# Sustainable building design under uncertain structural-parameter environment in seismic-prone countries

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## ABSTRACT

The structural member stiffness and strength of buildings are uncertain due to various factors resulting from randomness, material deterioration, temperature dependence etc. The concept of sustainable building design under such uncertain structural-parameter environment may be one of the most challenging issues to be tackled recently. By predicting the response variability accurately, the elongation of service life of buildings may be possible.

In this paper, it is shown that interval analysis in terms of uncertain structural parameters is an effective tool for evaluating the sustainability of buildings in earthquake-prone countries. All the combinations of uncertain structural parameters become huge numbers and this difficulty can be overcome by introducing the sensitivity or Taylor series expansion analysis.

In order to demonstrate the usefulness and reliability of the proposed method, a shear building model is used including passive viscous dampers with supporting members. It is demonstrated that the proposed method is actually useful for the development of the concept of sustainable building design under such uncertain structural-parameter environment.

Keywords: Sustainable buildings, Uncertainty, Earthquake engineering

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## **1. Introduction**

The structural control with passive dampers has a successful history in mechanical and aerospace engineering. This may result from the fact that these fields usually deal with predictable external loading and environment with little uncertainty. However, in civil engineering, it has a different situation (Housner et al., 1994; Housner et al., 1997; Soong and Dargush, 1997; Kobori et al., 1998; Srinivasan and McFarland, 2000; Casciati, 2002; Christopoulos and Filiatrault, 2006; Johnson and Smyth, 2006; de Silva, 2007; Cheng et al., 2008; Takewaki, 2009). Building and civil structures are often subjected to severe earthquake ground motions, wind disturbances and other external loading with large uncertainties (Takewaki 2007). It is therefore inevitable to take into account of these uncertainties in their structural design and application to actual structures.

While the structural control is a promising and smart tool for sustainable building design (Takewaki et al., 2011), it is also true that a lot of uncertainties should be quantified for reliable implementation of these techniques (Takewaki and Ben-Haim 2005). The sustainable building design under uncertain structural-parameter environment may be one of the most challenging issues in the building structural engineering. Even if all the design constraints are satisfied at the initial construction stage, some responses to external loadings (earthquakes, strong winds, etc.) during service life may violate such constraints due to various factors resulting from randomness, material deterioration, temperature dependence etc. To overcome such difficulty, response evaluation methods for uncertain structural-parameter environments are desired. By predicting the response variability accurately, the elongation of service life of buildings may be possible.

In this paper, it is shown that interval analysis (see, for example, Moore, 1966; Alefeld and Herzberger, 1983; Qiu et al, 1996; Mullen et al., 1999; Koyluoglu and Elishakoff, 1998; Qiu, 2003; Chen and Wu, 2004; Chen et al, 2009) in terms of uncertain structural parameters is an effective tool for evaluating the sustainability of buildings in earthquake-prone countries. All the combinations of uncertain structural parameters become huge numbers and this difficulty can be overcome by introducing the sensitivity or Taylor series expansion analysis.

In order to demonstrate the usefulness and reliability of the proposed method, a shear building model including passive viscous dampers with supporting members is subjected to a set of scaled earthquake ground motions and the time-history response analysis is used for simulating the earthquake response. The critical combination of interval parameters is found by introducing an assumption of 'inclusion monotonic' and the sensitivity information by Taylor series expansion. It is demonstrated that the proposed method is actually useful for the development of the concept of sustainable building design under uncertain structural-parameter environments.

The design earthquake ground motions change from time to time when a new class of ground motions (e.g. long-period ground motions due to surface waves) is observed or a new type of damage appears during severe earthquakes. Because the proposed method can easily add these earthquake ground motions, the flexibility of the proposed method is expected to be high.

# 2. Concept of sustainable building design under uncertain structural-parameter emvironment

The concept of sustainable building design under uncertain structural-parameter environments is illustrated in Fig.1. The member stiffness and strength of buildings are uncertain due to various factors resulting from randomness, material deterioration, temperature dependence etc. The damping coefficients of structural members and/or passive dampers may also be uncertain (Takewaki and Ben-Haim 2005). The time variation of Young's modulus and damping coefficients are shown in Fig.1 as representative examples. Karbhari and Lee (2010) discusses the service life estimation and extension of civil engineering structures from the viewpoints of material deterioration. These member and/or damper uncertainties lead to response variability of buildings under earthquake ground motions. Efficient and reliable methods are desired for predicting the upper bound of such building response.

