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“Entrepreneurial Choice and Knightian
Uncertainty with Borrowing Constraints”

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Entrepreneurial Choice and Knightian Uncertainty with Borrowing Constraints

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Abstract

This paper studies the effects of Knightian uncertainty, or ambiguity, on entrepreneurial choice. By distinguishing between risk and ambiguity, we first show that ambiguity aversion makes it less likely that an individual will become an entrepreneur. It is also shown that an increase in ambiguity unambiguously reduces the amount of investment. In the presence of borrowing constraints, the *less* ambiguity averse is the individual, the more likely is his or her investment to be constrained. More importantly, constrained wage workers, who would become entrepreneurs in the absence of credit market imperfection, emerge if and only if the market wage is high enough. These individuals are characterized by an intermediate degree of ambiguity aversion. When interpreting these constrained wage workers as managerial and professional workers, our model predicts the rise of such workers in the process of economic development.

Keywords: Entrepreneurship; Knightian Uncertainty; Risk; Borrowing Constraints.

JEL classification: L26; D8.

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1 Introduction

Starting a new business is fraught with uncertainty. First and foremost, entrepreneurs must organize and manage their businesses, and above all, they must take full responsibility. This does not involve the repetition of well-known daily work tasks. As Knight (1921, p.299) states, “the entrepreneur ... takes over all the *uncertainty* of the business along with control over it” (emphasis added). He also claims that “[t]he inseparability of the *uncertainty* problem and the managerial problem” is “especially important in the discussion of *entrepreneurship*” (p.259; emphasis added) and that “*true uncertainty* ... accounts for the peculiar income of the entrepreneur” (p.232).

It is well known that by uncertainty, Knight (1921) meant *unmeasurable* uncertainty, which he proposed should be distinguished from *risk*, or what he called *measurable* uncertainty. While it seems difficult to distinguish empirically between risk and ambiguity,¹ Engle-Warnick, Escobal, and Laszlo (2011) find that it is ambiguity aversion that better predicts farmers’ decisions about the adoption of new technology.² Their empirical findings thus naturally motivate us explicitly to take *real* uncertainty into account in studying entrepreneurship as a nonroutine task in the spirit of Knight (1921). Formal analysis, however, has yet to be developed, mainly because a tractable way of dealing with Knightian uncertainty in applied economic research has been developed only recently.

In this paper, we propose a formal, yet tractable, model of entrepreneurial choice based on the idea of Knight (1921). What are the effects of uncertainty on individuals’ decisions about whether to become entrepreneurs? Our theoretical model,

¹In the past, there has been some skepticism about this distinction in the context of entrepreneurship. For example, Schultz (1980) stated that “(Milton) Friedman does not believe that this is a valid distinction. He follows, as I do, L. J. Savage in his view of personal probability, which denies any valid distinction along these lines.” (p.440)

²Engle-Warnick, Escobal, and Laszlo (2011) study the OLS estimates of the risk aversion variable and of the ambiguity aversion variable in a linear regression model, using experimental data of Peruvian farmers. For experimental studies on ambiguity aversion, see, e.g., Ahn, Choi, Gale, and Kariv (2011), who consider a portfolio choice problem, and Asano, Okudaira, and Sasaki (2011), who investigate the validity of Nishimura and Ozaki’s (2004) model of job search under Knightian uncertainty.

in which there is no credit market imperfection, predicts that ambiguity aversion makes it less likely that an individual will become an entrepreneur. It is also shown that an increase in ambiguity unambiguously reduces the amount of investment. We then incorporate borrowing constraints into the model. This is motivated by our recognition that in reality, individuals cannot borrow as much as they want. In the presence of borrowing constraints, the less ambiguity averse is the individual, the more likely is his or her investment to be constrained. More importantly, constrained wage workers, who would otherwise become entrepreneurs, emerge if and only if the market wage is high enough. These individuals are characterized by an intermediate degree of ambiguity aversion. When interpreting these constrained wage workers as managerial and professional workers, our model predicts the rise of such workers in the process of economic development.

The distinction between risk and uncertainty made by Knight (1921) is perhaps most vividly illustrated by the following *Ellsberg paradox*, which casts doubt on the validity of the Subjective Expected Utility (SEU) Theory axiomatized by Savage (1972). Consider a bet on drawing a blue ball from two urns. Two urns include 100 balls whose colors are blue or red. One urn includes 50 blue balls and 50 red balls. In the other urn, the numbers of blue and red balls are unknown. Ellsberg (1961) provides experimental evidence that, in this situation, most people prefer the first urn to the second one. The reason is that most people dislike a situation in which probabilities are not uniquely assigned. Ellsberg (1961) points out that people prefer bets with known probabilities to those with unknown probabilities. In the spirit of Knight (1921), uncertainty captured by a set of probability measures is called *Knightian uncertainty* or *ambiguity* (these terms are used interchangeably throughout the paper). On the other hand, uncertainty captured by a unique probability measure is called *risk*. In order to overcome the shortcomings of standard expected utility theory, *nonexpected utility theories* have been proposed in the literature.³ A seminal paper is by Gilboa and Schmeidler (1989). Since the publication of

³For introductory accounts of non-expected utility theories, see, e.g., Mas-Colell, Whinston, and Green (1995, Chapter 6), Hansen and Sargent (2008, Chapters 1 and 19), Gilboa (2009) and Wakker (2010).

