KIER DISCUSSION PAPER SERIES

KYOTO INSTITUTE
OF
ECONOMIC RESEARCH

Discussion Paper No.780

"Global Liquidity Trap"

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June 2011

KYOTO UNIVERSITY

KYOTO, JAPAN

Global Liquidity Trap[*]

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June 30, 2011

Abstract

Using a two-country New Open Economy Macroeconomics model, we analyze how monetary policy should respond to a "global liquidity trap," where the two countries may fall into a liquidity trap simultaneously. We first characterize optimal monetary policy, and show that the optimal rate of inflation in one country is affected by whether or not the other country is in a liquidity trap. We next examine how well the optimal monetary policy is approximated by relatively simple monetary policy rules. We find that the interest-rate rule targeting the producer price index performs very well in this respect.

JEL Class: E52; E58; F41

Keywords: Zero interest rate policy; two-country model; international spillover; monetary policy coordination

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[*]For helpful comments and discussions, we thank Klaus Adam, Joshua Aizenman, Michele Cavallo, Larry Christiano, Michael Devereux, Marty Eichenbaum, Yiping Huang, Takatoshi Ito, Jinill Kim, Michael Klause, Giovanni Lombardo, Bartosz Mackowiak, Shinichi Nishiyama, Paolo Pesenti, Andrew Rose, Lars Svensson, Kazuo Ueda, Tack Yun, Tsutomu Watanabe, Mike Woodford, as well as seminar participants at University of Tokyo, Osaka University, Far East and South Asia Meeting of the Econometric Society 2009, Bank of Canada, Federal Reserve Board, Summer Workshop on Economic Theory 2009 at Otaru University of Commerce, Bank of Italy, ECB-IFS-Bundesbank seminar, Bank of Japan, and NBER-EASE. Nakajima gratefully acknowledges financial support from the Murata Science Foundation and JSPS. Views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

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1 Introduction

The world financial crisis in the late 2000s has triggered the largest economic downturn since World War II. To prevent further economic deterioration, most central banks in developed economies, including the United States, the United Kingdom, and Japan, have reduced their policy interest rates to unprecedentedly low levels. As a result the zero bound on nominal interest rates has now become a serious concern that is shared internationally. That is to say, the world economy is facing a *global liquidity trap*.

The implications of the zero bound on nominal interest rates have been studied extensively in closed economy models, such as Reifschneider and Williams (2000), Eggertsson and Woodford (2003), Jung et al. (2005), Kato and Nishiyama (2005), Adam and Billi (2006, 2007), and Nakov (2008). Optimal monetary policy in two-country models have been studied by Clarida et al. (2001) and Benigno and Benigno (2003), among others. However, the problem of optimal monetary policy in a two-country world when both countries may confront the zero bound has not yet been studied. In a stylized sense, that is the problem the world has faced over the last few years, and that is the problem our paper addresses.

Our model is a standard New Open Economy Macroeconomics (NOEM) model with producer currency pricing and complete international asset markets. Consistent with previous findings by Clarida et al. (2001) and Benigno and Benigno (2003), in our model, the first best can be attained if the producer price index (PPI) inflation rate in each country is set to zero at all times. However, the zero bound on the nominal interest rates might prevent monetary policy from achieving it.

For the closed economy, Eggertsson and Woodford (2003) describe the condition under which the economy falls into a liquidity trap using the notion called the "natural rate of interest," which is defined as the real interest rate that would prevail in the flexible-price equilibrium. For the open economy, however, it is not enough. We define an additional variable, called the "natural rate of change in the terms of trade," which is defined as the rate of change in the terms of trade in the flexible-price equilibrium. We use these two rates to describe when the zero bound binds in each country. Depending on their realizations, none or one or both countries will be in a liquidity trap.

Our numerical example is an open-economy extension of Eggertsson and Woodford (2003). The world economy is initially in the non-stochastic steady state with zero inflation. Then unexpected shocks arrive so that both countries fall into a liquidity trap. When each country gets out of the liquidity trap is determined stochastically. How should monetary policy be conducted in such a situation? We begin by analyzing the optimal monetary policy that is designed to maximize the world welfare. In the presence of a global liquidity trap, the optimal policy has two notable features: *history dependence* and *international dependence*. The importance of the history dependence has been noted in previous studies on the closed economy. The adverse effect of the liquidity trap can be mitigated if the monetary authority commits to generate some inflation in the future. Such a mechanism is also at work in a global liquidity trap.

The international dependence is a new feature. In the open economy, of course, countries are naturally interrelated through trades in goods and services. In the context of optimal monetary policy, several aspects of such interdependence have been analyzed in previous studies such as Clarida et al. (2001) and Benigno and Benigno (2003). But a global liquidity trap brings about a new form of interdependence in the optimal monetary policy. Namely, the optimal rate of inflation in one country is affected by whether or not the other country is caught in a liquidity trap. We show that the direction in which the optimal rate of inflation is affected is determined by whether goods produced in the two countries are Edgeworth complements or substitutes.

As our numerical example illustrates, the equilibrium paths under the optimal policy are in general very complicated. It suggests a difficulty in attaining the strict optimum in reality. So the next question we ask is how well, in terms of welfare, the optimal policy can be approximated by relatively simple monetary policy rules. There are related studies for the closed economy. For instance, when there is no possibility of a liquidity trap, Schmitt-Grohe and Uribe (2007) show that the optimal policy is replicated fairly well by the interest-rate rule that responds only to the inflation rate. Taking the liquidity trap into consideration, Eggertsson and Woodford (2003) argue that a simple price-level targeting rule performs well.

Given these previous results, we start with comparing the two classes of interest-rate rules, one with inflation targets (ITR) and the other with price-level targets (PLTR).[1]Not surprisingly, the ITR is not a good policy in our model just as Eggertsson and Woodford (2003) argue for the closed economy. This is because such a policy rule does not allow for history dependence. When both countries adopt the ITR, its welfare loss relative to the optimal policy is 0.46 percent of consumption in our numerical example.[2] What is probably surprising is how close the PLTR comes to the optimal policy. In our example, when both countries adopt the PLTR, its welfare loss relative to the optimal policy is only 0.00042 percent of consumption. We also want to emphasize that the benefit of the PLTR is obtained only when both countries adopt it. And this is not only true in terms of the world welfare, but also for the welfare of each country. For instance, in our example, when the home country adopts the PLTR and the foreign country adopts the ITR, the loss of the world welfare is 0.24 percent of consumption, the welfare loss for the home country is 0.28 percent, and that for the foreign country is 0.20 percent.

We then consider the interest-rate rules augmented by the nominal exchange rate. Such a rule may in principle be beneficial given that the optimal policy exhibits a particular form of international dependence as discussed above. In our numerical example, however, we see that its effect is quantitatively small.

In the previous literature, Coenen and Wieland (2003) and Svensson (2001) suggest a benefit of depreciating the currency of a country that falls into a liquidity trap. Here we do not find such a benefit. One reason for this is that the welfare relevant price levels in our framework are the PPI's rather than the CPI's.[3] Nakajima (2008) studies the optimal

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[1]Here, in both ITR and PLTR we consider, the relevant prices are the PPI's rather than the CPI's.

[2]We measure the welfare cost of a given policy rule in a way similar to Schmitt-Grohe and Uribe (2007). Thus it is given by the fraction of consumption in the equilibrium under the optimal policy that needs to be reduced in order to equate the world welfare under the optimal policy to that under the policy in consideration.

[3]Our assumption of producer currency pricing is crucial for this. As shown by Devereux and Engel (2003), under local currency pricing, the nominal exchange rate should also be stabilized. Extending our analysis to the case with local currency pricing is left for future research.

monetary policy in an open-economy model similar to ours, but he restricts attention to the case where only one country falls into a liquidity trap. Jeanne (2009) studies the role of monetary and fiscal policy in a global liquidity trap. He, however, assumes that prices are set one-period in advance. As a result, the liquidity trap does not last more than one period in his model. Cook and Devereux (2010) consider a global liquidity trap with staggered price setting, but they do not study the optimal monetary policy.

The rest of the paper is organized as follows. Section 2 describes our two-country model and defines the natural rates that are relevant for the global liquidity trap. In section 3, we analyze the optimal policy problem, emphasizing the history and international dependence as the key features of the optimal policy. Section 4 considers simple interest-rate rules, and examines how well they can approximate the optimal policy in the face of a global liquidity trap. Section 5 concludes.