### 3. Interval analysis methods for uncertain structural parameters

Fig.2 shows the relationship between the variation of the objective function f (response quantity) and a structural parameter combination for the cases of 'inclusion monotonic' and 'inclusion non-monotonic'.

Based on the assumption of "*inclusion monotonic*", we can derive the upper and lower bounds of f by iterative calculations with all end-point combinations  $(2^{N_x} \text{ for } N_x \text{ interval}$ parameters), i.e. the upper and lower bounds of interval parameters. However, when the number  $N_x$  of interval parameters is extremely large, this primitive approach needs much computational time caused by a large combination number of interval parameters.

From the practical point of view, a more efficient methodology is desired which can estimate the upper and lower bounds of the objective function without a hard computational task. The interval analysis methodology using the approximation of Taylor series expansion has been developed so far (Chen and Wu 2004; Chen et al., 2009). The formulation of Taylor series expansion in the interval analysis and the achievements of second-order Taylor series expansion proposed in the reference (Chen et al., 2009) are explained in this section.

#### 3.1 Interval analysis method based on approximation of first-order Taylor series expansion

Let  $\mathbf{X} = \{X_i\}, \mathbf{X}^c = \{X_i^c\}, \Delta \mathbf{X} = \{\Delta X_i\}$  denote the interval parameters, nominal parameter values and half intervals. Let ()<sup>*I*</sup> and [*a*, *b*] denote the definition of an interval parameter where *a* and *b* are the lower and upper bounds of the interval parameter, respectively. Then  $X_i^I = [X_i^c - \Delta X_i, X_i^c + \Delta X_i]$ .

The upper and lower bounds  $\overline{f}, \underline{f}$  of the objective function by the interval analysis method using first-order Taylor series expansion can be expressed as

$$\overline{f} \cong f\left(\mathbf{X}^{c}\right) + \sum_{i=1}^{N_{X}} \left| f_{,X_{i}} \Delta X_{i} \right|$$
(1)

$$\underline{f} \cong f\left(\mathbf{X}^{c}\right) - \sum_{i=1}^{N_{X}} \left| f_{,X_{i}} \Delta X_{i} \right|$$
(2)

where ( ),  $_{X_i}$  and  $N_X$  denote differentiation  $\partial f(\mathbf{X})/\partial X_i|_{X_i=X_i^c}$  of the objective function at the nominal value and the number of uncertain parameters, respectively. Therefore  $f_{,X_i}$ corresponds to a gradient of the objective function f with respect to *i*-th interval parameter  $X_i$  for the nominal model.

### 3.2 Interval analysis method based on approximation of second-order Taylor series expansion

Although an approximation using first-order Taylor series expansion can be achieved without hard task, the result by this approximation may include a large error especially for a wide range of interval parameters. So as to enhance the accuracy of the interval analysis method, an approximation using second-order Taylor series expansion has been developed in the reference (Chen et al., 2009). An approximate objective function  $f^*$  using second-order Taylor series expansion around the nominal model can be described as

$$f^{*}(\mathbf{X}) = f(\mathbf{X}^{c}) + \sum_{i=1}^{N_{X}} f_{,X_{i}}(X_{i} - X_{i}^{c}) + \frac{1}{2} \sum_{i=1}^{N_{X}} \sum_{j=1}^{N_{X}} f_{,X_{i}X_{j}}(X_{i} - X_{i}^{c})(X_{j} - X_{j}^{c})$$
(3)

where ( ),  $x_i x_j$  denotes second-order differentiation of the objective function at the nominal value. Therefore,  $f_{X_i X_j}$  corresponds to the Hessian matrix of the objective function f with respect to the *i*-th and *j*-th interval parameters  $X_i$ ,  $X_j$  for the nominal model. Based on the general interval analysis method, the upper and lower bounds of Eq.(3) can be evaluated by calculating all the end-point combinations of interval parameters and judging whether the objective function is the maximum or minimum value. The number of calculations in this approach is also  $2^{N_X}$  which is the same number of calculation as that in the primitive interval

analysis. However, the computational load for evaluating the objective function for each combination of interval parameters can be greatly reduced by using sensitivities around the nominal model.