the paper by Gilboa and Schmeidler (1989), who provide the axiomatic foundations to analyze behavior under ambiguity, the distinction between risk and ambiguity has been considered important for explaining a number of economic phenomena.⁴ Gilboa and Schmeidler (1989) axiomatize the *Max-min Expected Utility* (MEU) theory, under which the beliefs of a decision maker (DM) are represented by his or her set of probability measures, and in which his or her preferences are represented by the minimum of his or her expected utilities over his or her set of probability measures.⁵ Therefore, MEU theory is appropriate for analyzing a DM's cautious or conservative behavior if this behavior cannot be explained by SEU theory.⁶

In this paper, we follow Evans and Jovanovic (1989) to model entrepreneurial choice, and we follow Miao (2004) to model Knightian uncertainty. To the best of our knowledge, this paper is the first to consider the effects of borrowing constraints on entrepreneurial choice in the presence of Knightian uncertainty. We show that the presence of borrowing constraints affects entrepreneurial choice in a nontrivial way.⁷ While risk-neutrality is assumed as in Evans and Jovanovic (1989), we assume that individuals are heterogenous in terms of ambiguity aversion. We find that there is a negative correlation between ambiguity aversion and entrepreneurship, which is reminiscent of van Praag and Cramer (2001) who take into account *risk aversion* and entrepreneurial ability to analyze the individual decision to become an entrepreneur.

Rigotti, Ryan, and Vaithianathan (2011) also share with us the notion that

⁴There is a large literature on the effects of ambiguity aversion in finance: see, e.g., Dow and Werlang (1992), Epstein and Wang (1994), Epstein and Miao (2003), and Asano (2006, 2010a).

⁵From another point of view, Schmeidler (1989) proposes the *Choquet Expected Utility* Theory, under which a DM's beliefs are represented by a *nonadditive measure* and in which, if the DM satisfies a set of axioms, his or her preferences are represented by the *Choquet integral*.

⁶Recent topics of research that apply the notion of Knightian uncertainty include job search (Nishimura and Ozaki (2004)), sharing rules for partnership (Kelsey and Spanjers (2004)), irreversible investment (Nishimura and Ozaki (2004) and Asano (2010b, 2010c)), Cournot and Bertrand oligopolies (Eichberger, Kelsey and Schipper (2008)), and the tragedy of commons (Diamantaras and Gilles (2011)).

⁷In a classic paper, Kihlstrom and Laffont (1979) propose an equilibrium model in which an individual chooses between becoming an entrepreneur and becoming a wage earner. Their main prediction is similar to both ours and those of Rigotti, Ryan, and Vaithianathan (2011): the less risk averse is an individual, the more likely is he or she to become an entrepreneur. Although their analysis is motivated by Knight (1921), Kihlstrom and Laffont (1979) develop a model using the framework of risk and standard expected utility. Thus, their notion of heterogeneity in individual attitudes toward risk does not capture ambiguity aversion in the sense of Knight (1921).

“entrepreneurial characteristics” are related to tolerance to ambiguity (rather than to risk). Capturing the notions of optimism and pessimism through the use of belief functions over returns that depend on new and old technologies, Rigotti, Ryan, and Vaithianathan (2011) analyze individuals’ choices of occupation and technology. That is, Rigotti, Ryan, and Vaithianathan (2011) analyze how individuals choose between starting an entrepreneurial firm and working in such a firm, and how they choose between adopting a well-known traditional technology and adopting a new innovative technology. Rigotti, Ryan, and Vaithianathan (2011) show that three types of firms appear in equilibrium: (i) *entrepreneurial firms* operating with technology that generates ambiguous returns, in which both owners and workers are optimists; (ii) *traditional firms* in which optimistic owners hire pessimistic workers and operate with technology that generates more certain returns; and (iii) *bureaucratic firms* operating with well-known technology in which both owners and workers are pessimists.⁸

Whereas Rigotti, Ryan, and Vaithianathan (2011) consider a model of occupational choice based on the Arrow–Hurwicz (1972) criterion⁹ and analyze the effects of heterogeneity in ambiguity tolerance, we study individuals’ occupational choices based on Gilboa and Schmeidler’s (1989) formulation and analyze the effects on occupational choice of the degree of ambiguity. Unlike Rigotti, Ryan, and Vaithianathan (2011), we take into account borrowing constraints.

The organization of the paper is as follows. In the next section, we introduce our model. In Section 3, we present analytical results and numerical examples. After the basic model is analyzed, we incorporate borrowing constraints into the basic

⁸Rigotti, Ryan, and Vaithianathan (2008) propose a general equilibrium model of occupational choice in which new and old technologies are considered for the production of a single consumption good. Assuming that the returns from the new technology are ambiguous and that the returns from the old technology are risky but not ambiguous, Rigotti, Ryan, and Vaithianathan (2008) analyze the effects of heterogeneity in ambiguity tolerance. They show that (i) the most ambiguity-tolerant individuals own the innovative firms operating with the new technology; (ii) the least ambiguity-tolerant individuals work in firms operating with the old technology; (iii) individuals who tolerate ambiguity to an intermediate degree enter firms operating with the new technology.

