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**Kyoto University**
Sum Rate Maximizing Superposition Coding Scheme for a Two-User Wireless Relay System

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I. INTRODUCTION

Many works in wireless relay systems have shown the efficiency of cooperative diversity, where the broadcast nature of the wireless medium is taken advantage of by combining the different received signals at the destination. Recently, [1] introduced a relaying scheme based on Superposition Coding (SC) that increases spectral efficiency, for a single user system with a Base Station (BS) and a Relay Station (RS) based on Decode-and-Forward (DF) half-duplexing. SC was first introduced in broadcast channels, where several nodes are served by the BS [2]. In the Single-User SC (SU-SC) scheme of [1], BS creates two messages (basic and superposed), both destined to the single user, and transmits them in Step 1. In Step 2, after decoding both messages, RS only forwards the superposed one, which is also used to retrieve the basic message from the signal received in Step 1. This scheme outperforms the Single-User Multi-Hop (SU-MH) scheme where the message is sent via the relayed link with optimal time division, and the cooperative DF scheme [1]. A two-user SC-based scheme was proposed in [3], where the signals of the two users are superposed but only the message to one user is forwarded.

We aim at devising new scheduling algorithms in a multi-user relay system that improve throughput. We first focus on the Downlink (DL) of a two-user relay system, or Relay Broadcast Channel (RBC), as users MS\(_1\) and MS\(_2\) are served by a BS and a RS as in Fig. 1. We assume two types of relayed users, MS\(_1\) with a direct and relayed link, and MS\(_2\) with only a relayed link. The achievable rate region of RBC has been studied in [4] for discrete memoryless channels and in [5] for Gaussian channels and half-duplex RS, where two equally divided orthogonal subchannels are required to partition BS and RS transmissions and no resource optimization is performed among them. Our goal is not to investigate the RBC capacity region, but to design allocation schemes that enhance existing schedulers. We propose the 3-SC Layer scheme, where the signals of the two users are superposed into 3 layers: 2 layers for MS\(_1\) and one for MS\(_2\).

II. SYSTEM MODEL

We consider the system in Fig. 1. In step 1, BS transmits a vector \(x\) of \(M\) complex baseband symbols \(x[m], m \in \{1, \ldots, M\}\). The received signals at RS and MS\(_1\) are \(y_R = h_R x + z_R\) and \(y_{D1} = h_{D1} x + z_1\). In step 2, RS transmits a vector \(x_R\) of \(M_R\) complex baseband symbols. The received signal at MS\(_i\), \(i \in \{1, 2\}\) is \(y_{Ri} = h_{Ri} x_R + z'_{i}\). \(h_R, h_{D1}, h_{Ri}\) are the complex channel coefficients of the BS-RS, BS-MS\(_i\), and RS-MS\(_i\) channels. \(z_R, z_1, z'_{i}\) are vectors of complex additive white Gaussian noise with a circular-symmetric distribution \(CN(0, \sigma^2 I)\). Transmitted symbols satisfy \(E\{x[m]\} = 0\) and \(E\{|x[m]|^2\} = 1\). Link SNRs are given by \(\gamma_S = \frac{|h_{Ri}|^2}{\sigma^2}, S \in \{D1, R, R1, R2\}\). The capacity of each link is given by

\[
C(\gamma_S) = \log_2(1 + \gamma_S)[\text{bits/s}]/\text{Hz}
\]

in a bandwidth of 1 Hz.

Fig. 1. System Model and Steps of the Proposed Scheme

In the analysis, we assume constant link SNRs ordered as

\[
\gamma_{D1} < \gamma_{R1} < \gamma_{R2} < \gamma_R. \tag{1}
\]

In the scenario of multiple users and random channel fading, our scheduler will select user pairs satisfying (1).

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III. PROPOSED SCHEME: 3-SC Layer Scheme

We describe the steps of the proposed scheme, while the equations for each signal are given in Table I.

**Step 1:** The BS sends message $x$ composed of 3 superposed messages: $x_{b1}$, $x_{s1}$ for MS1 and $x_{s2}$ for MS2 with power allocation ratios $\alpha_{b1}$, $\alpha_{s1}$ and $\alpha_{s2} \in [0,1]$, respectively. We refer to $x_{b1}$ as the basic message for MS1 and $x_{s1}$ as the superposed message for MS1. The sum of power allocation ratios is equal to one. Then, the RS receives $y_{1R}$ from which it decodes each message one by one, treating the other messages as noise by Successive Interference Cancelation (SIC). The decoding order follows the order of increasing link SNRs $\gamma_{D1} < \gamma_{R1} < \gamma_{R2}$ as in [6], i.e., $x_{b1} \rightarrow x_{s1} \rightarrow x_{s2}$ at RS, shown to be very effective by our numerical results. On the other hand, MS1 receives $y_{D1}$ and keeps it in memory.