By using the approximation of Taylor series expansion, iterative response analyses can be avoided. However, the computation of full elements of the Hessian matrix requires much time when  $N_X$  is large, especially for numerical sensitivity analysis, i.e. the finite difference analysis using gradient vectors. For this reason, a more simple approach has been proposed in the reference (Chen et al., 2009) where the non-diagonal elements of the Hessian matrix are neglected. An approximate objective function  $f^{**}$  using second-order Taylor series expansion with only diagonal elements can be rewritten from Eq.(3) as

$$f^{**}(\mathbf{X}) = f(\mathbf{X}^{c}) + \sum_{i=1}^{N_{X}} \left\{ f_{,X_{i}}(X_{i} - X_{i}^{c}) + \frac{1}{2} f_{,X_{i}X_{i}}(X_{i} - X_{i}^{c})^{2} \right\}$$
(4)

From Eq.(4), we can evaluate the increment of the objective function by using first and second-order Taylor series expansion approximation as the sum of the increments of the objective function in the one-dimensional domain. If we regard all interval parameters except  $X_i$  as nominal values in the incremental term in Eq.(4), the perturbation  $\Delta f_i(\mathbf{X})$  of the objective function by the variation of  $X_i$  can be described as

$$\Delta f_i \left( X_1^c, \dots, X_{i-1}^c, X_i, X_{i+1}^c, \dots, X_{N_X}^c \right) = f_{X_i} \left( X_i - X_i^c \right) + \frac{1}{2} f_{X_i X_i} \left( X_i - X_i^c \right)^2 \tag{5}$$

In Eq.(5), the interval extension  $\Delta f_i^I$  of the one-dimensional perturbation can be derived as

$$\Delta f_i^{I} = \begin{bmatrix} \min\left[\Delta f_i\left(X_1^c, \dots, \overline{X}_i, \dots, X_{N_X}^c\right), \Delta f_i\left(X_1^c, \dots, \underline{X}_i, \dots, X_{N_X}^c\right)\right], \\ \max\left[\Delta f_i\left(X_1^c, \dots, \overline{X}_i, \dots, X_{N_X}^c\right), \Delta f_i\left(X_1^c, \dots, \underline{X}_i, \dots, X_{N_X}^c\right)\right] \end{bmatrix}$$
(6)

Finally, substituting  $\Delta f_i^I$   $(i = 1, \dots, N_X)$  into Eq.(4), the interval extension of the approximate objective function  $f^{**}$  can be obtained as

$$f\left(\mathbf{X}^{I}\right) \approx \left[f\left(\mathbf{X}^{c}\right) + \sum_{i=1}^{N_{X}} \underline{\Delta f_{i}}\left(X_{i}^{I}\right), f\left(\mathbf{X}^{c}\right) + \sum_{i=1}^{N_{X}} \overline{\Delta f_{i}}\left(X_{i}^{I}\right)\right]$$
(7)

It is remarkable that the number of calculations in Eq.(7) is reduced to  $2 \times N_X$  compared with  $2^{N_X}$  in Eq.(3). For this reason, the computational load can be dramatically reduced by neglecting non-diagonal elements of the Hessian matrix. Fig.3 shows the concept of the interval analysis method using Taylor series approximation.

# 4. Advanced interval analysis method based on the information of the approximation of Taylor series expansion

When the degree of uncertainty of interval parameters is large, the result of the interval analysis applying the approximation of Taylor series expansion of the objective function may include numerical errors. On the other hand, a reliable result can be derived by reanalyzing the objective function with the obtained structural parameters. In this section, the advanced interval analysis method is presented using reanalysis based on the information of interval parameter set derived by the Taylor series approximation.