⁹The Arrow–Hurwicz (1972) criterion conceptualizes the idea that one’s preference is expressed by the convex combination of the maximum and the minimum of the expected utility from an act. With the criterion, ambiguity aversion is characterized by the weights of the convex combination.

model. Section 4 concludes the paper.

2 The Model

In this section, we present and then parameterize our model. We follow Evans and Jovanovic (1989) to model entrepreneurial choice, and we incorporate ambiguity aversion à la Miao (2004).

2.1 Setup

Let S be the set of states of the world, let \mathcal{F} be a σ -algebra on S , and let P be a reference probability on (S, \mathcal{F}) . No information asymmetry exists. An individual is endowed with the beginning-of-period wealth $z > 0$.¹⁰ This individual, by inelastically supplying a unit of labor, decides whether to become an entrepreneur (working for himself or herself) or to become a wage earner (working for someone else). We assume that these two alternatives are mutually exclusive.¹¹ Conditional on his or her occupational choice, the individual's (gross) income is given by:

$$\tilde{y} \ (\equiv \tilde{y}(k)) = \begin{cases} w & \text{if he or she is a wage worker} \\ f(k, \tilde{\epsilon}) & \text{if he or she is an entrepreneur,} \end{cases}$$

where $w > 0$ is the (nonstochastic) wage that can be earned as a wage worker in the competitive labor market,¹² $k \geq 0$ is the amount of physical capital invested in the entrepreneurial project, and $\tilde{\epsilon}$ is a *technological shock* or a *nonsystematic interruption to production*, which is a random variable on a probability space (S, \mathcal{F}, P) (specified in the next subsection).¹³ We term $f(\cdot, \cdot)$ the *entrepreneurial production function* in

¹⁰The beginning-of-period wealth may come from wages obtained as a worker and/or from an endowment from parents.

¹¹We assume away disutility from labor and any labor market imperfection. Thus, having no employment (or “being unemployed”) is not an option for the individual.

¹²We include hired managers in this category in the spirit of Knight (1921), who claims that “[w]e must refuse to be misled by the superficial similarity between the daily work of the *hired manager* and that of the *man in business on his own account*” (p.297; emphasis added) and that “[w]ages of management are not different in principle from wages for routine work” (p.309).

¹³We assume that the production contribution by the individual as an entrepreneur is separable from that contributed by any other individuals who work for him or her. By making this assump-

the spirit of Knight (1921) and Lucas (1978).¹⁴ We assume that for any realization of ϵ , $f(\cdot, \epsilon)$ is twice continuously differentiable and satisfies the Inada conditions ($f(0, \epsilon) = 0$, $\lim_{k \downarrow 0} \partial f(k, \epsilon) / \partial k = \infty$, and $\lim_{k \rightarrow \infty} \partial f(k, \epsilon) / \partial k = 0$). An individual who has decided to become an entrepreneur must invest a positive amount of physical capital. Hence, the individual becomes an entrepreneur if and only if $k > 0$. The amount of consumption is given by:

$$\tilde{c} = \tilde{c}(k) \equiv \tilde{y} + (1 - \delta)k + (1 + r)(z - k),$$

where $\delta \in (0, 1)$ is the depreciation rate and $r \geq 0$ is the risk-free interest rate.¹⁵

We interpret $\tilde{\epsilon}$ in the production function as a source of *ambiguity*. There are a number of possible sources of ambiguity in the business project. Knight (1921) points out that “[t]he main uncertainty which affects the entrepreneur is that connected with the sale price of his product” (p.317). This type of uncertainty typically comes from the inevitable time lag between the (contractual) purchase of capital (i.e., $(1 + r)k$)¹⁶ and the sale of the final output (\tilde{y}).¹⁷ The real uncertainty is considered to reflect the subjective evaluation of the production technology by the individual. On the other hand, it would be reasonable to assume that wage work has no such ambiguity. Furthermore, for tractability, w is assumed to be a nonrandom variable.

In order to model the entrepreneur’s profit maximization problem under Knight-

tion, we can count as entrepreneurs small business owners or managers who hire wage workers as well as the self-employed.

¹⁴Knight (1921, p.305) states: “The independent entrepreneur is not yet by any means an extinct species. Such a person typically furnishes both property and labor services to a business, meaning by labor services personal activities which might be hired and paid for with a fixed wage. The entrepreneur income in a case of this sort contains an element of wages as well as an element of interest.”

¹⁵We assume that the interest rate for k is also r . That is, there is no distinction between lending and borrowing rates. However, as Evans and Jovanovic (1989, p.813) point out, because financial intermediaries “do not appear to fine-tune risk premia to individual borrowers”, this is a simplifying assumption.

¹⁶If an entrepreneur also hires wage workers (not modeled in this paper), to this cost is added wage payments.

