j/\varepsilon_j) \to \omega_0 \quad \text{strongly in } H.$$

Proposition 2.7 states that if d, δ, ε are sufficiently small, then a Palais-Smale sequence in $\mathcal{M}_\varepsilon \cap \mathcal{S}^d_{\varepsilon,\delta}$ has a strongly convergent subsequence. On the other hand Proposition 2.8 states the behavior of non-trivial solutions of (10). From Proposition 2.8, we can see that if we find a critical point u_ε in $\mathcal{M}_\varepsilon \cap \mathcal{S}^d_{\varepsilon,\delta}$ for each small ε, then both components of (u_ε) concentrate to the same point $P_0 \in \mathbf{R}^N$.

Thus in order to prove Theorem 1.3, we shall show that the minimizing problem (11) has a minimizer for each small $\varepsilon > 0$.

2.3 Solvability of (11) and Proof of Theorem 1.3

In this subsection, we prove that the minimizing problem is solvable for a small $\varepsilon > 0$ and complete a proof of Theorem 1.3.

To ensure the existence of minimizers for (11), the following proposition is a key.

Proposition 2.9. *There exist $d_3, \delta_3, \varepsilon_2 > 0$ which satisfies the following. For any $\nu > 0$, there exists an $R_\nu > 0$ such that for any $u \in \mathcal{M}_\varepsilon \cap \mathcal{S}^d_{\varepsilon,\delta}$, there exists a $v \in \mathcal{M}_\varepsilon \cap \mathcal{S}^d_{\varepsilon,\delta}$ which satisfies*

$$I_\varepsilon(v) \leq I_\varepsilon(u), \quad \|v\|_{H(|x-P/\varepsilon|>R_\nu)} \leq \nu$$

for all $d \in (0, d_3]$, $\delta \in (0, \delta_3]$, $\varepsilon \in (0, \varepsilon_2)$. Here P satisfies

$$u(x) = \omega_P(x - \tilde{P}/\varepsilon) + \varphi(x), \quad P \in K, \quad \omega_P \in \mathcal{S}_{V_1(P),V_2(P)}.$$

Let $d_0 = \min\{d_1/2, d_2/2, d_3/2\}$ and $\delta_0 = \min\{\delta_1, \delta_2, \delta_3\}$. By the aid of Proposition 2.9, we can prove the following:

Lemma 2.10. *For any $d \in (0, d_0]$ and $\delta \in (0, \delta_0]$, $\liminf_{\varepsilon \to 0} m_\varepsilon = m_0$.*

Now we prove that the minimizing problem (11) has a minimizer for small $\varepsilon > 0$.

Proposition 2.11. *(i) There exists a $\rho_0 > 0$ such that*

$$\liminf_{\varepsilon \to 0} \inf_{u \in \mathcal{M}_\varepsilon \cap \mathcal{S}^{d_0}_{\varepsilon,\delta_0} \setminus \mathcal{S}^{d_0/2}_{\varepsilon,\delta_0}} I_\varepsilon(u) \geq m_0 + \rho_0.$$

(ii) There exists an $\varepsilon_3 > 0$ such that for each $\varepsilon \in (0, \varepsilon_3)$, there exist $u_\varepsilon \in \mathcal{M}_\varepsilon \cap \mathcal{S}^{d_0}_{\varepsilon,\delta_0}$ which satisfies

$$I_\varepsilon(u_\varepsilon) = \inf_{u \in \mathcal{M}_\varepsilon \cap \mathcal{S}^{d_0}_{\varepsilon,\delta_0}} I_\varepsilon(u).$$