**Step 2:** RS sends message $x_{R}$ which superposes $x_{R1}$ and $x_{R2}$, the decoded and remodulated signals of $x_{s1}$ and $x_{s2}$, respectively, with the power allocation ratio $\beta \in [0,1]$. Then, MS1 decodes $y_{R1}$ from which it decodes $x_{R1}$ (as noise), treating $\sqrt{\beta}x_{R2}$ as noise (as $\gamma_{R1} < \gamma_{R2}$). From $y_{D1}$ received in Step 1, MS1 cancels $x_{s1}$, giving $y'_{D1}$. Finally, MS1 decodes $x_{b1}$ with noise $\sqrt{\alpha_{s2}}x_2$. In the same way, MS2 receives $y_{R2}$, from which it decodes $x_{R1}$. Canceling $x_{R1}$ from $y_{R2}$, MS2 gets $y'_{R2}$, from which $x_{s2}(x_{s2})$ is decoded.

We denote by $R_{b1}$, $R_{s1}$ the rates of the basic and superposed messages for MS1; by $R_{R1}, i = \{1,2\}$, the rate of the relayed messages $x_{R1}$. The constraints on these rates are given in Table II. In Step 1, the constraints ensure that RS can decode $x_{b1}$, $x_{s1}$, $x_{s2}$, respectively. In Step 2, the first two constraints ensure that MS1 decodes $x_{R1}$ and $x_{s1}$, and the first and last constraints ensure that MS2 decodes $x_{R1}$ (as $\gamma_{R1} < \gamma_{R2}$) and $x_{R2}$. We define $R_{R1}^A$, $R_{R1}^B$ to denote the two constraints on $R_{R1}$ to be satisfied, so $R_{R1} = \min(R_{R1}^A, R_{R1}^B)$. All the other rates are equal to their capacity expressions in Table II.

BS transmits $M(R_{b1} + R_{s1} + R_{R2})$ bits in Step 1. In Step 2, RS forwards $M(R_{R1} + R_{R2})$ bits. The transmission time $M_R$ at Step 2 is the larger one between the times to MS1 and MS2, i.e., $M_R = M_{11}$ with $I_1 = R_{R1}^B$, and $M_{22} = M_{12}$ with $I_2 = R_{R2}$. With the constraints in Table II, the sum rate becomes

$$R_{3L} = \frac{\min(R_{b1}^A, R_{R1}^B) + R_{s1} + R_{R2}}{1 + \max(I_1, I_2)}.$$  

(2)

Next, we determine the power allocation ratios that maximize the sum rate. Defining $\alpha_{s2}^* = \frac{1}{\gamma_{R2}} - \frac{1}{\gamma_{S2}}$, we distinguish two cases (we assume $\alpha_{s2}^* \leq 1$; otherwise multi-hop transmission may be used for MS1 as in [1]).

**Case 1:** $R_{b1}^A \leq R_{R1}^B \Leftrightarrow \alpha_{s1} \geq \alpha_{s2}^*$, we have $R_{3L}$ =

in Table I.

**TABLE I**

| Step 1 | BS sends $x = \sqrt{\alpha_{b1}}x_{b1} + \sqrt{\alpha_{s1}}x_{s1} + \sqrt{\alpha_{s2}}x_{s2}$ with $\alpha_{b1} + \alpha_{s1} + \alpha_{s2} = 1$. RS receives $y_{R} = R_{(1)}(\sqrt{\alpha_{s1}}x_{b1} + \sqrt{\alpha_{s1}}x_{s1} + \sqrt{\alpha_{s2}}x_{s2}) + x_{b1}$.
| Step 2 | RS sends $x_{R} = \sqrt{\beta}x_{R1} + \sqrt{\beta}x_{R2}$ with $\beta \in [0,1]$. MS1 decodes $y_{R1} = h_{R_1}(\sqrt{\beta}x_{R1} + \sqrt{\beta}x_{R2}) + x_{b1}$, $y_{R2} = h_{R_2}(\sqrt{\beta}x_{R1} + \sqrt{\beta}x_{R2}) + x_{R2}$.