# 4.1 Reanalysis approach based on the structural parameter set derived by the Taylor series approximation

From Eqs.(5) and (6), the combination of the end-points  $\hat{\mathbf{X}}$  of the interval parameters  $\mathbf{X}$  which maximizes the perturbation  $\Delta f_i \left( X_1^c, \dots, X_{i-1}^c, X_i, X_{i+1}^c, \dots, X_{N_X}^c \right)$   $(i = 1, \dots, N_X)$  of the objective function can be derived as

$$\hat{\mathbf{X}} = \left\{ X_i \text{ so as to } \max\left[\Delta f_i\left(X_i\right)\right], i = 1, \cdots, N_X \right\}$$
(8)

The upper bound of the objective function can be evaluated using a reliable response analysis method (time-history response analysis) for a regenerated structural model with the critical combination of interval parameters set (Eq.(8)). It should be mentioned that the objective

function evaluated by the time-history response analysis method will not exceed the feasible domain of the objective function. The flowchart of this proposed methodology is as follows.

- **Step 1** Calculate the gradient vector  $f_{X_i}$   $(i=1,\dots,N_X)$  and diagonal element of Hessian matrix  $f_{X_iX_i}$   $(i=1,\dots,N_X)$  of the objective function for the nominal model.
- **Step 2** Evaluate Eq.(6) for the upper and lower bounds of the interval parameter  $X_i$ .
- **Step 3** Derive the target end-point combinations  $\hat{\mathbf{X}}$  of interval parameters corresponding to the upper bound of Eq.(6).
- **Step 4** Evaluate the objective function by the time-history response analysis for given structural parameters  $\hat{\mathbf{X}}$ .

#### 4.2 Varied evaluation point (VEP) method considering the influence of initial value dependency

In order to obtain the reliable result of the response variability by using the proposed advanced interval analysis methodology in the previous section, it is important that the assumption of "*inclusion monotonic*" is satisfied for the objective function. If the objective function, e.g. the maximum interstorey drift of a damped structure subjected to a ground motion as shown in the numerical examples (Section 5), has a property of non-monotonic variation for the variation of the interval parameters, the interval analysis method using the first and second-order sensitivities at the nominal model will not provide a reliable response variability. To overcome this difficulty, an additional numerical procedure should be introduced to search for the evaluation point set for the evaluation of first and second-order sensitivities. In this paper, it is shown in the later numerical example that the evaluation point selected randomly for the combination of the interval parameters is effective. This is called the varied evaluation point (VEP) method. Fig.4 shows the concept of the VEP method.

### 4.3 Search of the exact solution

In numerical examples, an approximate candidate of the exact solution of the maximum or

minimum value of the objective function in a feasible domain of interval parameters is calculated by solving the original problem with the Sequential Quadratic Programming (SQP) method. In this analysis for the exact solution, the approximation by Taylor series expansion is not employed and the time-history response analysis for successive points of interval parameters is conducted. The problem of finding an approximate candidate of the exact solution of the maximum or minimum value of the objective function may be described by

Find 
$$\mathbf{X} = \{c_{d_1}, \dots, c_{d_N}, k_{b_1}, \dots, k_{b_N}, k_{f_1}, \dots, k_{f_N}\}^T$$
  
so as to maximize  $f(\mathbf{X})$   
subject to  $\mathbf{X} \in \mathbf{X}^I$  (9)

### 5. Numerical examples

Numerical examples are presented for 20-storey shear building models with passive viscous dampers to demonstrate the validity and accuracy of the proposed methodology. Fig.5 presents the shear building model with passive viscous dampers including supporting members. The properties of the nominal structural parameters are shown in Table 1. The floor masses are identical in all the storeys. The frame stiffness distribution in the nominal model is given by Eq.(10) which can be derived from the straight-line shape of fundamental eigenmode of the main frame.

$$k_{f_i} = \frac{1}{2} \left\{ N \left( N + 1 \right) - i \left( i - 1 \right) \right\} m \omega_1^2 \quad \left( i = 1, \cdots, N \left( = 20 \right) \right)$$
(10)

where  $\omega_1$  denotes the fundamental natural circular frequency of the frame and *m* is the floor mass. The nominal values of damping coefficients of passive viscous dampers are constant and are shown in Table 1. The ratio of the nominal value of supporting member stiffness to the nominal value of frame stiffness is assumed to be 1.0 in every storey.