2 The model

The model economy is an open-economy version of the sticky-price model developed by Woodford (2003), and closely related to the ones considered by Clarida et al. (2001), and Benigno and Benigno (2003).[4]

The world economy consists of two countries: the home country (H), and the foreign country (F). The size of population in country $j \in \{H, F\}$ is n_j, where $n_H + n_F = 1$. A set of differentiated products are produced in each country and they are traded between the two countries. Let N_j denote the set of those products. We assume that $N_H = [0, n_H]$, and $N_F = (n_H, 1]$.

Time is discrete and indexed by t. There are no stochastic shocks prior to period 1, and the world economy is at the non-stochastic steady state with zero inflation in period 0. At the beginning of period 1, unexpected shocks hit the world economy. Although these shocks are not expected at all before period 1, all agents immediately understand their stochastic nature

[4]Since our model is quite standard in the literature, we describe it only briefly here. A more detailed description is given in Appendix.

at the beginning of period 1. Thus, we let period 1 be the initial period in the description of our economy below.

2.1 Households

A representative household in the home country H has preferences given by

$$U_{H,1} = \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \tilde{u}(C_t) - \frac{1}{n_H} \int_{N_H} \tilde{v}\left[\ell_t(i)\right] di \right\}, \tag{1}$$

where $0 < \beta < 1$, E_t is the conditional expectation operator at time t, C_t is the consumption index defined below, and $\ell_t(i)$, $i \in N_H$, is the supply of type-i labor, which is used to produce differentiated product i. We assume that $\tilde{u}$ and $\tilde{v}$ have constant elasticity:

$$\tilde{u}(C) \equiv \frac{C^{1-\sigma}}{1-\sigma}, \qquad \tilde{v}(\ell) \equiv \frac{1}{1+\omega}\ell^{1+\omega},$$

where $\sigma > 0$, and $\omega > 0$. The consumption index for the home household, C_t, is given by

$$C_t = \left(\frac{C_{H,t}}{n_H}\right)^{n_H} \left(\frac{C_{F,t}}{n_F}\right)^{n_F}, \tag{2}$$

where $C_{H,t}$ and $C_{F,t}$ are the consumption indexes for home and foreign goods consumed by the home household, respectively. They are defined by

$$C_{j,t} = \left[n_j^{-\frac{1}{\theta}} \int_{N_j} c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \qquad j = H, F. \tag{3}$$

Here, $\theta > 1$ and $c_t(i) \in N_j$ is the home household's consumption of good i produced in country $j \in \{H, F\}$. It is convenient to define the function $u(C_H, C_F)$ by

$$u(C_H, C_F) \equiv \tilde{u}\left(\left[\frac{C_H}{n_H}\right]^{n_H} \left[\frac{C_F}{n_F}\right]^{n_F} \right).$$

The lifetime utility of a representative household in the foreign country F takes the same form as that of the home household:

$$U_{F,1} = \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \tilde{u}(C_t^*) - \frac{1}{n_F} \int_{N_F} \tilde{v}\left[\ell_t^*(i)\right] di \right\}. \tag{4}$$

The consumption indexes for the foreign household, $\{C_t^*, C_{H,t}^*, C_{F,t}^*\}$, are similarly defined as in equations (2) and (3).

Corresponding to the consumption indexes in the home country, C_t, $C_{j,t}$, $j = H, F$, the prices indexes, P_t, $P_{j,t}$, $j = H, F$, are defined as

$$P_t = P_{H,t}^{n_H} P_{F,t}^{n_F}, \qquad P_{j,t} = \left[\frac{1}{n_j} \int_{N_j} p_t(i)^{1-\theta} \, di \right]^{\frac{1}{1-\theta}}, \quad j = H, F,$$

where $p_t(i)$, $i \in N_j$, $j \in \{H, F\}$, is the price of good i produced in country j quoted in the home currency. The price indexes in the foreign country, P_t^*, $P_{j,t}^*$, $j = H, F$, are defined similarly by individual good prices, $p_t^*(i)$, $i \in N_j$, $j \in \{H, F\}$, quoted in the foreign currency.

We assume that the law of one price holds:

$$p_t(i) = \mathcal{E}_t p_t^*(i),$$

for all $i \in N_j$, $j \in \{H, F\}$, where $\mathcal{E}_t$ is the nominal exchange rate, defined as the price of foreign currency in terms of home currency. It follows that $P_{j,t} = \mathcal{E}_t P_{j,t}^*$, $j = H, F$, and $P_t = \mathcal{E}_t P_t^*$.

Let $y_t(i)$, $i \in N_H$, and $y_t^*(i)$, $i \in N_F$, denote the supply of home and foreign products, respectively. The corresponding production indexes are defined as

$$Y_{H,t} \equiv \left[n_H^{-\frac{1}{\theta}} \int_{N_H} y_t(i)^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}}, \qquad Y_{F,t} \equiv \left[n_F^{-\frac{1}{\theta}} \int_{N_F} y_t^*(i)^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}},$$

and $Y_t \equiv \left(\frac{Y_{H,t}}{n_H} \right)^{n_H} \left(\frac{Y_{F,t}}{n_F} \right)^{n_F}$.

We assume worldwide complete financial markets. The flow budget constraint for the home household is

$$P_t C_t + \mathrm{E}_t[Q_{t,t+1} W_{t+1}] = W_t + \int_{N_H} \left[w_t(i) \ell_t(i) + \Pi_t(i) \right] di + T_t,$$

where $Q_{t,t+1}$ is the stochastic discount factor between dates t and $t+1$ for nominal payoffs in the home country, W_{t+1} is the portfolio of one-period state-contingent bonds, $w_t(i)$ is the date-t nominal wage rate for type $i \in N_H$ labor, $\Pi_t(i)$ is the date-t nominal profits from sales of good $i \in N_H$, and T_t is the nominal lump-sum transfer from the home government. The flow budget constraint for the foreign household is given analogously.

Under the standard assumption that the representative households of the two countries are equally wealthy in the initial period, their equilibrium consumption levels are identical for all t:

$$C_{H,t} = C^*_{H,t} = Y_{H,t}, \qquad C_{F,t} = C^*_{F,t} = Y_{F,t}, \qquad C_t = C^*_t = Y_t.$$

2.2 Aggregate supply

The technology to produce each good is linear in labor:

$$y_t(i) = A_t n_H \ell_t(i), \quad i \in N_H, \qquad y^*_t(i) = A^*_t n_F \ell^*_t(i), \qquad i \in N_F,$$

where A_t and A^*_t represent country-specific technology shocks. Each product is produced by a monopolist. For simplicity, however, we assume that the monopoly distortions are completely eliminated by subsidizing each monopolist's revenue at the rate $\eta \equiv \frac{1}{\theta-1}$.

Following Woodford (2003), we define the "natural rates of output" at each date t, $Y^n_{H,t}$ and $Y^n_{F,t}$, as the levels of home and foreign output which would prevail in the flexible-price equilibrium. Since the monopolistic distortions are absent, the flexible-price equilibrium is the first best.

Suppose that goods prices are adjusted at random intervals as in Calvo (1983). Let α be the probability that each good price remains unchanged in each period. We assume that this probability is identical in the two countries. Consider the price adjustment in the home country. Suppose that the price of good $i \in N_H$ can be adjusted at date t. The supplier of that good chooses $p_t(i)$ to maximize its expected discounted profits:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left\{ \left[(1+\eta) p_t(i) - \frac{w_T(i)}{A_T} \right] \frac{Y_{H,T}}{n_H} \left[\frac{p_t(i)}{P_{H,T}} \right]^{-\theta} \right\}.$$

The price adjustment problem for the foreign country is described similarly.