**TABLE II**

| Step 1 | $R_{b1} \leq C\left(\frac{\alpha_{b1}\gamma}{1 + (\alpha_{b1} + \alpha_{s1} + \alpha_{s2})\gamma}\right)$, $R_{s1} \leq C\left(\frac{\alpha_{s1}\gamma}{1 + \alpha_{s1} + \alpha_{s2}}\right)$, $R_{b2} \leq C(\gamma\alpha_{s2})$.
| Step 2 | $R_{b1} \leq C\left(\frac{1 + \beta\gamma_{R1}}{1 + \beta\gamma_{R1}}\right)$, $R_{s1} \leq C\left(\frac{1 + \alpha_{s1}\gamma_{R1}}{1 + \alpha_{s1}\gamma_{R1}}\right)$, $R_{b2} \leq C(\beta\gamma_{R2})$.

**In both cases,** maximizing sum rate reduces to the problem of minimizing the transmission time $M_R$ of Step 2. Thus, we solve the following problem for each Case $i \in \{1,2\}$, s.t. $\alpha_{b1} + \alpha_{s1} + \alpha_{s2} = 1$, $0 \leq \beta \leq 1$. (3)

We denote $\max(I_1, I_2)$ by $I^{(i)}_{\alpha_{b1}, \alpha_{s1}}$ as it has only 2 variables $\alpha_{b1}$, $\alpha_{s1}$. That is, in Case 1, we have $\alpha_{s2}^* = \alpha_{s1}$ and $\alpha_{s1}$ is fixed by constraint (3). In Case 2, $\alpha_{s1} = 1 - \alpha_{s2}$ and $\alpha_{s1} = 0$. The global solution $\alpha_{b1}^*, \alpha_{s1}^*$ is given by the minimum $I^{(i)}_{\alpha_{b1}^*, \alpha_{s1}^*}$ between the 2 cases. For boundary values, $\alpha_{s2} = 0$ implies $\beta = 0$ since there is nothing to forward to MS2. Thus, $I^{(i)}_{\alpha_{b1}^*, \alpha_{s1}^*}(0,0) = \log_2(1 + \alpha_{s1}^*\gamma_{R1})$. If $\alpha_{s2} = 1$ (in only Case 2), then $\beta = 0$ as nothing is forwarded to MS1, so $I^{(i)}_{\alpha_{b1}^*, \alpha_{s1}^*}(1, 1) = \log_2(1 + \gamma_{R1})$. Removing these cases, the domain of (3) is $0 < \alpha_{s2} < 1$, $0 < \beta < 1$, i.e., $I_1 > 0$ and $I_2 > 0$.

We show that for any given $\alpha_{s2}$ in $[0,1]$, $I_1$ is minimized for $\beta$ in $[0,1]$, such that $I_1 = I^2_2$, as $I_1$ and $I_2$ are monotonic increasing and decreasing functions of $\beta$, respectively. We define $f(\beta) = R_{b1}R_{b2} - R_{b1}R_{R1}$. For any $\alpha_{s2}$ in $[0,1]$, there exists a unique $\beta$ such that $f(\beta) = 0$ which is equivalent to $I_1 = I_2$, as $f(0) < 0$, $f(1) > 0$ and $\frac{df}{d\beta} > 0$. This gives a bijection from $\alpha_{s2}$ to $\beta_2$, so we just need to find the optimum $\beta_2$ over the reduced domain in each case, if it exists, and then compare $I_1$ for $\beta_2$ with the boundary values, to determine the minimum over the whole domain. If it exists, $\beta_2$ is found by setting the derivative of $I_1$ and $I_2$ with respect
to $\alpha_2$ to zero (with $I_1 = I_2$ from $\beta_2$), which gives in Case 1,
\[
\gamma_R \frac{\gamma_R}{1 + \alpha_2 \gamma_R} C(\beta_2 \gamma_R) - \frac{\gamma_R}{1 + \beta_2 \gamma_R} \frac{\partial \beta_2}{\partial \alpha_2} C(\alpha_2 \gamma_R) = 0,
\]
\[
- (1 + \alpha_2 \gamma_R)(1 + (\alpha_2 + \alpha_2 \gamma_R) \log_2 (1 + \gamma_R)(1 + \beta_2 \gamma_R)) + \frac{\gamma_R}{1 + \beta_2 \gamma_R} \frac{\partial \beta_2}{\partial \alpha_2} \log_2 \frac{1 + (\alpha_2 + \alpha_2 \gamma_R)}{1 + \alpha_2 \gamma_R} = 0.
\]
Eliminating $\frac{\partial \beta_2}{\partial \alpha_2}$ and using the equation $f(\beta) = 0$, we obtain
\[
\hat{\alpha}_2(1) = \left( \frac{1}{\gamma_{D1}} - \frac{1}{\gamma_R} \right) \frac{\gamma_R}{1 + \beta_2 \gamma_R} \frac{\gamma_R}{1 + \beta_2 \gamma_R} - \frac{1}{\gamma_{D1}}, \quad (4)
\]
where $\beta_2^*$ is the value of $\beta_2$ for $\hat{\alpha}_2(1)$. By inserting Eq. (4) into $f(\beta) = 0$, we get $\beta_2^*$ numerically by Newton's method, as there exists a unique $\beta_2$ for any $\alpha_2$, so in particular for $\hat{\alpha}_2(1)$.