¹⁷Knight (1921, p.309) also mentions “true uncertainty” in estimating “human capacity”. This interpretation would apply not only to the evaluation of any wage workers hired by the entrepreneur but also to himself or herself. Takii (2003) proposes the broader concept of the entrepreneur’s prediction ability, which he interprets as entrepreneurial ability, in the framework of standard expected utility theory. See subsection 2.3 for a discussion of how the work of Takii (2003) relates to ours.

ian uncertainty, we follow Gilboa and Schmeidler (1989).¹⁸ Let \mathcal{M} be the set of all probability measures on (S, \mathcal{F}) , and let $\mathcal{P} \subseteq \mathcal{M}$ be a set of probability measures that the individual thinks plausible for the realization of $\tilde{\epsilon}$. The individual is assumed to maximize:

$$\min_{Q \in \mathcal{P}} E^Q[\tilde{c}(k)]$$

with respect to $k \geq 0$ (occupational choice). If this set of priors \mathcal{P} is reduced to a singleton, i.e., $\mathcal{P} = \{P\}$, then the individual faces only risk, and thus, ambiguity disappears.

2.2 Parameterization

To proceed further, we make parametric assumptions about the randomness in entrepreneurial production. We assume that the entrepreneurial production function has the Cobb–Douglas form: $f(k, \tilde{\epsilon}|\alpha, \theta) = \tilde{A}k^\alpha$, where $\tilde{A} \equiv \theta\tilde{\epsilon}$ represents the *production shock*, and $\alpha \in (0, 1)$ is the *rate of return on business capital*.¹⁹ We assume that \tilde{A} is multiplicatively decomposed into (i) $\theta \in (0, \bar{\theta})$, which is interpreted as the individual’s (nonrandom) *entrepreneurial ability*,²⁰ and (ii) the *technological shock*, which is the only source of ambiguity (and risk) for the individual.

Next, we introduce our parametric assumptions about decision maker’s set of priors. In this paper, an individual’s set of probability measures describes ambiguity itself *and* ambiguity aversion. As clearly seen in the objective function of an agent,

¹⁸Within the framework of MEU theory, a DM is supposed to maximize the following objective function:

$$\min_{Q \in \mathcal{P}} \int_S u(f(s))Q(ds),$$

where \mathcal{P} is his or her set of probability measures, f is some function from the state space S into some set of outcomes X , and u is his or her felicity function.

¹⁹Here we assume that the error term appears in an exponential term. Why we use this specification is that it allows the log-linearization

$$\ln y = \ln \theta + \alpha \ln k + \ln \tilde{\epsilon},$$

and that it is also adopted by Evans and Jovanovic (1989). In fact, some of the results depends on this functional form.

²⁰The range of values for θ is restricted to ensure that the second-order condition is satisfied. There may be ambiguity about θ , as suggested by Knight (1921) (see footnote 17 above). However, in this paper, for analytical simplicity, we do not allow heterogeneity in ability.

a larger set of probability measures provides a lower expected utility to the agent. Our interpretation is based on Ghirardato and Marinacci (2002) who propose the definition of *absolute* and *comparative* ambiguity aversion: they provide a behavioral foundation for interpreting an agent’s set of probability measures as the degree of ambiguity. This interpretation is adopted by Nishimura and Ozaki (2004) and Miao and Wang (2011). Similarly, *the parameter ϕ can be interpreted as the degree of Knightian uncertainty*. In the following analysis, we interpret, as Cao, Wang, and Zhang (2005) do, the variation in ϕ as *heterogeneity among individuals* with respect to attitude toward ambiguity, rather than the intrinsic ambiguity to which a business project may pertain. The reference probability P is thus interpreted as a reflection of the intrinsic nature of a business opportunity, which is assumed to be common for all individuals.

Another related work is Ghirardato, Maccheroni, and Marinacci (2004). Ghirardato, Maccheroni, and Marinacci (2004) provide a normative foundation to distinguish a decision maker’s ambiguity and her attitude toward it. In their framework, *α -Maxmin Expected Utility* (henceforth, α -MEU) is derived as a special case.²¹ In Ghirardato, Maccheroni, and Marinacci’s (2004) model, a decision maker’s set of priors captures ambiguity itself, and the parameter α captures her attitude toward ambiguity. These two notions are clearly distinguished. If her set of priors is singleton (i.e., $\mathcal{C} = \{P\}$), then ambiguity disappears, and thus the standard expected utility follows. If $\alpha = 1$, then her preferences are represented by “maxmin” preferences. On the other hand, if $\alpha = 0$, then her preferences are represented by “maxmax” preferences. Ghirardato, Maccheroni, and Marinacci (2004, Proposition 12) provide a normative justification for this way of interpreting the parameter α as

²¹Ghirardato, Maccheroni, and Marinacci (2004) provide a more general axiomatization theorem than α -MEU (see also Jaffray (1989)). More specifically, let S be a set of states of the world, Σ be an algebra of subsets of S , X be a set of consequences, $f : S \rightarrow \mathbb{R}$ be a finite-valued Σ -measurable function, and $B_0(\Sigma)$ be the set of all real-valued Σ -measurable simple functions. Ghirardato, Maccheroni, and Marinacci (2004) show that if a set of axioms is satisfied, then there exist some set \mathcal{C} of probability measures on (S, Σ) , a nonconstant affine function $u : X \rightarrow \mathbb{R}$ and $\alpha \in [0, 1]$ such that decision maker’s preferences are represented by some functional $I : B_0(\Sigma) \rightarrow \mathbb{R}$ defined by

$$I(u(f)) = \alpha \min_{P \in \mathcal{C}} \int_S u(f) dP + (1 - \alpha) \max_{P \in \mathcal{C}} \int_S u(f) dP.$$

ambiguity attitude.