Proof. (i) We argue indirectly and assume that there exist (ε_j) and $(u_j) \in \mathcal{M}_{\varepsilon_j} \cap \mathcal{S}_{\varepsilon_j,\delta_0}^{d_0} \backslash \mathcal{S}_{\varepsilon_j,\delta_0}^{d_0/2}$ such that

$$I_{\varepsilon_j}(u_j) \to m_0.$$

By Ekeland's variational principle and Lemma 2.10, there exist a sequence (v_j) such that

$$v_j \in \mathcal{M}_{\varepsilon_j} \cap \mathcal{S}_{\varepsilon_j,\delta_0}^{2d_0} \backslash \mathcal{S}_{\varepsilon_j,\delta_0}^{d_0/4}, \quad I_{\varepsilon_j}(v_j) \le I_{\varepsilon_j}(u_j), \quad I_{\varepsilon_j}'(v_j) \to 0 \text{ in } H^*.$$

By Proposition 2.8, there exist a subsequence (we still denote (j)) (ε_j), $(P_j) \subset \mathbf{R}^N$, $P_0 \in K$ and $\omega_0 \in \mathcal{S}_{V_1(P_0),V_2(P_0)}$ such that

$$P_j \to P_0, \quad v_j(x + P_j/\varepsilon_j) \to \omega_0 \quad \text{strongly in } H.$$

Thus (v_j) satisfies

$$\|v_j(x) - \omega_0(x - P_j/\varepsilon_j)\|_H \to 0.$$

Therefore $v_j \in \mathcal{S}_{\varepsilon_j,\delta_0}^{d_0/4}$ for sufficiently large j. However, this is a contradiction, which implies that there exists a $\rho_0 > 0$ such that

$$\liminf_{\varepsilon \to 0} \inf_{u \in \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta_0}^{d_0} \backslash \mathcal{S}_{\varepsilon,\delta_0}^{d_0/2}} I_\varepsilon(u) \ge m_0 + \rho_0.$$

(ii) By (i), there exists an $\varepsilon_3 > 0$ such that if $0 < \varepsilon < \varepsilon_3$, then

$$\inf_{u \in \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta_0}^{d_0} \backslash \mathcal{S}_{\varepsilon,\delta_0}^{d_0/2}} I_\varepsilon(u) \ge m_0 + \rho_0. \tag{12}$$

On the other hand, by Lemma 2.10, if ε_3 is sufficiently small, we can show that if $0 < \varepsilon < \varepsilon_3$, then $m_\varepsilon < m_0 + \rho_0$.

Let $0 < \varepsilon < \varepsilon_3$ and $(u_n) \subset \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta_0}^{d_0}$ be a minimizing sequence:

$$I_\varepsilon(u_n) \to \inf_{u \in \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta_0}^{d_0}} I_\varepsilon(u).$$

From (12), we can suppose $(u_n) \subset \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta_0}^{d_0/2}$. By Ekeland's variational principle, there exists a sequence $(v_n) \subset \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta_0}^{d_0}$ such that

$$I_\varepsilon(v_n) \to \inf_{u \in \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta_0}^{d_0}} I_\varepsilon(u), \quad I_\varepsilon'(v_n) \to 0 \quad \text{in } H^*.$$

By Proposition 2.7, (v_n) has a strongly convergent subsequence (v_n) and there exists a critical point $v_0 \in \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta_0}^{d_0}$ and v_0 satisfies

$$I_\varepsilon(v_0) = \inf_{u \in \mathcal{M}_\varepsilon \cap \mathcal{S}_{\varepsilon,\delta_0}^{d_0}} I_\varepsilon(u).$$

This completes the proof. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Proof of Theorem 1.3. From Propositions 2.8 and 2.11, there exist a family of non-trivial solutions (u_ε). Furthermore for any sequence (ε_n), there exist a subsequence (ε_j) (we still denote (j)), $(P_j) \subset \mathbf{R}^N$, $P_0 \in \mathbf{R}^N$, $\omega_0 \in \mathcal{S}_{V_1(P_0),V_2(P_0)}$ such that

$$P_j \to P_0, \quad u_{\varepsilon_j} \to \omega_0 \quad \text{strongly in } H.$$

Thus $v_\varepsilon(x) = u_\varepsilon((x + P_j)/\varepsilon_j)$ is a non-trivial solution of the equations of (1) and after taking a subsequence, both components of (v_ε) concentrate to the same point $P_0 \in K$. Therefore we only prove that (u_ε) is a positive function if $\varepsilon > 0$ is sufficiently small.