Log-linearizing the first-order conditions for the price adjustment problems yields the New Keynesian aggregate-supply relations:

$$\pi_{H,t} = \gamma_H x_{H,t} + \gamma_{HF} n_F x_{F,t} + \beta \mathrm{E}_t \pi_{H,t+1}, \tag{5}$$

$$\pi^*_{F,t} = \gamma_{HF} n_H x_{H,t} + \gamma_F x_{F,t} + \beta \mathrm{E}_t \pi^*_{F,t+1}. \tag{6}$$

Here $\pi_{H,t} \equiv \ln P_{H,t} - \ln P_{H,t-1}$ and $\pi^*_{F,t} \equiv \ln P^*_{F,t} - \ln P^*_{F,t-1}$ are the inflation rates for goods produced in the home and foreign countries, respectively: $x_{j,t} \equiv \ln Y_{j,t} - \ln Y^n_{j,t}$ is the output gap in country $j = H, F$; and the coefficients are given by $\gamma_H \equiv \zeta\big[1+\omega+(\sigma-1)n_H\big]$, $\gamma_{HF} \equiv \zeta(\sigma-1)$, $\gamma_F \equiv \zeta\big[1+\omega+(\sigma-1)n_F\big]$, and $\zeta \equiv \frac{1-\alpha}{\alpha}\frac{1-\alpha\beta}{1+\omega\theta}$.

2.3 Welfare approximation

Our welfare criterion is the average lifetime utility of the representative households in the two countries evaluated in period 1:[5]

$$U_{W,1} \equiv n_H U_{H,1} + n_F U_{F,1},$$

where $U_{H,1}$ and $U_{F,1}$ are the lifetime utility levels of the home and foreign households given by (1) and (4), respectively. Following Woodford (2003), a second-order approximation of the world welfare around the zero-inflation steady state is obtained as

$$U_{W,1} \approx -\nu_0 \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} L_t + \nu_1, \tag{7}$$

where $\nu_0 > 0$ and ν_1 are constants independent of policy, and L_t is a quadratic measure of the world-welfare loss given by

$$L_t \equiv \frac{1}{2} x_t' \Lambda x_t + \frac{n_H}{2}(\pi_{H,t})^2 + \frac{n_F}{2}(\pi_{F,t}^*)^2.$$

Here, $x_t \equiv (x_{H,t}, x_{F,t})'$, and

$$\Lambda \equiv \frac{1}{\theta}\begin{bmatrix} \gamma_H n_H & \gamma_{HF} n_H n_F \\ \gamma_{HF} n_H n_F & \gamma_F n_F \end{bmatrix}.$$

Notice that our welfare measure (7) implies that what must be stabilized are the PPI inflation rates, $\pi_{H,t}$ and $\pi_{F,t}^*$, rather than the consumer price index (CPI) inflation rates, π_t and π_t^*, where $\pi_t \equiv \ln P_t - \ln P_{t-1}$ and $\pi_t^* \equiv \ln P_t^* - \ln P_{t-1}^*$. That is, it is efficient for price dispersion to be eliminated within but not across countries. This is consistent with previous findings by Clarida et al. (2001), and Benigno and Benigno (2003), among others.

2.4 IS equations and the zero lower bound

As demonstrated by Eggertsson and Woodford (2003), a negative shock to the real interest rate triggers a liquidity trap in the closed economy framework. In our open economy model

[5] A detailed derivation is given in Appendix.

with complete markets, the real interest rate is equalized across the two countries. Does that mean that the two countries get into and out of a liquidity trap simultaneously? The answer is no. The key is that it is the PPI inflation rates that have to be stabilized, as our welfare measure (7) indicates.

To see this, let us first consider how the nominal interest rate in each country is related to its CPI inflation rate, π_t and π_t^*. Let $i_{j,t}$ be the nominal interest rate in period t in country $j = H, F$. The Euler equations imply that

$$i_{H,t} = r_t + \mathrm{E}_t(\pi_{t+1}), \quad \text{and} \quad i_{F,t} = r_t + \mathrm{E}_t(\pi_{t+1}^*). \tag{8}$$

Here r_t denotes the real interest rate in period t:

$$r_t \equiv -\ln \left\{ \mathrm{E}_t \left[\beta \frac{\tilde{u}_c(Y_{t+1})}{\tilde{u}_c(Y_t)} \right] \right\}.$$

where $\tilde{u}_c$ denotes the derivative of the function $\tilde{u}$. Equations (8) show that if $\pi_{t+1} = \pi_{t+1}^*$, then $i_{H,t} = i_{F,t}$. That is, if it were the CPI inflation rates that must be stabilized, then the desired nominal interest rates would be identical between the two countries.

To derive the relationship between the PPI inflation rates and the nominal interest rates, let δ_{t+1} denote the rate of change of the terms of trade of the home country:

$$\delta_{t+1} \equiv \ln \left(\frac{P_{H,t+1}}{P_{F,t+1}} \right) - \ln \left(\frac{P_{H,t}}{P_{F,t}} \right).$$

It follows that

$$\pi_{t+1} = \pi_{H,t+1} - n_F \delta_{t+1}, \quad \text{and} \quad \pi_{t+1}^* = \pi_{F,t+1}^* + n_H \delta_{t+1}.$$

It then follows from equations (8) that the relationship between the nominal interest rates and the PPI inflation rates is given as

$$i_{H,t} = r_t - n_F \mathrm{E}_t(\delta_{t+1}) + \mathrm{E}_t(\pi_{H,t+1}), \tag{9}$$

$$i_{F,t} = r_t + n_H \mathrm{E}_t(\delta_{t+1}) + \mathrm{E}_t(\pi_{F,t+1}^*). \tag{10}$$

Note that $i_{H,t}$ and $i_{F,t}$ are in general different even when $\pi_{H,t+1} = \pi_{F,t+1}^* = 0$. This clarifies why the desired nominal interest rates differ across countries as long as the PPI inflation rates have to be stabilized.

Now let us define the natural rates for r_t and δ_{t+1}. The natural rate of interest, r_t^n, is the real interest rate that would prevail in the flexible-price equilibrium, that is,

$$r_t^n \equiv -\ln\left\{\mathrm{E}_t\left[\beta\frac{\tilde{u}_c(Y_{t+1}^n)}{\tilde{u}_c(Y_t^n)}\right]\right\},$$

where Y_t^n is the natural rate of output. To define the natural rate of change in the terms of trade, notice that in equilibrium

$$\frac{P_{H,t}}{P_{F,t}} = \frac{u_H(Y_{H,t}, Y_{F,t})}{u_F(Y_{H,t}, Y_{F,t})} = \frac{n_H}{n_F}\frac{Y_{F,t}}{Y_{H,t}},$$

where u_H and u_F denote the derivative of the function $u(C_H, C_F)$ with respect to C_H and C_F, respectively. Thus it follows that the natural rate of change in the terms of trade of the home country, δ_{t+1}^n, is given by

$$\delta_{t+1}^n \equiv \ln\left(\frac{Y_{F,t+1}^n}{Y_{F,t}^n}\right) - \ln\left(\frac{Y_{H,t+1}^n}{Y_{H,t}^n}\right).$$

Then equations (9)-(10) can be rewritten as

$$i_{H,t} = \mathrm{E}_t\Big\{\big[1+(\sigma-1)n_H\big](x_{H,t+1}-x_{H,t}) + (\sigma-1)n_F(x_{F,t+1}-x_{F,t}) + \pi_{H,t+1}\Big\} + r_{H,t}^n, \quad (11)$$

$$i_{F,t} = \mathrm{E}_t\Big\{\big[1+(\sigma-1)n_F\big](x_{F,t+1}-x_{F,t}) + (\sigma-1)n_H(x_{H,t+1}-x_{H,t}) + \pi_{F,t+1}^*\Big\} + r_{F,t}^n, \quad (12)$$

where $r_{H,t}^n$ and $r_{F,t}^n$ are the composite natural rate shocks defined as

$$r_{H,t}^n \equiv r_t^n - n_F\mathrm{E}_t(\delta_{t+1}^n), \quad \text{and} \quad r_{F,t}^n \equiv r_t^n + n_H\mathrm{E}_t(\delta_{t+1}^n).$$

A competitive equilibrium attains the first best outcome (up to a first-order approximation) if $\pi_{H,t} = \pi_{F,t}^* = x_{H,t} = x_{F,t} = 0$ at all dates and under all contingencies. The IS relations (11) and (12) imply that the nominal interest rates in such an equilibrium should satisfy $i_{j,t} = r_{j,t}^n$, for $j \in \{H, F\}$. It becomes infeasible, however, if the composite natural rates, $r_{H,t}^n$ and $r_{F,t}^n$, become negative, because of the zero bounds for the nominal interest rates:

$$i_{H,t} \geq 0, \quad \text{and} \quad i_{F,t} \geq 0. \quad (13)$$

We say that a country $j \in \{H, F\}$ is in a liquidity trap if the zero bound condition (13) is binding in country j.