Based on Kogan and Wang (2002) and Miao (2004), we specify a structure for \mathcal{P} . This specification enables us to derive clear analytical results.²² We consider the following set of priors:

$$\mathcal{P}(P, \phi) = \left\{ Q \in \mathcal{M} \mid E^Q \left[\log \left(\frac{dQ}{dP} \right) \right] \leq \phi^2 \right\}, \quad (1)$$

where \mathcal{M} denotes the set of all probability measures on (S, \mathcal{F}) , dQ/dP is the density of Q with respect to P , and $\phi \geq 0$ captures an individual's *degree of Knightian uncertainty*, as explained below. As explained by Kogan and Wang (2002), the intuition for the specification $\mathcal{P}(P, \phi)$ is as follows. An individual is not perfectly certain about the reference probability measure P . This reference probability measure P may be interpreted as the true probability measure. However, his or her set of priors is not so large that it covers the set of all probability measures. Because the individual faces such ambiguity, he or she assumes that each element in $\mathcal{P}(P, \phi)$ is a possible alternative to the reference probability P .

Let Q be an element in $\mathcal{P}(P, \phi)$, and let dQ/dP be its density. One way to evaluate the alternative is to use the *relative entropy index*, $E^Q[\log(dQ/dP)]$. This index can be interpreted as an approximation to the empirical log-likelihood value. We then assume that the set of priors for $\tilde{\epsilon}$ is the class of normal distributions (for tractability). Specifically, we let the reference probability P be the normal distribution with mean μ_ϵ and variance σ_ϵ^2 . Furthermore, we assume that an individual has a set of probability measures defined by set (1), all of which have normal distributions with a variance σ_ϵ^2 and a mean $\mu_\epsilon - v$ for some $v \in \mathbb{R}$. Note that v can take negative values. Therefore, the mean $\mu_\epsilon - v$ is not always less than that μ_ϵ . As proved by Kogan and Wang (2002), the set $\mathcal{P}(P, \phi)$ is isomorphic to the following set:

$$\mathcal{V}(\phi) = \left\{ v \in \mathbb{R} \mid \frac{1}{2} v^2 \sigma_\epsilon^{-2} \leq \phi^2 \right\}.$$

²²Based on the approach proposed by Kogan and Wang (2002), Cao, Wang, and Zhang (2005) analyze limited market participation under Knightian uncertainty and show that limited market participation can be explained endogenously by considering Knightian uncertainty and heterogeneous uncertainty-averse investors.

This formulation leads to the following property: the larger ϕ , the larger $\mathcal{V}(\phi)$, which expands the individual's set of priors $\mathcal{P}(P, \phi)$. On the other hand, as in standard analyses, we assume that the notion of risk is captured by the variance σ_ϵ^2 .

Two comments are in order. First, as do Cao, Wang, and Zhang (2005) and Wang (2004), in this paper, we consider a situation in which the entrepreneur has a precise estimate of the variance σ_ϵ^2 but is not perfectly certain about the mean of $\tilde{\epsilon}$ because of the presence of Knightian uncertainty. Second, the case of $\phi = 0$ corresponds to the situation in which an individual is *ambiguity neutral*.

2.3 Interpretation of the Parameter ϕ

In this paper, we interpret ϕ as reflecting heterogeneity among individuals with respect to attitudes toward ambiguity. This is because the larger ϕ , the larger the individual's set of priors $\mathcal{P}(P, \phi)$, which implies that the individual assumes various scenarios including the best and worst ones. Based on this interpretation, the inverse ratio, $1/\phi$, can be interpreted as a measure of “prediction ability”. Remember that the larger is ϕ , the less confident is the individual about what the actual probability is. The source of this anxiety may be one's innate or learned characteristics. Under this interpretation, individuals differ in their prediction abilities. According to Takii (2003), prediction ability reflects how accurately an entrepreneur interprets a realized signal in predicting productivity. By contrast, in our model, prediction ability directly reflects ambiguity and its relation to entrepreneurship.

3 Analysis

Remember that the individual becomes an entrepreneur if and only if the optimal amount of the capital investment $k > 0$. In this section, we first solve for the optimal k and then study its characteristics. We then analyze how the existence of borrowing constraints changes the results.

3.1 Entrepreneurial Investment and Occupational Choice under Ambiguity

The individual is assumed to face model uncertainty and to be ambiguity averse. We also assume that the individual must decide on the investment level (i.e., whether to become an entrepreneur) *before* the realization of $\tilde{\epsilon}$. Let the entrepreneur's (multiprior expected) *profit or net business income* (given $k > 0$) be denoted by:

$$\begin{aligned} V(k) &\equiv \min_{Q \in \mathcal{P}(P, \phi)} E^Q[\pi(k, \tilde{\epsilon})] \\ &= \theta \min_{Q \in \mathcal{P}(P, \phi)} E^Q[\tilde{\epsilon}]k^\alpha + (1 - \delta)k - (1 + r)k \\ &= \theta k^\alpha \min_{v \in \mathcal{V}(\phi)} (\mu_\epsilon - v) - (\delta + r)k \\ &= \bar{A}k^\alpha - (\delta + r)k, \end{aligned}$$