The structural parameters  $\mathbf{c}_{d}$ ,  $\mathbf{k}_{b}$  and  $\mathbf{k}_{f}$  are dealt with as interval parameters and the ratios of half the intervals to the nominal values are defined as follows.

$$\{\alpha_1, \dots, \beta_1, \dots, \gamma_1, \dots\} = \left\{ \frac{\Delta c_{d_1}}{c_{d_1}^c}, \dots, \frac{\Delta k_{b_1}}{k_{b_1}^c}, \dots, \frac{\Delta k_{f_1}}{k_{f_1}^c}, \dots \right\}$$
(11)

The degrees of uncertainties of interval parameters are given by  $\alpha_i = \beta_i = 0.5$  and  $\gamma_i = 0.3$  for all *i*. This means that the degree of performance of passive dampers may be rather large and that of mainframe may be relatively small.

Fig.6 shows representative recorded ground motions, El Centro NS 1940, Taft EW 1952 and Hachinohe NS 1968, whose maximum velocities are normalized by 50[cm/s]. These earthquake ground motions are used for structural design (Level 2 of large earthquake ground motion) of high-rise and base-isolated buildings in Japan.

Fig.7 illustrates the comparison of evaluated bounds of the top horizontal displacement under uncertain structural parameters derived by Taylor series approximations with those derived by the SQP method. The SQP method has been applied to two cases. The first case has the nominal value as the initial value and the second case has randomly generated parameter combinations giving the top three maximum responses as the initial value. The second case has been introduced to guarantee the search of global optimum one. The first-order Taylor and reanalysis means that the critical combination is determined by the first-order Taylor series expansion and the upper bound of response is evaluated by the time-history response analysis for the determined combination. It can be observed from Fig.7 that the first-order Taylor and reanalysis provides an accurate estimate for the maximum top horizontal displacement for all the three ground motions. Furthermore the result of the SQP method with the nominal value as the initial value coincides fairly well with that of the SQP method with randomly generated parameter combinations giving the top three maximum responses as the initial value.

Fig.8 shows the comparison of critical interval parameters for the upper bound of the top horizontal displacement derived by the first-order approximation with those by the SQP method with the nominal value as the initial value. It can be observed that, while a little difference is seen for El Centro and Hachinohe, most parameters coincide well. Fig.9 presents the comparison of evaluated bounds of the maximum interstorey drift under uncertain structural parameters derived by Taylor series approximations and reanalysis method with those derived by the SQP method where the initial value is given by the nominal model. Furthermore, since the SQP method is known as the gradient-based optimization algorithm, the result by the SQP method may depend on the initial value and may attain a local maximum. For confirming whether the result by the SQP method is the global optimum solution or not as implemented in Fig.7, the upper bound of the objective function for El Centro NS (1940) is derived by the SQP method where the initial value is given by three different randomly generated evaluation points making the objective function maximum (top three). It can be seen that the method with the nominal model as the initial model is not sufficient and another method is desired for the maximum interstorey drift.

Fig.10 illustrates the comparison of critical interval parameters for the upper bound of the maximum interstorey drift derived by the first-order approximation with those by the SQP method where the initial value is given by the nominal model. It can be observed from Fig.10(b) that some of critical structural parameters exist except at the end-points of the interval. However, the critical structural parameters by the first-order approximation with the nominal value as the initial value exhibit a distribution similar to those by the SQP method.

Fig.11 shows the comparison of the maximum interstorey drifts under uncertain structural parameter sets derived by first-order Taylor series approximation with those by the SQP method employing the nominal model as the initial value combination for three ground motions, El Centro NS 1940, Taft EW 1952 and Hachinohe NS 1968. It can be observed that the maximum interstorey drift of the nominal model occurs in the first storey for all the three ground motions. On the other hand, while the varied maximum interstorey drift under uncertain structural parameters occurs in the first storey for El Centro NS 1940 and Taft EW 1952, that occurs in the top storey for Hachinohe NS 1968. In the case where the storey indicating the maximum interstorey drift changes from the nominal model like Hachinohe NS

1968, the sensitivities of the objective function (maximum interstorey drift in this case) with respect to uncertain parameters exhibit largely different values. This leads to large errors in evaluating the response variability (see Fig.9).