Let us briefly discuss how the nominal exchange rate should evolve in the first best equilibrium. As we have noted, in the first best equilibrium, the CPI inflation rates are not zero, and the nominal exchange rate fluctuates in general. Indeed, in the first best equilibrium, since $P_{H,t}$ and $P^*_{F,t}$ are constant, δ^n_{t+1} is equal to the rate of change in the nominal exchange rate. In this sense, δ^n_{t+1} represents the efficient rate of change in the nominal exchange rate as long as the zero bound does not bind.

3 Optimal monetary policy

In this section we study the optimal monetary policy that maximizes the world welfare function (7). In period 0, the world economy is at the non-stochastic steady state with zero inflation, but stochastic shocks arrive in period 1 so that it is no longer possible to set the PPI inflation rates equal to zero at all dates. In response to such shocks, here we assume that the two monetary authorities coordinate with each other and choose their policies with perfect commitment in period 1 in order to maximize the world welfare. The equilibrium in this case is obtained by solving the Ramsey problem, that is, by maximizing the world-welfare function (7) subject to the constraints (5), (6), (11), (12), and (13). We refer to this equilibrium as the Ramsey equilibrium, and the optimal policy as the Ramsey policy.[6]

3.1 Properties of the optimal policy

Consider first the case in which the zero bound conditions (13) never bind. Then the Ramsey equilibrium can be implemented by the following targeting rules:

$$\pi_{H,t} + \frac{1}{\theta}\left(x_{H,t} - x_{H,t-1}\right) = 0, \tag{14}$$

$$\pi^*_{F,t} + \frac{1}{\theta}\left(x_{F,t} - x_{F,t-1}\right) = 0. \tag{15}$$

These rules are inward looking in the sense that the monetary authority in each country only needs to look at the PPI inflation rate and the output gap in its own country. Thus, in this

[6]Our optimal policy exactly corresponds to the one studied by Eggertsson and Woodford (2003) for the closed economy. In particular, the initial conditions of the Lagrange multipliers for the Ramsey problem are set to zero as in Eggertsson and Woodford (2003).

case, the world welfare is maximized by purely inward-looking policies. This point has been previously made, for instance, by Clarida et al. (2001). Note that this inward-looking feature of the optimal monetary policy does not depend on the value of σ.

However, the optimal monetary policy can no longer be described by inward-looking rules if the zero bound conditions bind with a positive probability. Even with the producer currency pricing, foreign variables must be included in the domestic targeting rule. The degree of influence from foreign variables is determined by σ. Denoting the Lagrange multipliers associated with inequalities (13) by $\phi_{H,t}$ and $\phi_{F,t}$, the first order conditions for the Ramsey problem yield the following targeting rules:

$$\pi_{H,t} + \frac{1}{\theta}\left(x_{H,t} - x_{H,t-1}\right) = z_{H,t}, \tag{16}$$

$$\pi^*_{F,t} + \frac{1}{\theta}\left(x_{F,t} - x_{F,t-1}\right) = z_{F,t}, \tag{17}$$

where $z_{H,t}$ and $z_{F,t}$ are defined by

$$\begin{bmatrix} z_{H,t} \\ z_{F,t} \end{bmatrix} = Z(L) \begin{bmatrix} \phi_{H,t} \\ \phi_{F,t} \end{bmatrix}. \tag{18}$$

Here L is the lag operator and $Z(L)$ is given by

$$Z(L) \equiv -\begin{bmatrix} \frac{\sigma+\omega+(\sigma-1)\omega n_H}{\zeta(1+\omega)(\omega+\sigma)} & \frac{(\sigma-1)\omega n_F}{\zeta(1+\omega)(\omega+\sigma)} \\ \frac{(\sigma-1)\omega n_H}{\zeta(1+\omega)(\omega+\sigma)} & \frac{\sigma+\omega+(\sigma-1)\omega n_F}{\zeta(1+\omega)(\omega+\sigma)} \end{bmatrix} (1-L)(1-\beta^{-1}L) + \begin{bmatrix} \beta^{-1} & 0 \\ 0 & \beta^{-1} \end{bmatrix} L. \tag{19}$$

Comparing the targeting rules (14)-(15) and (16)-(17), we see that when the zero bound binds, its effect is summarized by the term $z_t = (z_{H,t}, z_{F,t})$. Suppose that country $j \in \{H, F\}$ is in a liquidity trap in some period $\hat{t}$, so that $\phi_{j,\hat{t}} > 0$. Then it affects z_t for three periods: $t = \hat{t}, \hat{t}+1, \hat{t}+2$, as shown by equation (19). If $\phi_{H,t} = \phi_{F,t} = 0$ for all t, the optimal targeting rules (16)-(17) reduce to the inward-looking rules (14)-(15).

To understand better the effect of a liquidity trap on the optimal policy, Figure 1 plots how z_H and z_F respond to a one-time increase in ϕ_H for different values of σ in equation (18). Specifically it shows how $z_{H,t}$ and $z_{F,t}$ vary when $\phi_{H,t} = 0$ for all $t \neq 1$ and $\phi_{H,1} = 1$ with $\phi_{F,t} = 0$ for all t.[7] Let us look at the upper panel, which shows how the optimal targeting rule

[7] The parameter values used to plot the figure are summarized in Table 1.

for the home country is affected when it falls into a liquidity trap in period 1. In the period when the zero bound binds, the monetary authority has to allow for deflation and a negative output gap, so that the targeting rule shifts downward: $z_{H,1} < 0$. However, such a downward shift in the targeting rule is alleviated by promising an upward shift in the targeting rule in the future, $z_{H,2} > 0$. In other words, a country caught in a liquidity trap can reduce the damage it sustains if the monetary authority commits itself to generating some inflation and positive output gaps in the future. This feature of the optimal monetary policy is the *history dependence* that is emphasized in previous studies on the closed economy, such as Eggertsson and Woodford (2003) and Jung et al. (2005).

The possibility of a global liquidity trap adds an additional feature to the optimal policy: *international dependence*. Mathematically, such interdependence can be seen by the fact that either $\phi_{H,t}$ or $\phi_{F,t}$ affects both $z_{H,t}$ and $z_{F,t}$, as shown in equation (18) provided that $\sigma \neq 1$.[8] For instance, if the home country is in a liquidity trap in period t, then $\phi_{H,t} > 0$; this will affect not only the home country's targeting rule (16), but also the foreign country's rule (17) through its influence on $z_{H,t}$ and $z_{F,t}$. The optimal rate of inflation for each country is affected by whether or not the other country is caught in a liquidity trap. Economic efficiency is no longer attained simply by 'keeping one's house in order.'

The lower panel of Figure 1 shows how a liquidity trap in the home country affects the optimal targeting rule for the foreign country. The direction of the effect depends on whether σ is greater or less than unity. This follows from the fact that the source of the international dependence in our model is the dependence of the marginal utility from consuming the composite good produced in one country on the consumption of the composite good produced in the other country. When $\sigma > 1$, for instance, home goods and foreign goods are Edgeworth substitutes, i.e., $u_{HF} = u_{FH} < 0$. In this case, a change in the output of the composite good produced in country $j \in \{H, F\}$, Y_j, affects both u_H and u_F in the same direction,

[8] It is clear from equation (18) that if $\sigma = 1$, then ϕ_j only affects z_j for each $j = H, F$. Thus, the targeting rules (16)-(17) do not exhibit the form of international dependence discussed here. In what follows, whenever we emphasize the international dependence of the optimal monetary policy, we are implicitly assuming that $\sigma \neq 1$.

because $u_{HH} < 0$ and $u_{FF} < 0$. Thus, a shift of the optimal targeting rule in one country is transmitted into a shift of the optimal targeting rule in the other country in the *same* direction. This can be seen in the figure that $\phi_{H,t}$ affects $z_{F,t}$ and $z_{H,t}$ in the same direction when $\sigma = 2$. To the contrary, when $\sigma < 1$, home goods and foreign goods are Edgeworth complements: $u_{HF} = u_{FH} > 0$. Thus, a change in Y_j for $j \in \{H, F\}$ affects u_H and u_F in opposite directions. As a result, the optimal targeting rule in the two countries shift in the *opposite* directions. This is consistent with the figure in the case of $\sigma = 0.5$.