where the first equality follows from the definition of $\pi(k, \tilde{\epsilon}) \equiv \theta k^\alpha \tilde{\epsilon} + (1 - \delta)k - (1 + r)k$, the second equality follows because the set $\mathcal{P}(P, \phi)$ is isomorphic to the set $\mathcal{V}(\phi)$, and $\tilde{\epsilon}$ is distributed with a mean $\mu_\epsilon - v$ and a variance σ_ϵ^2 , and \bar{A} , defined by:

$$\bar{A} = \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta) \equiv \left(\mu_\epsilon - \sqrt{2}\sigma_\epsilon\phi \right) \theta,$$

is the production coefficient. We assume that the variance is sufficiently small that $\sigma_\epsilon^2 < \mu_\epsilon^2 / (2\phi^2)$. Note that the entrepreneur imagines the *worst* prospects for his or her business (i.e., $v = \sqrt{2}\sigma_\epsilon\phi$). Our interpretation is that individual heterogeneity in the population comes from the recognition that what constitutes the worst prospects varies across individuals depending on their own sets of priors, although all individuals have some conception of what are the worst prospects. The entrepreneur then solves $\max_{k \geq 0} V(k)$. The first-order condition for an interior optimum is:

$$\alpha \bar{A} k^{\alpha-1} - (\delta + r) = 0.$$

The solution is given by:

$$k^+(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) = \left(\frac{\alpha \bar{A}}{\delta + r} \right)^{1/(1-\alpha)}.$$

This k^+ is the level of capital that is optimally chosen by the individual who has (not necessarily optimally) decided to become an entrepreneur. What are the effects

of risk (σ_ϵ) and of ambiguity (ϕ) on the optimal level of capital investment (k^+)? As intuition suggests, an increase in either risk or in ambiguity leads to a lower level of investment. This is because it is reasonable for greater fluctuations to make an individual's decision more conservative. Not only risk but uncertainty also captures the conservativeness in investment.

Proposition 1. *Both an increase in risk or an increase in ambiguity reduce the optimal level of investment ($\partial k^+(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha)/\partial \sigma_\epsilon = -(\sqrt{2}\phi\theta/(1-\alpha)\bar{A})k^+ < 0$ and $\partial k^+(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha)/\partial \phi = -(\sqrt{2}\sigma_\epsilon\theta/(1-\alpha)\bar{A})k^+ < 0$).*

The multiprior expected net business income under k^+ is now given by:

$$\begin{aligned} V^*(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) &\equiv V(k^+) \\ &= \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta) \left(\frac{\alpha \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta)}{\delta + r} \right)^{\alpha/(1-\alpha)} \\ &\quad - (\delta + r) \left(\frac{\alpha \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta)}{\delta + r} \right)^{1/(1-\alpha)}. \end{aligned}$$

The individual becomes an entrepreneur if and only if the wage that can be earned is no greater than multiprior expected net business income (remember that we ignore labor disutility). Thus, the optimal amount of investment is given by:²³

$$k^* = \begin{cases} k^+(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) & \text{if } w \leq V^*(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) \\ 0 & \text{if } w > V^*(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha). \end{cases}$$

Note that if $\phi = 0$, the set of priors \mathcal{P} is a singleton, and the preceding analyses capture the situation with no ambiguity. Our formulation allows one to interpret

²³The effects of risk aversion on the optimal amount of investment within the framework of the mean-variance preferences of Markowitz (1952) and Tobin (1958) are worth being analyzed. Note that mean-variance preferences are represented by the following:

$$E^P[u(k)] = E^P[\pi(k, \tilde{\epsilon})] - \frac{1}{2}\eta \text{Var}[\pi(k, \tilde{\epsilon})],$$

where P is the reference probability measure and η is a measure of risk aversion. Generally, it is not always ensured that within the framework of MEU, DM's preferences can be written in an additive form as above. Thus, in this paper, we do not consider the interaction between ambiguity aversion and risk aversion within the framework of mean-variance preferences, which is clearly an interesting future topic. See Maccheroni, Marinacci, and Rustichini (2006) and Maccheroni, Marinacci, Rustichini, and Taboga (2009) for this issue.

ϕ as a subjective assessment of the degree of ambiguity. However, in what follows, the variation in ϕ is interpreted as heterogeneity among individuals with respect to attitudes toward ambiguity. This enables us to analyze how individual differences in ambiguity affect occupational choice. First, we derive the following proposition, which establishes a threshold for ambiguity aversion, below which individuals become entrepreneurs and above which they become wage earners.

Proposition 2. *Suppose that the individual chooses to become an entrepreneur if he or she is ambiguity neutral ($V^*(0, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) > w$). Then, there exists a unique threshold, $\hat{\phi} = \hat{\phi}(w, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) > 0$ such that individuals with $\phi \leq \hat{\phi}$ become entrepreneurs and those with $\phi > \hat{\phi}$ become wage workers, where $\hat{\phi}$ satisfies $V^*(\hat{\phi}, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) = w$.*

Proof. It can be verified that $V^*(\cdot, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha)$ is continuous and strictly decreasing in $\phi > 0$ because:

$$\frac{\partial V^*(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha)}{\partial \phi} = -\sqrt{2}\sigma_\epsilon\theta \cdot \left(\frac{\alpha \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta)}{\delta + r} \right)^{\alpha/(1-\alpha)} < 0,$$

and that $\lim_{\phi \uparrow \infty} V^*(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) = 0$, which gives the desired result. *Q.E.D.*

This proposition can be interpreted as a formal statement of Knight's (1921) insights into entrepreneurship, as discussed in the Introduction: "ambiguity-bearing" is a key determinant of entrepreneurial choice.²⁴ The next proposition establishes an important characteristic of the threshold, $\hat{\phi}$.