In Case 2, we find $\hat{\alpha}_2(1) = \emptyset$. Finally, the global minimum $(\hat{\alpha}_2^*, \beta_2^*)$ is determined comparing $I_R(1)(0, 0)$, $I_R(2)(1, 1)$ and $I_R(1)(\hat{\alpha}_2(1), \beta_2^*)$.

IV. NUMERICAL RESULTS

The sum-rates of our 3-SC Layer and benchmark schemes are plotted in Fig. 2 for two users and specified SNR values. The benchmark scheme from [1] allocates the user with the highest rate for a given set of SNRs, which is achieved by MS1 with SU-MH for 2dB $\leq \gamma_{D1} < 6$dB, and by MS1 with SU-SC for $\gamma_{D1} \geq 6$dB. Fig. 2 shows that this benchmark scheme is largely outperformed by 3-SC Layer for all $\gamma_{D1}$. Note that the scheme in [3] under sum rate maximization achieves the same rate as SU-SC, as the superposed message is sent at rate $C(\gamma_{R1})$. Finally, our scheme improves over the sum rate of the RBC in [5] that requires two equally divided orthogonal subchannels, stressing the benefit of our resource optimization.

Next, we evaluate the impact of our scheme for multi-user scheduling. We assume 20 users, half of them supported by direct and relayed links (type 1), and half without a direct link (type 2). The BS-RS link SNR is fixed to 30dB, but other channel SNRs are generated by the exponential distribution with mean $\gamma_{D1} = 5$dB for direct links of type 1 users and $\gamma_{R1} = \gamma_{R2} = 15$dB for relayed links of both types of users. Benchmarks schedulers are based on the Max SINR scheme of [7], where the user with the highest link quality is allocated all resources in each channel, a strategy commonly taken for throughput maximization in relayed networks with multiple users. Equivalently, benchmark Schedulers 2 and 3 allocate the user with the highest rate among single user schemes. In Scheduler 3, only Direct transmission or SU-MH is available for MS1 and SU-MH for MS2. In Scheduler 2, additionally, SU-SC may be used for MS1. Scheduler 1 includes our 3-SC Layer scheme, which is applied to all user pairs of type 1 and 2, and the pair with the best sum-rate is scheduled if it outperforms the best single user rate. Table III shows the sum-rate in [b/s/Hz] of each scheduler and their fairness measured every ten frames by Jain’s Index [7], as well as the percentage that each scheme (Direct, SU-MH, SU-SC, 3SC-L) achieved the best sum-rate. Scheduler 1 outperforms both reference ones, owing to 3-SC Layer which achieves the best sum-rate in 82% of the cases. Comparing Schedulers 2 and 3, we see that SU-SC from [1] contributes to enhance sum-rate but decreases fairness as only type 1 user rates are improved. However, Scheduler 1 outperforms both sum-rate and fairness of Schedulers 2 and 3. The ability to improve two opposite measures such as system throughput and fairness makes our scheme very appealing for, e.g., best-effort traffics which are not fairness-constrained although fairness is a critical measure.

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<th>Sum-rate</th>
<th>Fairness</th>
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<th>SU-MH</th>
<th>SU-SC</th>
<th>3SC-L</th>
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<td>0%</td>
<td>0%</td>
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</tr>
<tr>
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<td>4.0</td>
<td>0.39</td>
<td>10%</td>
<td>90%</td>
<td>6%</td>
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V. CONCLUSION

We have proposed the 3-SC Layer scheme for allocating DL resources in a two-user wireless relay system. By splitting the messages to the two users into three superposed layers, this scheme takes advantage of the relayed links as in a broadcast channel. With the derived optimal power allocation under sum-rate maximization, our scheme outperformed benchmark schemes and the achievable sum rate for RBC in [5]. The benefits in sum-rate and fairness of the proposed scheme were also shown for a large number users, which makes it very promising for multi-user scheduling in general relay systems.

REFERENCES


1Cooperative diversity could be used for MS1, but such a scheduler achieves lower fairness than Scheduler 3 and lower sum-rate than Scheduler 2.