It can also be understood from Fig.11 that, although the storey indicating the maximum interstorey drift by first-order Taylor series approximation does not change from that by the SQP method, the solution by the SQP method employing the nominal model as the initial value combination may drop into a local maximum. For such case, the VEP method introduced in Section 4.2 seems to be effective. A numerical example using the VEP method will be shown in the following.

Fig.12 shows the distribution of the maximum interstorey drift for El Centro NS (1940) of damped structures given by randomly generated structural parameters. The horizontal axis indicates the lowest-mode damping ratio of the model with a respective set of randomly generated structural parameters. The number of samples is 10000.

Fig.13(a) illustrates the comparison of interval parameters, i.e. initial structural parameters (randomly generated one giving maximum response), critical structural parameters by the VEP method (first-order Taylor approximation) and critical structural parameters by the SQP method for randomly generated combinations of uncertain structural parameters giving top three maximum interstorey drifts. It can be observed that the critical structural parameters by the VEP method coincide fairly well with those by the SQP method. Fig.13(b) presents the result of the maximum interstorey drifts by the VEP method and the SQP method. It can be seen that the maximum interstorey drift by the VEP method coincides fairly well with that by the SQP method. This indicates the reliability and accuracy of the VEP method for the maximum interstorey drift.

The degrees of uncertainties  $\alpha_i = \beta_i = 0.5$  and  $\gamma_i = 0.3$  are rather large and the present numerical examples seem to give the upper bounds of errors of the proposed method.

## 6. Conclusions

The following conclusions have been obtained.

- Interval analysis in terms of uncertain structural parameters is an effective tool for evaluating the sustainability of buildings in earthquake-prone countries.
- (2) All the combinations of uncertain structural parameters become huge numbers and this difficulty can be overcome by introducing the sensitivity or Taylor series expansion analysis.
- (3) The VEP (varied evaluation point) method is a reliable method because the present problem is a non-convex problem and the change of the evaluation point or the initial design point is necessary for search of the global solution.
- (4) The necessity of use of the VEP method depends on the objective function. When the objective function is a top floor displacement, it may not be necessary. When the objective function is an interstorey drift, it appears necessary. The VEP method is an accurate and reliable method for the estimation of the maximum interstorey drift under uncertain structural parameter environments.

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#### References

- Alefeld, G. & Herzberger, J. (1983). *Introduction to interval computations*. New York: Academic Press.
- Casciati, F. (ed.) (2002). *Proceedings of 3rd world conference on structural control*. John Wiley & Sons: Como.
- Chen, SH & Wu, J. (2004). Interval optimization of dynamic response for structures with interval parameters. *Comp. Struct.*, **82**, 1-11.

- Chen, SH, Ma, L, Meng, GW & Guo, R. (2009). An efficient method for evaluating the natural frequency of structures with uncertain-but-bounded parameters. *Comp. Struct.*, 87: 582-590.
- Cheng, FY., Jiang, H. and Lou, K. (2008). Smart structures: Innovative systems for seismic response control. CRC Press.
- Christopoulos, C., & Filiatrault, A. (2006). *Principle of Passive Supplemental Damping and Seismic Isolation*. IUSS Press, University of Pavia, Italy.

de Silva, C.W. (ed.) (2007). Vibration damping, control, and design, CRC Press.

- Fujita, K., Moustafa, A. & Takewaki, I. (2010a). Optimal placement of viscoelastic dampers and supporting members under variable critical excitations. *Earthquakes and Structures/ An Int. J.*, 1(1), 43-67.
- Housner, GW, Masri, SF & Chassiakos, AG. (eds). (1994). Proceedings of 1st world conference on structural control. IASC: Los Angeles, CA.
- Housner, G. et al. (1997). Special issue, Structural control : past, present, and future. *J. Engng Mech ASCE* **123**(9), 897-971.
- Johnson, E., & Smyth, A. (eds). (2006). Proceedings of 4th world conference on structural control and monitoring, (4WCSCM). IASC: San Diego, CA.
- Karbhari, VM. & Lee, LS. (eds.) (2010). Service life estimation and extension of civil engineering structures. Woodhead Publishing.
- Kobori, T, Inoue, Y, Seto, K, Iemura, H & Nishitani, A. (eds). (1998). Proceedings of 2nd world conference on structural control. John Wiley & Sons: Kyoto.
- Koyluoglu, HU and Elishakoff, I. (1998). A comparison of stochastic and interval finite elements applied to shear frames with uncertain stiffness properties. *Comp. and Struct.* 67: 91-98.
- Moore, RE. (1966). Interval analysis, Englewood Cliffs, New Jersey: Prentice-Hall.
- Qiu, Z.P. (2003). Comparison of static response of structures using convex models and interval

