Wave-number spectral characteristics of drift wave micro-turbulence with large-scale structures

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Abstract

Wave-number spectral characteristics of drift wave micro-turbulence with large-scale structures (LSSs) including zonal flows (ZFs) and Kelvin-Holmheltz (KH) mode are investigated based on 3-dimensional gyrofluid simulations in a slab geometry. The focus is on the property of the wave-number spectral scaling law of the ambient turbulence under the back reaction of the self-generated LSSs. The comparison of the spectral scaling laws between ion/electron temperature gradient (ITG/ETG) driven turbulence is presented. It is shown that the spectral scaling of the ITG turbulence with robust ZFs are fitted well by an exponential-law function $\langle \phi^2/2 \rangle_E \propto e^{-\lambda k_x}$ in k_x and a power-law one $\langle \phi^2/2 \rangle_P \propto k_y^{-\beta}$ in k_y. However, the ETG turbulence is characterized by a mixing Kolmogorov-like power-law and exponential-law $\left\langle \phi^2/2 \right\rangle \propto e^{-\lambda k_{x,y}} k_{x,y}^{-3} \big/ (1+k_{x,y}^2)^2$ scaling for both k_x and k_y spectra due to the ZFs and KH mode dynamics. Here λ and β are the slope index factors. The underlying physical mechanism is understood as the spectral scattering caused by the back-reaction of the LSSs on the ambient turbulence. These findings may provide helpful guideline to diagnose the plasma fluctuations and flow structures in experiments.

Keywords: Spectral scaling law, zonal flows, micro-turbulence, gyrofluid simulation

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1. Introduction

Experimental measurement of plasma flows and turbulence is of essential importance in transport and confinement study [1,2]. Recently the diagnoses of the edge fluctuations and flows including the zonal flows(ZFs) and geodesic acoustic mode(GAM) in tokamak and stellarator plasmas have been extensively carried out to characterize the structure formation such as the internal transport barriers (ITBs) and to understand the suppression mechanism of the turbulent transport by sheared mean flows and ZFs [3,4]. Plasma fluctuations are of wide spatio-temporal scale, which is conventionally described by wave-number and frequency spectra. Various linear and nonlinear instabilities and coherent large-scale structures (LSSs) such as the ZFs, streamers and the generalized Kelvin-Holmheltz (GKH) modes are categorized by the spectral characteristics [5]. The spectral analysis is extensively applied to investigate the nonlinear interaction processes through the basic three-wave coupling. Furthermore, the turbulence spectra are some of the measurable quantities in experiments, which are generally diagnosed and analyzed to characterize the fluctuation and the coherent structures.

Specifically, the drift wave turbulence is likely one of the most common fluctuations in tokamak plasmas, which has a big family including all modes originated from plasma gradients as well as the magnetic field inhomogeneity [6]. In past years, the ion temperature gradient (ITG) driven drift wave turbulence has attracted much attention [7]. A key physical mechanism involved is ascribed to the suppression of the turbulent fluctuation and ion heat transport by the ZFs [3]. Similarly, the electron temperature gradient (ETG) driven turbulence is expected hopefully to produce large electron transport [5]. Furthermore the trapped electron mode (TEM) is also proposed to be responsible to the electron transport recently. Theoretically speaking, the ZF is robust in drift wave turbulence so that the relevant problem is frequently referred to as drift wave-ZF turbulence [3]. The ZF is generated nonlinearly through three-wave triad couplings of higher-*k* components in ambient turbulence and may be saturated by the excitation of a long wavelength KH mode [5,8,9]. These LSSs have been extensively testified by employing the bispectral analysis in experiments and simulations [1,2]. Meanwhile, their back reaction on the ambient turbulence may be also generally considered as the result of the nonlinear mode coupling to dissipation range at smaller scales. The demonstration of such processes in experiments requires detailed measurements of the drift wave turbulence at wide scale. Furthermore it is also necessary to identify the type of the ambient turbulence such as the ITG or TEM in experiments.