Another way to understand how monetary policy should be conducted in a global liquidity trap is to look at the dynamic IS curves (11)-(12) with the zero bound conditions (13). First, suppose that $r^n_{H,t_0} < 0$ so that the the home country is in a liquidity trap in period t_0: $i_{H,t_0} = 0$. The optimal policy attempts to relax the degree to which the zero bound binds. Equation (11) reveals that there are several ways to do this. One way is for the monetary authority in the home country to commit to future stimulation of the home economy. Such a commitment makes $\mathrm{E}_{t_0}\pi_{H,t_0+1} > 0$ and $\mathrm{E}_{t_0}(x_{H,t_0+1} - x_{H,t_0}) > 0$. Both of these would offset at least partially the depressing effect of the liquidity trap in the home country. Additionally, if the foreign monetary authority also commits to achieve $(\sigma - 1)\mathrm{E}_{t_0}(x_{F,t_0+1} - x_{F,t_0}) > 0$, then the severity of the liquidity trap in the home country would be weakened further. Thus, if $\sigma > 1$ (respectively, if $\sigma < 1$), a future expansion (contraction) of the foreign economy helps alleviate the severity of the current liquidity trap for the home economy. In this way, policy commitment by each of the two monetary authorities acts to reduce the welfare loss associated with the home country's liquidity trap.

Next consider the IS curve for the foreign country (12) at the date $t_1 > t_0$, when r^n_{H,t_1} turns back to positive. As discussed above, given the home monetary authority's policy commitment, the home economy experiences a temporary boom in period t_1, $x_{H,t_1} > 0$, which implies that $\mathrm{E}_{t_1}(x_{H,t_1+1} - x_{H,t_1}) < 0$. From the perspective of the foreign monetary authority, if $\sigma > 1$ ($\sigma < 1$), this tends to lower (raise) the right hand side of equation (12). Thus, for $\sigma > 1$ (for $\sigma < 1$), the foreign monetary authority tends to lower (raise) $i_{F,t}$ when $r^n_{H,t}$ becomes positive. Notice also that such a response by the foreign monetary authority tends to raise (lower) x_{F,t_1} when $\sigma > 1$ ($\sigma < 1$); this is indeed consistent with the foreign monetary authority's commitment to generate $(\sigma - 1)\mathrm{E}_{t_0}(x_{F,t_0+1} - x_{F,t_0}) > 0$ during periods when the home country is in the liquidity trap.

3.2 Numerical example

In order to further analyze the properties of the optimal policy, let us consider a numerical example, which extends the closed-economy experiment of Eggertsson and Woodford (2003) to our open-economy model. Table 1 summarizes the parameter values assumed here. In the initial period $t = 0$, the world economy is in the non-stochastic steady state with zero inflation: $\pi_{H,0} = \pi^*_{F,0} = x_{H,0} = x_{F,0} = 0$ and $i_{H,0} = i_{F,0} = r^n_{H,0} = r^n_{F,0} = \overline{r}^n \equiv \frac{1-\beta}{\beta}$. Then, in period $t = 1$, both $r^n_{H,1}$ and $r^n_{F,1}$ drop unexpectedly to a negative level $\underline{r}^n < 0$. We assume that the natural rates evolve according to the following stochastic process: (i) $r^n_{H,1} = r^n_{F,1} = \underline{r}^n$; (ii) if $r^n_{j,t} = \underline{r}^n$ in period t in country $j \in \{H, F\}$, then

$$r^n_{j,t+1} = \begin{cases} \overline{r}^n, & \text{with probability } p_{j,t}, \\ \underline{r}^n, & \text{with probability } 1 - p_{j,t}, \end{cases}$$

where $p_{j,t} = p_j$ for $1 \leq t \leq \overline{S} - 1$ and $p_{j,t} = 1$ for $t \geq \overline{S}$; (iii) if $r^n_{j,t} = \overline{r}^n$, then $r^n_{j,t+1} = \overline{r}^n$ with probability one, for each $j \in \{H, F\}$ and for all $t > 1$. Here, $\overline{S}$ is a large positive integer that determines the maximal number of periods for which a country's natural rate may remain negative.

Let T_H and T_F be the stopping times defined respectively as the last periods in which $r^n_{H,t} = \underline{r}^n$ and $r^n_{F,t} = \underline{r}^n$. For each $(\tau_H, \tau_F) \in \{1, \ldots, \overline{S}\}^2$, the probability that $(T_H, T_F) = (\tau_H, \tau_F)$ is $(1 - p_H)^{\tau_H - 1} p_H (1 - p_F)^{\tau_F - 1} p_F$. For a given monetary policy, the equilibrium is described by a set of stochastic processes $\{i_{H,t}, i_{F,t}, \pi_{H,t}, \pi^*_{F,t}, x_{H,t}, x_{F,t}\}_{t=1}^{\infty}$, each of which is adapted to the filtration generated by the stopping times (T_H, T_F). The optimal monetary policy chooses this set of stochastic processes so as to solve the Ramsey problem described in the previous subsection. The details of the numerical algorithm are given in the Appendix.

Figure 2 plots the path of the Ramsey equilibrium corresponding to the realization $T_H = 15$ and $T_F = 10$ (that is, where $r^n_{H,t}$ and $r^n_{F,t}$ return to $\overline{r}^n$ when $t = 16$ and $t = 11$, respectively). It is clear that the optimal policy exhibits the history dependence discussed in the previous subsection: for each country j, the nominal interest rate remains set to zero for a while even after its natural rate becomes positive, and both the inflation rate and the output gap are positive in the period when its natural rate changes from $\underline{r}^n$ to $\overline{r}^n$.

Furthermore, the international dependence of the optimal policy can also be clearly seen. For instance, look at what happens to the foreign country's nominal interest rate $i_{F,t}$ after the home country's natural rate returns to $\overline{r}^n$ (i.e., $t = 16, 17$). The home country's output gap increases temporarily in period 16, as a result of which its expected growth rate from t to $t+1$ is negative for $t = 16, 17$. Given that our example has $\sigma = 2$, the negative growth of the home output gap tends to lower the right hand side of (12). This is why the foreign nominal interest rate $i_{F,t}$ declines for the periods $t = 16, 17$. Analogously, the negative expected growth rate implied by the foreign output gap in the period when the foreign natural rate returns to $\overline{r}^n$ ($t = 11$) acts as a negative shock to (11). In that period, however, the home country is still caught in a liquidity trap and the home nominal interest rate cannot be lowered further. As a result, its effect appears as a decline in the home output gap in period 11. Yet another form of the international dependence appears in the term $\mathrm{E}_t(x_{F,t+1} - x_{F,t})$ in equation (11) for periods $t \leq T_H$. When $r^n_{F,t}$ returns to $\overline{r}^n$ in period 11, the foreign output gap rises at first, and then declines for a few periods ($t = 12, 13$). After this, the foreign output gap starts to increase gradually (for $t = 14, 15, 16$). Although quantitatively small, this behavior of the foreign output gap for $t = 14, 15, 16$ is enough to yield $\mathrm{E}_t(x_{F,t+1} - x_{F,t}) > 0$ during those periods, which helps to alleviate the severity of the liquidity trap that the home country is caught in.[9]

4 Simple monetary policy rules

In the last section we have examined the properties of the Ramsey equilibrium. In particular, we have seen that the state contingent paths of the Ramsey equilibrium can be very complicated as is illustrated in Figure 2. Thus it may be very difficult to attain the Ramsey

[9]See the working paper version (Fujiwara et al., 2011) for more detailed analysis on the properties of the optimal policy.

equilibrium in reality. In this section, therefore, we examine the extent to which the optimal monetary policy is approximated by simple monetary policy rules.[10] Indeed, we show that the PLTR is quite successful in approximating the Ramsey equilibrium in terms of the welfare.