Now we prove the following proposition and this completes the proof of Theorem 1.3.

Proposition 2.12. *There exists an $\varepsilon_4 > 0$ such that if $0 < \varepsilon < \varepsilon_4$, then both of components of u_ε are positive.*

Proof. We prove indirectly and assume that there exist (ε_n) and (u_n) such that $\varepsilon_n \to 0$ and $\|u_{n,1}^-\|_{L^2} > 0$, where $u_{n,1}^-(x) = \min\{0, u_{n,1}(x)\}$. Moreover we can assume that

$$u_n(x) = \omega_n(x - P_n/\varepsilon_n) + \varphi_n(x), \quad P_n \to P_0,$$
$$\omega_n(x) \to \omega_0(x), \quad \varphi_n \to 0 \quad \text{strongly in } H$$

where

$$P_0 \in K, \quad \omega_n(x - P_n/\varepsilon_n) \in \mathcal{S}_{\varepsilon_n,\delta_0}, \quad \omega_0 \in \mathcal{S}_{V_1(P_0),V_2(P_0)}.$$

Let $\tilde{u}_n(x) = u_n(x + P_n/\varepsilon)$. Then $\tilde{u}_n$ satisfies

$$\begin{cases} -\Delta \tilde{u}_{n,1} + V_1(\varepsilon_n x + P_n)\tilde{u}_{n,1} = \mu_1 \tilde{u}_{n,1}^3 + \beta \tilde{u}_{n,1}\tilde{u}_{n,2} \\ -\Delta \tilde{u}_{n,2} + V_2(\varepsilon_n x + P_n)\tilde{u}_{n,2} = \beta \tilde{u}_{n,1}^2 \tilde{u}_{n,2} + \mu_2 \tilde{u}_{n,2}^3 \end{cases} \tag{13}$$

and $\tilde{u}_{n,i} \to \omega_{0,i}$ in H. Thus by the elliptic regularity theory, we have

$$\tilde{u}_{n,i} \to \omega_{0,i} \quad \text{in } C^2_{\text{loc}}(\mathbf{R}^N)$$

and there exists $C_5, C_6 > 0$ such that for all $x \in \mathbf{R}^N$ and n, it follows that

$$|\tilde{u}_{n,1}(x)| + |\tilde{u}_{n,2}(x)| + |\nabla \tilde{u}_{n,1}(x)| + |\nabla \tilde{u}_{n,2}(x)| \leq C_5 \exp(-C_6|x|)$$

Let $R_0 > 0$. We take an $n_{R_0} \in \mathbf{N}$ such that if $n \geq n_{R_0}$, then $\tilde{u}_{n,1}(x) > 0$ on $|x| \leq R_0$. We multiply the fist equation in (13) by $\tilde{u}_{n,1}^-$ and integrate on

$\{|x| \geq R_0\}$. Then there exists an $\alpha > 0$ which depends only on $V_1(x)$ such that

$$\|\nabla \tilde{u}_{n,1}^-\|_{L^2(|x|\geq R_0)}^2 + \alpha \|\tilde{u}_{n,1}^-\|_{L^2(|x|\geq R_0)}^2$$

$$\leq \|\nabla \tilde{u}_{n,1}^-\|_{L^2(|x|\geq R_0)}^2 + \int_{|x|\geq R_0} V_1(\varepsilon_n x + P_n)(\tilde{u}_{n,1}^-)^2 dx$$

$$= \mu_1 \|\tilde{u}_{n,1}^-\|_{L^4(|x|\geq R_0)}^4 + \beta \|\tilde{u}_{n,1}^- \tilde{u}_{n,2}\|_{L^2(|x|\geq R_0)}^2$$

$$\leq C_7 \exp(-2C_5 R_0)\|\tilde{u}_{n,1}^-\|_{L^2(|x|\geq R_0)}^2.$$

Thus if $R_0 > 0$ and n_{R_0} are sufficiently large, we can conclude that if $n \geq n_{R_0}$, then $\tilde{u}_{n,1}^- \equiv 0 \equiv u_{n,1}^-$. However, this contradicts to $\|u_{n,1}^-\|_{L^2} > 0$, which completes the proof. $\square$