Usually the drift wave turbulence is vaguely assumed to generate the ZFs as *a priori* in tokamak plasmas based on theoretical argument [1-4]. The statistical dispersion relation of the fluctuations with the Doppler frequency shift effect due to the radial electric field produced $\vec{E}\times\vec{B}$

shear flows is generally analyzed in experiments [10]. In fact the turbulence should be characterized by the nonlinear spectral scaling of the frequency and wave-number since highly complex nonlinearities make it significantly different from the linear theory. A fluid example is the well-known Kolmogorov power-law scaling of turbulent flows [11]. A typical counterpart in strong plasma turbulence is estimated roughly by Hasegawa and Mima (HM) based on three-wave interaction [12], which shows some deviation from the usual Kolmogorov power-law featured by a factor $k^{1.8}/(1+k^2)^{2.2}$. Recently a theoretical derivation based on Hasegawa-Wakatani (HW) turbulence model [13] predicts a wave-number spectrum featured by $e^{-\lambda k}k^{-3}/(1+k^2)^2$, which is also compared with the experimental observation [14]. Here λ is determined by the growth-damping profile, which represents the dissipation and the ZF dynamics. Hence the wave-number spectral scaling is of much importance in characterizing the turbulence structure and understanding the complex nonlinear interaction between the turbulent fluctuations and flows.

In this work, we present the wave-number spectral scaling of both ITG and ETG turbulence based on gyrofluid simulations. The underlying physical mechanism behind the scaling law is analyzed considering the back-reaction of the LSSs on the ambient turbulence. The comparison between the ITG and ETG turbulence spectra is carried out to clarify the role of the LSSs in forming the spectral scaling law of the drift wave turbulence. In Sec.II the physical model and numerical setting of 3D gyrofluid simulations are described in a slab geometry. The understanding of the back reaction of the ZFs on the ambient ITG turbulence is proposed through the feature of the spectral scaling in Sec.III. The spectral analysis of the ETG turbulence is given in Sec. IV. Finally the principal results are summarized in Sec. IV.

2. Gyrofluid model of drift wave turbulence simulations

Electrostatic ITG and ETG turbulence is of a perfect isomorphism except for different response of the adiabatic component to the ZFs. For the ITG turbulence, adiabatic electron response is assumed but no response of electron density perturbation to the zonal flow component [15,16]. On the other hand, the adiabatic ion response in the ETG turbulence corresponds to the Boltzmann distribution of the ions for all fluctuation components [5]. To investigate the fundamental spectral characteristics of the drift wave-ZF turbulence, the nonlinear evolution of ITG and ETG [or $\eta_{i,e}(=d\ln T_{i,e}/d\ln n)$] modes, typical drift wave fluctuations in tokamak plasmas, are simulated in sheared slab configuration for simplicity so that the simulation results can be compared with some theoretical prediction, which is usually derived from the HM or HW turbulence [12-14]. Here a set of modeling equations describing the ITG-ZF dynamics with adiabatic electron response is described as follows, i.e. [15],

$$\left(1-\delta-\nabla_\perp^2\right)\partial_t\phi=-\left(1+K\nabla_\perp^2\right)\partial_y\phi-\nabla_{//}\upsilon_{//}+\left[\phi,\nabla_\perp^2\phi\right]-\partial_x\bar{\phi}\partial_y\phi-\mu_\perp\nabla_\perp^4\phi \quad , \tag{1}$$

$$\partial_t\upsilon_{//}=-\nabla_{//}\phi-\nabla_{//}p-\left[\phi,\upsilon_{//}\right]+\eta_\perp\nabla_\perp^2\upsilon_{//} \quad , \tag{2}$$

$$\partial_t p=-K\partial_y\phi-(\Gamma-1)\sqrt{8/\pi}\left|k_{//}\right|(p-\phi)-\Gamma\nabla_{//}\upsilon_{//}-\left[\phi,p\right]+\chi_\perp\nabla_\perp^2 p \quad . \tag{3}$$