²⁴On the other hand, under our specification of a production function, it is that

$$\frac{\partial V^*(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha)}{\partial \sigma_\epsilon} = \frac{-\sqrt{2}\phi + \sigma_\epsilon}{1 - \alpha} V^*(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha),$$

which does not permit an interpretation as Proposition 2 above. If $\phi = 0$, then we first have $\partial V^*/\partial \sigma_\epsilon = \sigma_\epsilon V^*(\phi = 0, \sigma_\epsilon)/(1 - \alpha) > 0$. Because it is verified that

$$\begin{aligned} V^*(\phi = 0, \sigma_\epsilon = 0) &> 0 \\ \Leftrightarrow \alpha &< 1, \end{aligned}$$

we have a threshold $\hat{\sigma}_\epsilon$, above which an individual chooses to be an entrepreneur, if the market wage level w is high enough that $w > V^*(\phi = 0, \sigma_\epsilon = 0)$.

Proposition 3. *An increase in entrepreneurial ability increases the threshold; i.e., $\partial\widehat{\phi}(w, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha)/\partial\theta > 0$.*

Proof. It follows from the presentation in the Appendix that:

$$\frac{\partial\widehat{\phi}(w, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha)}{\partial\theta} = \frac{\mu_\epsilon - \sqrt{2}\sigma_\epsilon\theta}{\sqrt{2}\sigma_\epsilon\theta} > 0$$

Q.E.D.

This proposition states that individuals with great entrepreneurial ability are more likely to become entrepreneurs, *ceteris paribus*. This is because their entrepreneurial ability enables them to tolerate more ambiguity in their future net business profits. We also have $\partial\widehat{\phi}/\partial w < 0$ (see Appendix): the more attractive the outside opportunity (becoming a wage worker) becomes, the more likely are inframarginal entrepreneurs (slightly below $\widehat{\phi}$) to opt out of wage work. On the other hand, the effect on the threshold of a marginal increase in the rate of return from business capital ($\partial\widehat{\phi}/\partial\alpha$) is indeterminate. This is because the effects of a small increase in α on the multiprior expected entrepreneurial profit are twofold: it is associated with an increase in k^+ and, hence, an increase in the benefit $\bar{A}(k^+)^\alpha$, and at the same time, it also incurs the cost $(\delta + r)k^+$. As is expected, we have $\partial\widehat{\phi}/\partial\mu_\epsilon > 0$: a marginal increase in the mean production shock makes inframarginal wage workers (slightly above $\widehat{\phi}$) become entrepreneurs. However, the sign of $\partial\widehat{\phi}/\partial\sigma_\epsilon$ is indeterminate. Nevertheless, $\partial\widehat{\phi}/\partial\sigma_\epsilon > 0$ if and only if $\sigma_\epsilon > \sqrt{2}\phi$. This means that if the common variance of each possible distribution is large enough, then further increases encourage more individuals to become entrepreneurs. An immediate result that is relevant for the next subsection is that individuals' initial wealth holdings do not affect their (deterministic) prospects of becoming entrepreneurs.

Corollary 1. *In the absence of imperfection in the credit market, whether an individual chooses to become an entrepreneur is independent of his or her initial level of assets, z .*

This is because what matters to individuals is only whether they can obtain

returns in the future by becoming entrepreneurs. Therefore, the level of initial wealth does not affect entrepreneurial choice.

3.2 Incorporating Borrowing Constraints into the Basic Model

In practice, many prospective entrepreneurs cannot borrow as much as they want.²⁵ In the literature on entrepreneurial choice, much attention has been paid to the significance of borrowing constraints (or lack thereof).²⁶ To determine the effects of borrowing constraints on entrepreneurial choice in the presence of Knightian uncertainty, as do Evans and Jovanovic (1989), we assume that there is an upper bound for the amount of business capital: k must be no greater than λz , where $\lambda \geq 1$ is a constant.²⁷ Thus, if the individual (not necessarily optimally) becomes an entrepreneur, the actual level of capital investment is:

$$k^{\$} = \min \{ \lambda z, k^+(\phi, \sigma_{\epsilon}, \mu_{\epsilon}, \theta, \alpha) \}.$$

(Note that once $k^{\$} = k^+(\phi, \sigma_{\epsilon}, \mu_{\epsilon}, \theta, \alpha)$ is not chosen, then $k^{\$} = \lambda z$ is (sub)optimal because $V(k)$ is increasing for $k < k^+$.) The constraint is *binding* if and only if $k^{\$} = \lambda z$. If the constraint is *not* binding, then the optimal solution is the one given in the previous subsection. (Note that the limiting case of $\lambda \rightarrow \infty$ corresponds to the situation in which the individual faces no borrowing constraints.) Multiprior

²⁵Knight (1921, p.364) recognizes this situation by pointing out that “the entrepreneur, as society is organized, is almost always a property owner and must necessarily be the owner of productive power in some form.” (p.364)

²⁶While Evans and Jovanovic (1989) and subsequent studies (such as those of Holtz-Eakin, Joulfaian, and Rosen (1994a, 1994b), and Dunn and Holtz-Eakin (2000)) empirically emphasize the adversarial effects of borrowing constraints in entrepreneurial choice, Hurst and Lusardi (2004) present the opposite view that the significance of borrowing constraints may have been caused by only a small fraction of those with higher amounts of net worth. See also, e.g., Cressy (1999), Cressy (2000), and Harada and Kijima (2005), who also obtain results that contradict the (theoretical and empirical) significance of borrowing constraints.