Our measure of the welfare costs associated with a suboptimal policy is similar to the one used by Schmitt-Grohe and Uribe (2007). Let $U^o_{W,1}$ be the world welfare associated with the optimal monetary policy discussed in the previous section:

$$U^o_{W,1} = \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \tilde{u}(Y^o_t) - \int_{N_H} \tilde{v}[\ell^o_t(i)]\, di - \int_{N_F} \tilde{v}[\ell^{*\,o}_t(i)]\, di \right\},$$

where Y^o_t, ℓ^o_t, and $\ell^{*\,o}_t$ denote output and labor supplies under the optimal policy. Consider an alternative policy regime, denoted by a. The world welfare corresponding to it is given as

$$U^a_{W,1} = \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \tilde{u}(Y^a_t) - \int_{N_H} \tilde{v}[\ell^a_t(i)]\, di - \int_{N_F} \tilde{v}[\ell^{*\,a}_t(i)]\, di \right\}.$$

The welfare cost of adopting the suboptimal policy regime a is given by χ_W, which solves

$$U^a_{W,1} = \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \tilde{u}[(1-\chi_W)Y^o_t] - \int_{N_H} \tilde{v}[\ell^o_t(i)]\, di - \int_{N_F} \tilde{v}[\ell^{*\,o}_t(i)]\, di \right\}.$$

That is, χ_W is the fraction of consumption that has to be given up in the Ramsey equilibrium in order to make the world welfare identical between the two policy regimes. In this sense, it measures the welfare loss associated with the suboptimal policy a in consumption units.

It is sometimes of interest to examine the welfare in each country separately. For instance, let $U^p_{H,1}$ denote the welfare in the home country associated with the policy regime $p = \{o, a\}$:

$$U^p_{H,1} = \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \tilde{u}(Y^p_t) - \frac{1}{n_H} \int_{N_H} \tilde{v}[\ell^p_t(i)]\, di \right\}.$$

The welfare cost of adopting policy a is measured by χ_H which solves

$$U^a_{H,1} = \mathrm{E}_1 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \tilde{u}[(1-\chi_H)Y^o_t] - \frac{1}{n_H} \int_{N_H} \tilde{v}[\ell^o_t(i)]\, di \right\}.$$

Notice that χ_H can be negative, because our optimal policy is the one that maximizes the world welfare $U_{W,1}$ and not the welfare of individual countries, $U_{H,1}$ or $U_{F,1}$. The welfare cost for the foreign country, χ_F, is defined analogously.

[10] Related questions for the closed economy have been addressed by Schmitt-Grohe and Uribe (2007), and Eggertsson and Woodford (2003), among others.

We apply the second-order approximation to compute χ_W, χ_H, and χ_F, the details of which are described in the Appendix.

4.1 Basic rules

We start with the following two classes of interest rate rules: the ITR:

$$\begin{aligned}\tilde{\imath}_{H,t} &= \phi_\pi \left(\pi_{H,t} - \overline{\pi}_{H,t}\right) + \phi_x x_{H,t} + \overline{r}^n, \\ \tilde{\imath}_{F,t} &= \phi_\pi \left(\pi^*_{F,t} - \overline{\pi}^*_{F,t}\right) + \phi_x x_{F,t} + \overline{r}^n;\end{aligned} \tag{20}$$

and the PLTR:

$$\begin{aligned}\tilde{\imath}_{H,t} &= \phi_p \left(\ln P_{H,t} - \ln \overline{P}_{H,t}\right) + \phi_x x_{H,t} + \overline{r}^n, \\ \tilde{\imath}_{F,t} &= \phi_p \left(\ln P^*_{F,t} - \ln \overline{P}^*_{F,t}\right) + \phi_x x_{F,t} + \overline{r}^n.\end{aligned} \tag{21}$$

In what follows we assume that the target inflation rates in the interest-rate rules (20) are zero: $\overline{\pi}_{H,t} = \overline{\pi}^*_{F,t} = 0$; and that the target price levels in the interest-rate rules (21) are the date-0 price levels: $\overline{P}_{H,t} = P_{H,0}$ and $\overline{P}^*_{F,t} = P^*_{F,0}$. Due to the zero bound on nominal interest rates, the actual rates set by the monetary authorities are

$$i_{H,t} = \max\{\tilde{\imath}_{H,t}, 0\}, \qquad \text{and} \qquad i_{F,t} = \max\{\tilde{\imath}_{F,t}, 0\}.$$

Given that the exogenous shocks, $r^n_{H,t}$ and $r^n_{F,t}$, follow the stochastic process described in the previous section, we compare the expected world welfare (7) evaluated in period 1 under alternative policy rules. For the interest-rate rules (20)-(21), we restrict the policy parameters so that $1.1 \leq \phi_\pi \leq 5$, $0 \leq \phi_x \leq 5$, and $1.1 \leq \phi_p \leq 5$.[11] Furthermore, this parameter space is discretized with a grid size of 0.25, when searching for the optimal parameter configuration.

Table 2 shows the welfare costs of adopting different policy rules under the benchmark parameter values given in Table 1. The label "ITR" denotes the interest-rate rule with inflation targets (20) and "PLTR" denotes the interest-rate rule with price-level targets (21). For each policy rule, the policy parameters (such as ϕ_π, ϕ_p, and ϕ_x) are chosen so as to maximize the world welfare within the given parameter space.

[11] This restriction guarantees the local determinacy. As is well known, however, there remains a problem of global indeterminacy. On this issue, see, for instance, Benhabib et al. (2001).

The first two rows of Table 2 show the result for these two policy rules. The best inflation targeting rule places a positive weight on the output gap. This is in contrast with what Schmitt-Grohe and Uribe (2007) find for the closed economy without a liquidity trap. Our result here suggests that liquidity traps might provide a justification that the nominal interest rate should respond to the output gap.

More importantly, however, the table shows that the simple price-level targeting rule with $\phi_p = 5$ and $\phi_x = 0$ approximates the Ramsey equilibrium surprisingly well. The welfare cost of adopting this simple policy rule is only 0.00042 percent of consumption in the Ramsey equilibrium.[12] Adopting the best inflation targeting rule with $\phi_\pi = 5$ and $\phi_x = 0.5$ is one thousand times costly: its welfare cost is 0.46 percent of consumption in the Ramsey equilibrium. The difference between the two rules is due to the fact that the inflation targeting rule does not provide any history dependence, a key element in mitigating the severity of a liquidity trap. In contrast, with price-level targets, the nominal interest rate in each country is gradually adjusted to its steady-state level after the natural rate shocks are gone. This enables the price-level targeting rule to generate history dependence. Our result demonstrates that the claim made by Eggertsson and Woodford (2003) for the closed economy can be extended to the open economy.

The third row of Table 2 shows the importance of international cooperation in the presence of a global liquidity trap: the benefit of adopting the price-level targeting rule is obtained only if both countries adopt it. Suppose that the home country uses the price-level targeting rule while the foreign country adopts the inflation targeting rule. The best policy parameter configuration for this case is given by $(\phi_p, \phi_x) = (5, 0)$ for the home country and $(\phi_\pi, \phi_x) = (5, 0)$ for the foreign country. Then the welfare costs in terms of the world, home, and foreign welfare are 0.24, 0.28, and 0.20 percent of consumption in the Ramsey equilibrium, respectively. Thus the welfare levels of both the home and foreign households are much lower than those in the case where both countries adopt the price-level targeting rule. In this sense,

[12] Under the benchmark parameter values, the two countries are perfectly symmetric. With these two policy rules, therefore, $\chi_W = \chi_H = \chi_F$. In Tables 2-3, the decomposition of χ_W into χ_H and χ_F is reported only for the cases where the countries are not symmetric.

the international cooperation of monetary policy is crucial during a global liquidity trap.

Notice that the welfare cost for the home country is greater than that for the foreign country: $\chi_H > \chi_F$. Since the foreign country uses the inflation targeting rule, it suffers from a large drop in the output gap during the liquidity trap. Thanks to the complete asset markets, however, households in both countries decrease the amount of consumption of foreign goods by the same amount, which equally lowers the welfare of the two countries. On the other hand, the negative output gap in the foreign country reduces the labor supply there, which lowers the disutility of labor supply in the foreign country. This partially offsets the negative effect of the decrease in consumption of foreign goods for the foreign household. This is why χ_F can be lower than χ_H. This point is indeed closely related to the fact that a country has an incentive to adopt a deflationary policy in the open economy, as discussed, for instance, by Corsetti and Pesenti (2001).