Here $\nabla_\perp^2=\partial_{xx}^2+\partial_{yy}^2$, $\nabla_{//}=ik_{//}=\partial_z+\hat{s}x\partial_y$ with magnetic shear $\hat{s}=L_n/L_s$, $L_{n;s}$ are the characteristic length of plasma density and magnetic field, respectively. $K=1+\eta_i$, $\Gamma=5/3$. The term with $\partial\bar{\phi}/\partial x$ in Eq.(1) is from the correct adiabatic electron response to the self-generated ZFs in ITG turbulence. $\delta=1$ is for ZF component and $\delta=0$ for fluctuations [16]. The Poisson brackets $[f,g]=\hat{z}\cdot\nabla_\perp f\times\nabla_\perp g=\partial_x f\partial_y g-\partial_x g\partial_y f$ indicate the $\vec{E}\times\vec{B}$ convective nonlinear terms. The Landau damping is represented by closure model $q_{//}=-i\sqrt{8/\pi}\,k_{//}T/\left|k_{//}\right|$ (Ref.17). The cross-field dissipation terms with $\mu_\perp$, $\eta_\perp$ and $\chi_\perp$ are included to absorb the energy at short wavelength range [18]. $\vec{x}$ and $\vec{y}$ correspond to the radial and poloidal directions in a toroidal plasma, respectively. The perturbed quantities are conventionally normalized at ion-scale [15]. The modeling equations for the ETG version are similar in form with corresponding normalization at electron-scale and the adiabatic response [5].

The nonlinear equations (1-3) can be numerically solved by using an initial value code, in which Fourier spectral decomposition in both y and z directions and an implicit finite difference scheme for x variable are employed. The details of the code have well documented in previous publication [5,15]. 3D simulation is bounded in domain (L_x,L_y,L_z) with reference parameters typically: $\eta_i=2.5$, $\hat{s}=0.4$, $\mu_\perp=\eta_\perp=\chi_\perp=0.1$, $L_x=50\rho_i\sim200\rho_i$, $L_y=10\pi\rho_i$, $L_z=2\pi L_n$. The resolution and maximum wave-numbers of k_y in the simulations are $\Delta k_y=0.2$ and $k_y^{Max}=3.0$, respectively. In the following simulations, all zonal components of perturbed variables $(\phi;\upsilon_{//};p)$ have been included except for the cases with artificially excluding the ZFs.

3. Spectral scaling of ITG-ZF turbulence

It is well-known that the ZF in ITG turbulence is robust so that the ambient turbulence and ion heat transport are evidently suppressed [3]. While the suppression mechanism is usually understood through the flow shearing of the fluctuation, here we investigate the wave-number spectral characteristics under the back reaction of the ZFs to clarify the underlying interaction processes between the ZFs and turbulence. Fig.1 illustrates the k_x and k_y spectra of both the ITG-ZF turbulence and the ITG turbulence with artificially excluding the ZFs (hereafter referred to as ITG-no-ZF) for comparison. Simulations reveal several typical features [19]. First the k_x spectrum of ITG-ZF turbulence fits well with an exponential-law function, i.e, $\langle\phi^2/2\rangle_E=Be^{-\lambda k_x}$, at short wavelength regime $k_x \geq 1.0$, while the counterpart in ITG-no-ZF turbulence is described by a Kolmogorov-like power-law function $\langle \phi^2/2 \rangle_P = Ak_x^{-\alpha}/(1+k_x^2)^2$. Second the k_y spectra of both ITG-ZF and ITG-no-ZF turbulence are characterized by a pure power-law scaling $\langle \phi^2/2 \rangle_P \propto k_y^{-\beta}$ with different slope index. Here (λ, α, β) are slope index factors. Furthermore, parametric scans of the k_x spectral scaling laws show better agreement with the fitting functions for the ITG-ZF turbulence with weak magnetic shear, low viscosity damping and strong drive force η_i, as shown in Fig.2 as an example of the η_i scan, in which the ZF is stronger. It is shown that the spectral scaling laws do not change with such parameters in a certain range but the slope index of the scaling law depends on their values, showing that the zonal flow dynamics may dominate the turbulence property in the parametric range [19]. The spectral scaling laws of the ITG turbulence with and without the zonal flow dynamics observed in the simulations have been compared with the theoretical derivation [14] based on the Hasegawa-Wakatani drift wave model in detail in [19], showing a fair agreement with each other.