By Proposition 2.12, we can prove the positivity of the minimizer (u_ε). Thus we complete the proof of Theorem 1.3. $\square$

3 Observation of concentrating points

In this section, we discuss points to which non-trivial solution of (1) concentrate. We begin with the following definition.

Definition 3.1. Let (u_ε) be a family of solution of (1). We say that $P_0 \in \mathbf{R}^N$ is a concentrating point of (u_ε) if and only if there exist a subsequence (ε_n), $(P_n) \subset \mathbf{R}^N$ and a limit function $\omega_0 = (\omega_{0,1}, \omega_{0,2})$ such that

$$P_n \to P_0, \quad u_{\varepsilon_n}(\varepsilon_n x + P_j) \to \omega_0 \quad \text{strongly in } H.$$

Here ω_0 is a non-trivial positive solution of

$$\begin{cases} -\Delta v_1 + V_1(P_0)v_1 = \mu_1 v_1^3 + \beta v_1 v_2^2 & \text{in } \mathbf{R}^N, \\ -\Delta v_2 + V_2(P_0)v_2 = \beta v_1^2 v_2 + \mu_2 v_2^3 & \text{in } \mathbf{R}^N. \end{cases}$$

Remark 3.2. A family of solution we found in Theorem 1.3 satisfies the above condition.

Now we prove the following characterization of concentrating points, which is related to Proposition 1.5.

Proposition 3.3. *Let $\beta \in \mathbf{R}$ and $P_0 \in \mathbf{R}^N$ be a concentrating point of (u_{ε_n}). Furthermore, suppose that $V_1, V_2 \in C^1(\mathbf{R}^N)$ satisfy Assumption 1 and there exist $C_8, \alpha > 0$ such that*

$$|\nabla V_1(x)| + |\nabla V_2(x)| \leq C_8(|x|^\alpha + 1) \quad \text{for all } x \in \mathbf{R}^N.$$

Then P_0 satisfies

$$\nabla V_1(P_0) \int_{\mathbf{R}^N} \omega_{0,1}^2 dx + \nabla V_2(P_0) \int_{\mathbf{R}^N} \omega_{0,2}^2 dx = 0. \tag{14}$$

Here ω_0 is a limit function of (u_{ε_n}).

Remark 3.4. The conclusion of Proposition 3.3 is still valid if $V_1(x)$, $V_2(x)$ are unbounded and satisfy some growth conditions as $|x| \to \infty$. In that case, we replace the function space H by

$$\mathcal{H} = \{ u \in H \mid \int_{\mathbf{R}^N} V_1(x) u_1^2 dx < \infty, \quad \int_{\mathbf{R}^N} V_2(x) u_2^2 dx < \infty \}.$$

Proof. We set $v_n(x) = u_{\varepsilon_n}(\varepsilon_n x + P_n)$. Then v_n satisfies

$$\begin{cases} -\Delta v_{n,1} + V_1(\varepsilon_n x + P_n) v_{n,1} = \mu_1 v_{n,1}^3 + \beta v_{n,1} v_{n,2}^2 & \text{in } \mathbf{R}^N, \\ -\Delta v_{n,2} + V_2(\varepsilon_n x + P_n) v_{n,2} = \beta v_{n,1}^2 v_{n,2} + \mu_2 v_{n,2}^3 & \text{in } \mathbf{R}^N. \end{cases} \tag{15}$$