4.2 Exchange rates

We next examine if augmenting the policy rules (20)-(21) with the nominal exchange rate improves the welfare. From the household's first-order conditions it follows that the nominal exchange rate $\mathcal{E}_t$ satisfies

$$\mathcal{E}_t = \frac{P_{H,t}}{P^*_{F,t}} \frac{n_F}{n_H} \frac{Y_{H,t}}{Y_{F,t}}.$$

Let $\mathcal{E}^n_t$ denote the nominal exchange rate in period t that would prevail if the first best equilibrium were attained:

$$\mathcal{E}^n_t = \frac{\overline{P}_H}{\overline{P}^*_F} \frac{n_F}{n_H} \frac{Y^n_{H,t}}{Y^n_{F,t}}.$$

Then let ϵ_t denote the exchange rate gap, i.e., $\epsilon_t \equiv \ln \mathcal{E}_t - \ln \mathcal{E}^n_t$. Notice that

$$\epsilon_t = \left(\ln P_{H,t} - \ln \overline{P}_H\right) - \left(\ln P^*_{F,t} - \ln \overline{P}^*_F\right) + x_{H,t} - x_{F,t}. \tag{22}$$

Given this, let us modify the policy rules (20)-(21) as

$$\begin{aligned} \tilde{\imath}_{H,t} &= \phi_\pi \left(\pi_{H,t} - \overline{\pi}_{H,t}\right) + \phi_x x_{H,t} + \phi_{ex}\epsilon_t + \overline{r}^n, \\ \tilde{\imath}_{F,t} &= \phi_\pi \left(\pi^*_{F,t} - \overline{\pi}^*_{F,t}\right) + \phi_x x_{F,t} - \phi_{ex}\epsilon_t + \overline{r}^n; \end{aligned} \tag{23}$$

and

$$
\begin{aligned}
\tilde{\imath}_{H,t} &= \phi_p \left(\ln P_{H,t} - \ln \overline{P}_{H,t}\right) + \phi_x x_{H,t} + \phi_{ex}\epsilon_t + \overline{r}^n, \\
\tilde{\imath}_{F,t} &= \phi_p \left(\ln P^*_{F,t} - \ln \overline{P}^*_{F,t}\right) + \phi_x x_{F,t} - \phi_{ex}\epsilon_t + \overline{r}^n.
\end{aligned}
\tag{24}
$$

Here we restrict that $\phi_{ex} \in [0, 5]$ with a grid size of 0.25.

The welfare cost of adopting these policy rules are reported in the fourth and fifth rows of Table 2. The policy parameter for each rule is again chosen to maximize the world welfare within the given parameter space. We see that as long as the price-level targeting rules are adopted by the two countries letting the nominal interest rates respond to the nominal exchange rate does not improve welfare: the best value of ϕ_{ex} for the rules (24) is zero. On the other hand, when the inflation targeting rule is adopted, augmenting the policy rule with the exchange rate can improve welfare: the best value of ϕ_{ex} for the rules (23) is 0.25 and the welfare cost goes down from 0.46 to 0.10 percent of consumption. The intuition for this result is obtained by looking at equation (22). It shows that the nominal exchange rate is related to the PPI price levels, $P_{H,t}$ and $P_{F,t}$. Thus, letting the policy rate respond to the exchange rate is an indirect way to letting it respond to the price levels.

Our result here suggests what type of international cooperation is important in the presence of a global liquidity trap. It is an agreement across monetary authorities to use the price-level targeting rule. It is not that important to make the policy rule responsive to foreign variables.[13]

4.3 Sensitivity analysis

Table 3 provides some sensitivity analysis. It reports how the welfare costs of the inflation targeting and the price-level targeting rules vary for different values of p_j, σ, and (n_H, n_F). As before, for each parameter configuration, the policy parameters, ϕ_π, ϕ_p, ϕ_x, are chosen to maximize the world welfare.

The parameter p_j denotes the probability that the natural rate shock $r^n_{j,t}$ returns to the normal level $\overline{r}^n$ for country $j \in \{H, F\}$. Panel (a) of the table shows how the welfare costs

[13] In the working paper version of this paper (Fujiwara et al., 2011), we have considered interest-rate rules that are conditioned on foreign price levels and foreign output gaps. As we report there, the welfare gains obtained by considering such rules are quantitatively small.

of the two policy rules are affected by reducing p_j for both countries. Since lowering this probability makes the liquidity trap a more prolonged event on average, the welfare losses of suboptimal policy rules are expected to be greater, which is confirmed by the table. What is notable here is that the advantage of the price-level targeting rule over the inflation targeting rule becomes greater: compared to the benchmark case, the welfare cost of the inflation targeting rule increases from 0.46 percent to 1.76 percent, while that of the price-level targeting rule rises from 0.00042 percent to only 0.00048 percent.

Panel (b) shows the sensitivity analysis with respect to σ, the degree of risk aversion. When it is lowered from 2 to 0.5, the welfare losses of both policy rules become greater. Now the welfare cost of the inflation targeting rule becomes as much as 2 percent of consumption, while that of the price-level targeting rule remains small, 0.00097 percent.

Lastly we see how the welfare costs are affected when the two countries have different sizes. Panel (c) reports the case when the home country is twice as big as the foreign country, $n_H = \frac{2}{3}$ and $n_F = \frac{1}{3}$. It illustrates that a smaller country suffers more from a suboptimal policy. But quantitatively the difference appears to be not very large.

Overall, the analysis here confirms the robustness of our basic result: the price-level targeting rule is a very good policy regime in face of a global liquidity trap. Its welfare cost is minimal when compared to that of the inflation targeting rule.[14]

5 Concluding Remarks

In this paper we have studied how monetary policy should respond to a "global liquidity trap" using a two-country New Open Economy Macroeconomics model. We first analyze the optimal monetary policy, which is designed to maximize the world welfare. Compared to the closed economy case, a notable feature of the optimal policy in the presence of a global

[14] In this paper we assume that the natural rate shocks $r^n_{H,t}$ and $r^n_{F,t}$ are independent. It would be interesting to see what happens if this assumption is relaxed. Note that if they were perfectly correlated, then our model would effectively reduce to the closed economy model of Eggertsson and Woodford (2003), where the desirability of the price-level targeting rule has already been confirmed. The case where the two shocks are imperfectly correlated would be somewhere between these two cases. For this reason, we conjecture that our basic result would also be robust against this extension.

liquidity trap is its international dependence. The optimal rate of inflation for each country is affected by whether or not the other country is in a liquidity trap. We next examine how well, in terms of welfare, the optimal monetary policy can be approximated by relatively simple monetary policy rules. We find that the optimal policy is approximated surprisingly well when each country adopts an interest-rate rule that targets its own producer price index (PPI).

The model considered in this paper is of course very stylized, and the robustness of our findings needs to be tested under alternative assumptions. For instance, we have abstracted from strategic interactions between monetary authorities in different countries. An interesting venue for future research is to allow them to choose their policy in a non-cooperative way. Another extension of potential interest would be to consider how the results would be affected if we adopted local currency pricing, rather than producer currency pricing. It may also be interesting to augment the model with a debt-deflation mechanism, which will make deflation a more serious event. These extensions are left for our future research.

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Table 1: Parameter values

Parameters	Values	Explanation
β	0.99	Subjective discount factor
p_j	0.2	Prob$(r^n_{j,t+1} = \overline{r}^n \vert r^n_{j,t} = \underline{r}^n)$, for $j = H, F$.
σ	2	Inverse of the intertemporal elasticity of substitution
θ	7.88	Elasticity of substitution among differentiated goods
ω	0.47	Elasticity of the disutility of labor supply
n_j	0.5	Size of country $j \in \{H, F\}$
α	0.66	Probability of no price adjustment
$\underline{r}^n$	-0.02/4	Negative natural rate shock
$\overline{S}$	50	Maximal length of periods with $r^n_t = \underline{r}^n$

Table 2: Welfare costs of alternative policy rules: Benchmark parameters

Rules	Policy parameters	Welfare costs (%)
ITR	$\phi_\pi = 5$, $\phi_x = 0.5$	0.46
PLTR	$\phi_p = 5$, $\phi_x = 0$	0.00042
PLTR in the home country and ITR in the foreign country	$\phi_p = 5$, $\phi_x = 0$ and $\phi_\pi = 5$, $\phi_x = 0$	world: 0.24 home: 0.28 foreign: 0.20
ITR with the exchange rate	$\phi_\pi = 5$, $\phi_x = 0.5$, $\phi_{ex} = 0.25$	0.10
PLTR with the exchange rate	$\phi_p = 5$, $\phi_x = 0$, $\phi_{ex} = 0$	0.00042

Note: ITR denotes the interest-rate rule with inflation targets. PLTR denotes the interest-rate rule with price-level targets. For each policy rule, the policy parameters such as ϕ_π, ϕ_p, ϕ_x, and ϕ_{ex} are chosen to maximize the world welfare.