The comparison of the k_x spectral characteristics in ITG-ZF and ITG-no-ZF turbulence and the comparison between k_x and k_y spectra in ITG-ZF turbulence may exhibit the back reaction mechanism of the ZFs on the ambient turbulence since the ZF has only radial k_q spectrum. It could be understood that the ZFs may subsequently scatter the ITG spectrum from the most unstable range of $k_x \leq 1.0$ to the short wavelengths $k_x \approx 1.0 \sim 3.0$ through the nonlinear three-wave coupling with $k_{x2} = k_{x1} + k_q$. Then the k_x spectrum of the ITG-ZF turbulence is deformed at short wavelengths. The spectral scattering by the ZFs through $k_{x2} = k_{x1} + k_q$ can transfer the free energy of the ITG instability to the dissipation range. As a result, the total free energy, on one hand, is damped versus the viscosity, showing a dramatic suppression of ITG fluctuations. On the other hand, the subsequent ZF spectral scattering play a driving role in transferring the free energy to larger k_x range. Hence the k_x spectrum of the ITG turbulence could be featured by an exponential-law deformed from a Kolmogorov-like power-law under the back reaction of the ZFs.

4. Spectral scaling of ETG-ZF-KH turbulence

Generally speaking, the ZFs are relatively weak in ETG turbulence compared with those in ITG turbulence except for the weak magnetic shear case [20]. The difference may mainly originate from the density response of the corresponding adiabatic particles to the zonal flows in the ITG or ETG turbulence. In the ITG turbulence, the zonal flow is quite robust so that the other kind of large-scale structures like the streamers and KH mode with finite k_y are difficult to be observed. It has been demonstrated that weak shear can enhance the ZF generation in ETG turbulence, which is saturated by the excitation of long wavelength KH mode. In this sense, the ETG turbulence is a mixture of ETG, ZFs and KH fluctuations [5]. To understand the nonlinear interaction among them and further clarify the deformation mechanism of spectral scaling law due to the back reaction of the LSSs in k_x or k_y direction, 3D ETG simulation is performed with weak magnetic shear and larger η_e. Simulations show that after the ETG saturation, the ZF starts to grow up exponentially with small growth rate. The KH mode is then excited to saturate the ZFs when the ZF amplitude becomes higher than a threshold. The wave-number spectra at the quasi-steady state are plotted in Fig.3. A mixing Kolmogorov-like power-law and exponential-law scaling for both k_x and k_y spectra is observed in ETG-ZF-KH turbulence. It is noticed that this spectral scaling law is qualitatively in agreement with the theoretical prediction based on a Hasegawa-Wakatani turbulence model [14]. In ETG-no-ZF turbulence, the k_x spectrum is characterized by Kolmogorov-like power-law scaling and the k_y spectrum fits with a power-law scaling. This can be understood as the back reaction of the weaker ZFs and the KH mode in the k_x direction and the spectral scattering by the KH mode in the k_y direction.

To further manifest the back reaction interaction process, a 2D simulation with very high resolution in the y direction is performed as shown in Fig.4, in which the KH mode is slowly excited [21]. During the KH evolution from the creation to the saturation in ETG-ZF turbulence, the k_y spectral characteristic is changing from an approximate Kolmogorov power-law to a mixing Kolmogorov-like power-law and exponential-law scaling as shown in Fig.5. These evidences positively support the underlying mechanism behind the spectral scaling laws.

5. Summary

The wave-number spectral characteristics of the drift wave micro-turbulence at ion and electron scales have been investigated based on 3D gyrofluid simulation with an emphasis on the back reaction mechanism of the self-generated LSSs on the ambient turbulence. It is found that the k_x spectrum of the ITG-ZF turbulence is fitted well by an exponential-law scaling, which is deformed from the usual Kolmogorov-like power-law by the back reaction of the robust ZFs through the spectral scattering. On the other hand, the ETG turbulence with ZFs and long wavelength KH fluctuation is characterized by a mixing Kolmogorov-like power-law and exponential-law scaling for both k_x and k_y spectra due to the back reaction of both the ZFs and KH mode. These observations are qualitatively in agreement with the theoretically predicted spectral scaling in drift wave turbulence based on a reduced turbulence modeling analysis and fairly match with the experimental observation of the density fluctuation spectrum in Tore Supra tokamak [22]. Hence, they may be likely applied to the data analyses of turbulence and flow measurement in tokamak experiments.