analysis method. Int. J. Numer. Meth Engng. 56: 1735-1753.

- Qiu, ZP, Chen, SH & Song, D (1996). The displacement bound estimation for structures with an interval description of uncertain parameters. *C. Numer. Meth. Engng.*, **12**: 1-11.
- Mullen, RL & Muhanna, RL. (1999). Bounds of structural response for all possible loading combinations. J. Struct. Engrg. ASCE, 125: 98-106.
- Soong, T.T., & Dargush, G.F. (1997). Passive energy dissipation systems in structural engineering. John Wiley & Sons, Chichester.
- Srinivasan & McFarland (2000). Smart structures: Analysis and design, Cambridge University Press.
- Takewaki, I. (2007). *Critical excitation methods in earthquake engineering*. Elsevier, Amsterdam.
- Takewaki, I. (2009). Building control with passive dampers: -Optimal performance-based design for earthquakes-. John Wiley & Sons Ltd. (Asia), Singapore.
- Takewaki, I. & Ben-Haim, Y. (2005). Info-gap robust design with load and model uncertainties. J. Sound & Vibration, 288(3), 551-570.
- Takewaki, I., Fujita, K., Yamamoto, K., & Takabatake, H. (2011). Smart passive damper control for greater building earthquake resilience in sustainable cities. *Sustainable Cities and Society*, 1(1), 3-15.

	20-storey building
Floor mass [kg]	$1024 \times 10^{3}$
Storey stiffness [N/mm]	Eq.(10)
Damper capacity [Ns/mm]	$2.250 \times 10^{7}$
Supporting member stiffness [N/mm]	Ratio 1.0 to frame storey stiffness
Structural damping ratio (stiffness-proportional damping)	0.02
Fundamental natural circular frequency <sup>*1</sup> with damper [rad/s]	3.927

# Table I. Structural parameters of main frame

\*1 complex eigenvalue analysis



Fig.1 Concept of sustainable design considering varied structural performance caused by various uncertainties of structural parameters



Fig.2 Relationship between the variation of objective function and the structural parameter combination, (a) example of 'inclusion monotonic', (b) example of 'inclusion non-monotonic'



Fig.3 Concept of interval analysis method using Taylor series approximation



Fig.4 Concept of advanced interval analysis method using Taylor series approximation and random search technique for initial evaluation point



Fig.5 Structural model with passive dampers including supporting members



Fig.6 Recorded ground motions whose maximum velocities are normalized by 50[cm/s]



Fig.7 Comparison of evaluated bounds of top horizontal displacement under uncertain structural parameters derived by Taylor series approximations with those derived by SQP method



Fig.8 Comparison of critical interval parameters for upper bound of top horizontal displacement derived by first-order approximation with those of exact solution,

(a) first-order approximation, (b) SQP method with the nominal value as the initial value



Fig.9 Comparison of evaluated bounds of maximum interstorey drift under uncertain structural parameters derived by Taylor series approximations with those derived by SQP method



Fig.10 Comparison of critical interval parameters for upper bound of maximum interstorey drift derived by first-order approximation with those of exact solution (a)first-order approximation, (b) SQP method



Fig.11 Comparison of maximum interstorey drifts under uncertain structural parameter sets derived by first-order Taylor approximation with those by SQP method, (a) El Centro NS 1940, (b) Taft EW 1952, (c) Hachinohe NS 1968



Fig.12 Distribution of maximum interstorey drift of damped structures given by randomly generated structural parameters (El Centro NS (1940))





(a) comparison of interval parameters, (b) comparison of maximum interstorey drift