By the assumption about V_1, V_2 and the elliptic regularity theory, we can assume that there exist $C_9, C_{10} > 0$ such that for all n and $x \in \mathbf{R}^N$, it follows

$$v_{n,1}(x) + v_{n,2}(x) \le C_9 \exp(-C_{10}|x|).$$

Moreover, we can show that $v_{n,1}$, $v_{n,2} \in H^2(\mathbf{R}^N)$. We multiply $\partial v_{n,1}/\partial x_j$ by the first equation in (15) and integrate over $\mathbf{R}^N$, we obtain

$$\begin{aligned} \int_{\mathbf{R}^N} -\Delta v_{n,1} \frac{\partial v_{n,1}}{\partial x_j} dx &+ \int_{\mathbf{R}^N} V_1(\varepsilon_n x + P_n) v_{n,1} \frac{\partial v_{n,1}}{\partial x_j} dx \\ &= \int_{\mathbf{R}^N} \mu_1 v_{n,1}^3 \frac{\partial v_{n,1}}{\partial x_j} + \beta v_{n,1} v_{n,2}^2 \frac{\partial v_{n,1}}{\partial x_j} dx. \end{aligned} \tag{16}$$

In a similar way, we can obtain

$$\begin{aligned} \int_{\mathbf{R}^N} -\Delta v_{n,2} \frac{\partial v_{n,2}}{\partial x_j} dx &+ \int_{\mathbf{R}^N} V_2(\varepsilon_n x + P_n) v_{n,2} \frac{\partial v_{n,2}}{\partial x_j} dx \\ &= \int_{\mathbf{R}^N} \beta v_{n,1}^2 v_{n,2} \frac{\partial v_{n,2}}{\partial x_j} + \mu_2 v_{n,2}^3 \frac{\partial v_{n,2}}{\partial x_j} dx. \end{aligned} \tag{17}$$

Direct calculations show that

$$-\int_{\mathbf{R}^N} \Delta v_{n,j} \frac{\partial v_{n,j}}{\partial x_j} dx = \int_{\mathbf{R}^N} \nabla v_{n,j} \cdot \nabla \left(\frac{\partial v_{n,j}}{\partial x_j} \right) dx = \frac{1}{2} \int_{\mathbf{R}^N} \frac{\partial}{\partial x_j} |\nabla v_{n,j}|^2 dx = 0. \tag{18}$$

In a similar way, it follows that

$$\int_{\mathbf{R}^N} \mu_1 v_{n,1}^3 \frac{\partial v_{n,1}}{\partial x_j} + \beta v_{n,1} v_{n,2}^2 \frac{\partial v_{n,1}}{\partial x_j} + \beta v_{n,1}^2 v_{n,2} \frac{\partial v_{n,2}}{\partial x_j} + \mu_2 v_{n,2}^3 \frac{\partial v_{n,2}}{\partial x_j} dx$$
$$= \int_{\mathbf{R}^N} \frac{\partial}{\partial x_j} \left(\frac{\mu_1}{4} v_{n,1}^4 + \frac{\beta}{2} v_{n,1}^2 v_{n,2}^2 + \frac{\mu_2}{4} v_{n,2}^4 \right) dx = 0. \tag{19}$$

By the assumption about $V_i(x)$, we have

$$\int_{\mathbf{R}^N} V_i(\varepsilon_n x + P_n) v_{n,i} \frac{\partial v_{n,i}}{\partial x_j} dx = \frac{1}{2} \int_{\mathbf{R}^N} V_i(\varepsilon_n x + P_n) \frac{\partial}{\partial x_j} v_{n,i}^2 dx$$
$$= -\frac{\varepsilon_n}{2} \int_{\mathbf{R}^N} \frac{\partial V_i}{\partial x_j} (\varepsilon_n x + P_n) v_{n,i}^2 dx. \tag{20}$$