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Figures and Captions

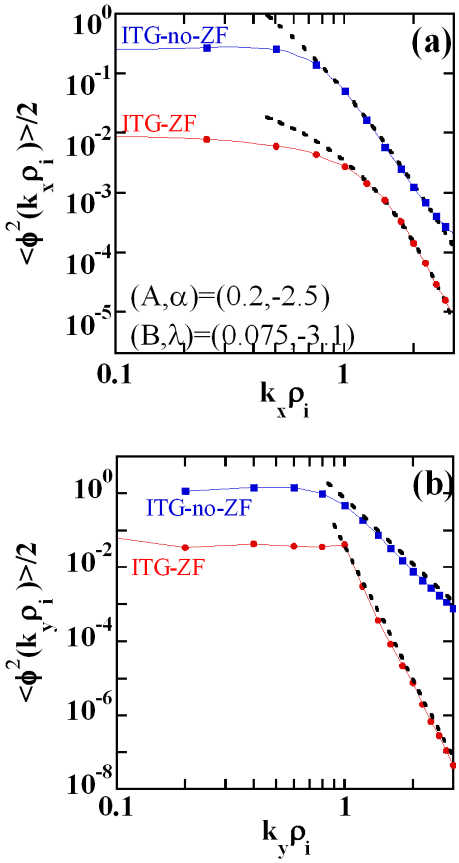

Fig.1 k_x (a) and k_y (b) spectra of both the ITG-no-ZF and the ITG-ZF turbulence. The dashed curves in (a) plot corresponding fitting functions $\langle\phi^2/2\rangle_P = Ak_x^{-\alpha}/(1+k_x^2)^2$ and $\langle\phi^2/2\rangle_E = Be^{-\lambda k_x}$. The dot straight lines in (b) are for reference.

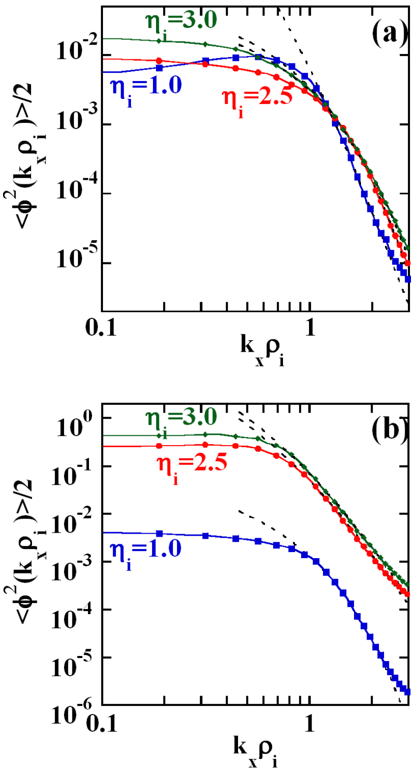

Figure 2: k_x spectra of both the ITG-ZF (a) and the ITG-no-ZF (b) turbulence for different η_i. The dashed curves plot corresponding best fitting functions $\langle \phi^2/2 \rangle_E = Be^{-\lambda k_x}$ in (a) and $\langle \phi^2/2 \rangle_P = Ak_x^{-\alpha}/(1+k_x^2)^2$ in (b). $\hat{s}=0.2$, $\mu_\perp = 0.1$.