Table 3: Welfare costs of policy rules: Sensitivity analysis

(a) $p_j = 0.18$ for $j \in \{H, F\}$

Rules	Policy parameters	Welfare costs (%)
ITR	$\phi_\pi = 5,\ \phi_x = 0.75$	1.76
PLTR	$\phi_p = 5,\ \phi_x = 0,$	0.00048

(b) $\sigma = 0.5$

Rules	Policy parameters	Welfare costs (%)
ITR	$\phi_\pi = 1.1,\ \phi_x = 5$	2.05
PLTR	$\phi_p = 5,\ \phi_x = 0,$	0.00097

(c) $n_H = \frac{2}{3}$ and $n_F = \frac{1}{3}$

Rules	Policy parameters	Welfare costs (%)
ITR	$\phi_\pi = 5,\ \phi_x = 0.5$	world: 0.45 home: 0.42, foreign: 0.49
PLTR	$\phi_p = 5,\ \phi_x = 0,$	world: 0.00041 home: 0.00027, foreign: 0.00069

Note: ITR denotes the interest-rate rule with inflation targets. PLTR denotes the interest-rate rule with price-level targets. For each policy rule, the policy parameters such as ϕ_π, ϕ_p, and ϕ_x are chosen to maximize the world welfare.

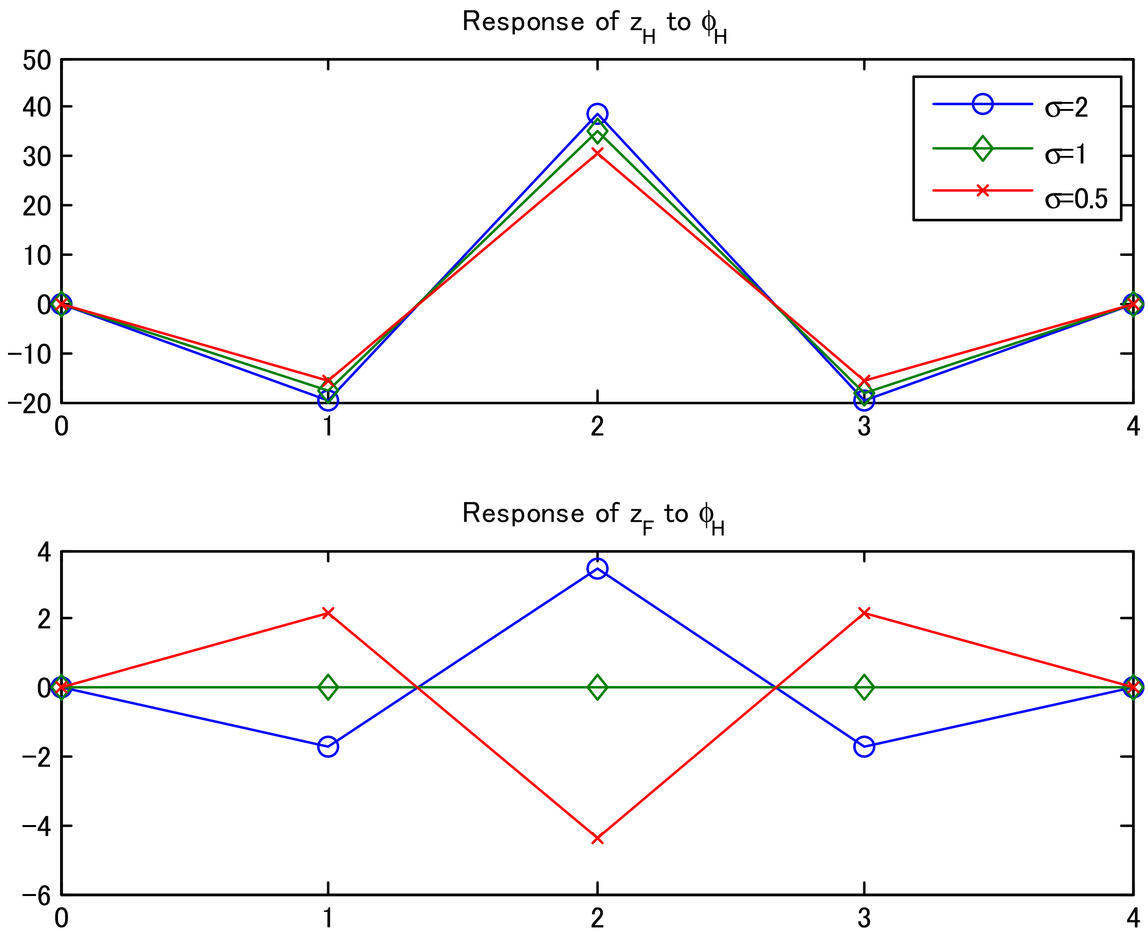

Figure 1: Response of $z_H$and z_F to a one-time increase in ϕ_H for different values of σ. The upper panel shows the dynamics of z_H and the lower panel shows the dynamics of z_F .

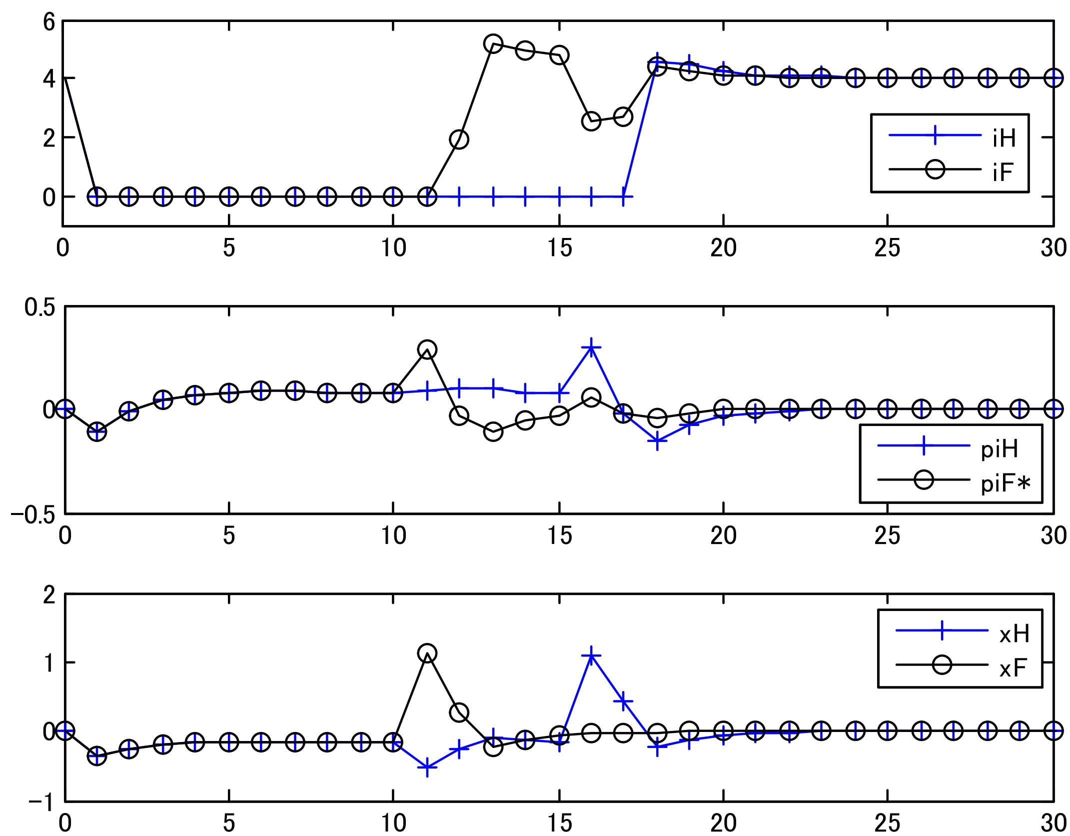

Figure 2: The optimal paths of i_H, i_F, π_H, π_F^*, x_H, and x_F when $T_H = 15$ and $T_F = 10$.