Combining (16)–(20), we have

$$\int_{\mathbf{R}^N} \frac{\partial V_1}{\partial x_j} (\varepsilon_n x + P_n) v_{n,1}^2 + \frac{\partial V_2}{\partial x_j} (\varepsilon_n x + P_n) v_{n,2}^2 dx = 0.$$

Let $n \to \infty$, then it follows that

$$\frac{\partial V_1}{\partial x_j}(P_0) \int_{\mathbf{R}^N} \omega_{0,1}^2 dx + \frac{\partial V_2}{\partial x_j}(P_0) \int_{\mathbf{R}^N} \omega_{0,2}^2 dx = 0.$$

Thus we complete the proof. □

In particular, in the case $\beta = 0$, we have the following corollary by the above proof.

Corollary 3.5. *Let $\beta = 0$ and $P_0, V_1(x), V_2(x)$ satisfy the same assumptions in Proposition 3.3. Then P_0 satisfies*

$$\nabla V_1(P_0) = \nabla V_2(P_0) = 0.$$

Now we discuss the relationships between (14) and the function $m(P)$. At first we consider the single nonlinear Schrödinger case, that is

$$-\varepsilon^2 \Delta \varphi_i + V_i(x)\varphi_i = \mu_i^3 \varphi_i^3 \quad \text{in } \mathbf{R}^N. \tag{21}$$

In this case, the least energy level $d_i(P)$ for constant coefficients case has the following form:

$$d_i(P) = \frac{V_i(P)^{(4-N)/2}}{\mu_1} d_0 \tag{22}$$

where d_0 is the least energy level for $-\Delta \varphi_1 + \varphi_1 = \varphi_1^3$ in $\mathbf{R}^N$. From (22), we can see that $P_0 \in \mathbf{R}^N$ is a critical point of $V_i(P)$ if and only if P_0 is the

one of $d_i(P)$ and if $P_0 \in \mathbf{R}^N$ is a strictly local minimum point or maximum point of V_i, P_0 is the one of d_i.

Concerning concentration points, we can show the following: If $P_0 \in \mathbf{R}^N$ is a concentration point of (21), then $\nabla d_i(P_0) = 0$, namely it holds $\nabla V_i(P_0) = 0$. Thus in the single nonlinear Schrödinger case, any concentrating point must be a critical point of V_i.

On the other hand, if a critical point P_0 of V_i has some nondegeneracy, then we can construct a family of solution of the single nonlinear Schrödinger equation which concentrates to P_0. See Ambrosetti, Badiale and Cingolani [1], Ambrosetti and Malchiodi [3], Byeon and Jeanjean [7], Byeon, Jeanjean and Tanaka [9], del Pino and Felmer [11], Floer and Weinstein [13], Jeanjean and Tanaka [17], Oh [23, 24] and references therein.

Now, we consider system case. In the system case, the function $m(P)$ plays an important role to characterize concentration points. In fact, in the case where $\beta > 0$ is sufficiently small, then we can prove that the function $m(P)$ is of class C^1 using the arguments in [15]. Furthermore, we can show that any concentrating point P_0 satisfies

$$\nabla m(P_0) = 0$$

and $\nabla m(P_0) = 0$ is equivalent to the equation (14). In the single case, we see that a concentration point P_0 is a critical point of the potential function and characterized the location explicitly, i.e., the local maximum point or minimum point and so on. However, in the system case, we do not know the explicit formula of the function $m(P)$ with using potential functions $V_1(x), V_2(x)$. Therefore, we do not have the specific location of concentration points except for the equation (14). Furthermore, in general, we do not know the regularity of the function $m(P)$. In using the Liapunov–Schmidt reduction method, it is important that it holds the uniqueness and nondegeneracy of non-trivial positive solution of (3). However, except for special cases, we do not know the uniqueness and nondegeneracy. Thus it seems difficult to apply the Liapunov–Schmidt reduction method for obtaining the solutions of (1) which has the properties in Theorem 1.3.

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