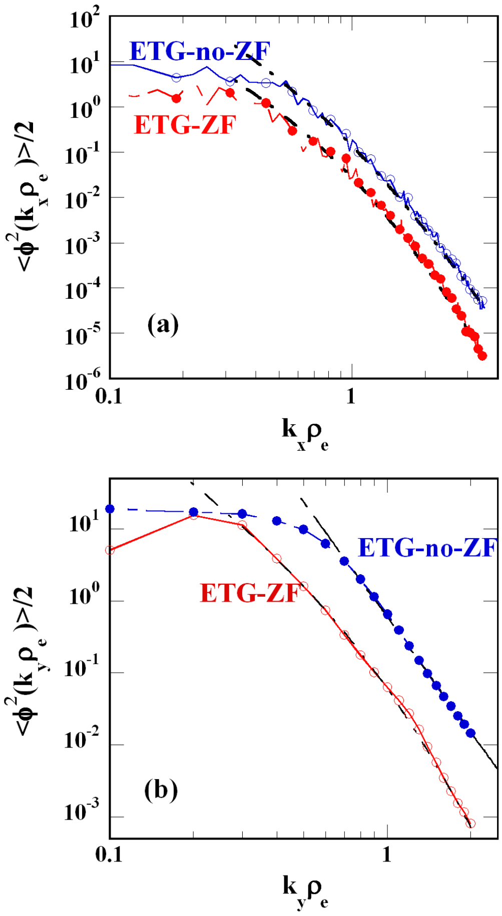

Fig.3 k_x (a) and k_y (b) spectra of both the ETG-ZF and the ETG-no-ZF turbulence. The dot-dashed curves matching with the spectra in ETG-ZF turbulence in both (a) and (b) plot corresponding best fitting functions $\langle\phi^2/2\rangle_P = Ae^{-\lambda k_{x,y}} k_{x,y}^{-\alpha} / (1+k_{x,y}^2)^2$ and the one to the ETG-no-ZF case in (a) corresponds to the best fitting function $\langle\phi^2/2\rangle_P = Ak_x^{-\alpha}/(1+k_x^2)^2$. The dashed straight line in (b) represents the power-law function $\langle\phi^2/2\rangle_P \propto k_y^{-\alpha}$ for reference.

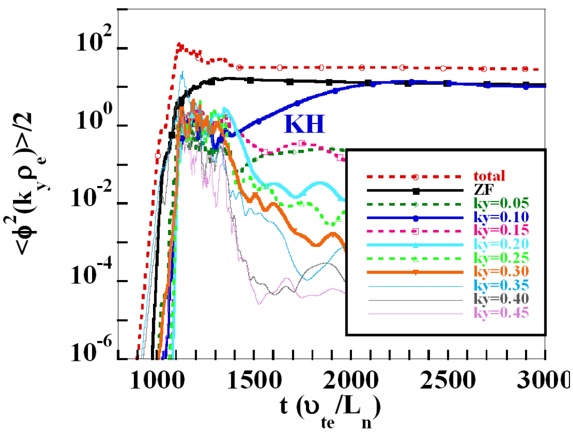

Fig.4 Time history of fluctuation energy $\langle\phi^2\rangle/2$ of representative k_y components in 2D ETG-ZF turbulence with high resolution. $\hat{s}=0.1$, $\eta_e=5.5$, $\mu_\perp=1.5$, $L_x=400\rho_e$, $L_y=160\pi\rho_e$.

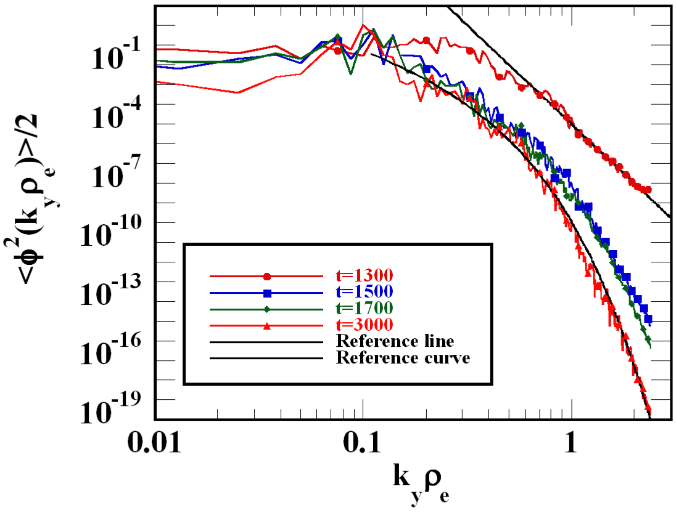

Fig.5 Time evolution of k_y spectrum in ETG-ZF turbulence of Fig.4. The change of the spectral scaling law is due to the KH creation after $t \approx 1500$. The reference line corresponds to fitting function $\langle \phi^2 / 2 \rangle_P \propto k_y^{-10.0}$, the reference curve is to $\langle \phi^2 / 2 \rangle_M \propto e^{-13.0 k_y} k_y^{-3} / (1 + k_y^2)^2$.