²⁷The lower bound is exogenously given. To endogenize the borrowing constraint, one must explicitly model how credit market imperfection induces the constraint. We assume that:

$$0 < \lambda z < k^+(0, \sigma_{\epsilon}, \mu_{\epsilon}, \theta, \alpha) = \left[\frac{\alpha \theta \exp(\mu_{\epsilon} + \sigma_{\epsilon}^2/2)}{\delta + r} \right]^{1/(1-\alpha)}$$

to ensure the existence and uniqueness of $\bar{\phi}$, which is defined below.

expected net business income is now given by:

$$\begin{cases} V^*(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) & \text{if the entrepreneur is unconstrained} \\ V^C(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha, \lambda, z) & \text{if the entrepreneur is constrained,} \end{cases}$$

where

$$\begin{aligned} V^C(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha, \lambda, z) &\equiv \min_{Q \in \mathcal{P}(P, \phi)} E^Q[\pi(\lambda z)] \\ &= \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta) (\lambda z)^\alpha - (\delta + r) (\lambda z). \end{aligned}$$

An individual characterized by $(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha, \lambda, z)$ becomes an *unconstrained* entrepreneur if his or her level of ambiguity aversion, ϕ , satisfies $\bar{\phi} \leq \phi \leq \hat{\phi}$, where:

$$\begin{aligned} \bar{\phi} &= \bar{\phi}(\sigma_\epsilon, \mu_\epsilon, \theta, \alpha, \lambda, z) \\ &\equiv \frac{\mu_\epsilon - \frac{(\lambda z)^{1-\alpha} (\delta + r)}{\alpha \theta}}{\sqrt{2} \sigma_\epsilon} \end{aligned}$$

(uniquely) solves $k^+(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) = \lambda z$ with respect to ϕ , so that $\bar{\phi}$ is the threshold that distinguishes between an individual whose investment is constrained when becoming an entrepreneur and one who can invest optimally even when there are borrowing constraints. The latter type of individual can become (not necessarily optimally) an entrepreneur by choosing $k^\$ = k^+(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha)$. However, for an individual with $\phi < \bar{\phi}$ who is courageous enough, his or her optimal level of capital investment cannot be chosen *if he or she (not necessarily optimally) becomes an entrepreneur*. This situation is depicted in Figure 1, where:

$$\bar{w} \equiv \frac{(1 - \alpha)(\delta + r)}{\alpha} (\lambda z) \quad (\equiv \bar{w}(\lambda z)).$$

It is also shown that $[V^*]'(\bar{\phi}) = [V^C]'(\bar{\phi})$ and that the second derivatives of both functions are positive.

An individual characterized by $(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha, \lambda, z)$ compares the multiprior expected payoff from becoming an entrepreneur with that from becoming a wage worker. Thus, he or she (optimally) becomes a *constrained* entrepreneur if he or she is sufficiently tolerant to ambiguity that his or her ϕ satisfies $\phi < \underline{\phi}$, where:

$$\underline{\phi} = \underline{\phi}(w, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha, \lambda, z)$$

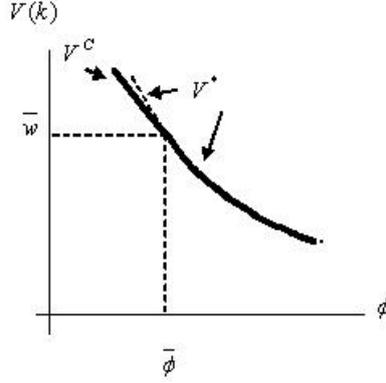


Figure 1: Multiple-prior expected entrepreneurial profit in the presence of borrowing constraints

$$= \frac{\mu_\epsilon - \frac{(\lambda z)(\delta + r) + w}{\theta(\lambda z)^\alpha}}{\sqrt{2}\sigma_\epsilon}$$

is the (unique) solution of $V^C = w$ with respect to ϕ .²⁸ This is because, for these individuals:

$$\min_{Q \in \mathcal{P}(P, \phi)} E^Q[\pi^C(\underline{\phi}, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha, \lambda, z)] \geq w.$$

Notice that although $\bar{\phi}$ is *independent* of w , $\underline{\phi}$ does depend on w . Now, we characterize occupational choice under Knightian uncertainty in the presence of borrowing constraints. First, we fix the value of the upper bound for business capital, λz . Then, we obtain the following proposition about the characteristics of occupational choice for alternative values of the wage. In particular, it shows that the monotonic relationship between ambiguity aversion and occupational choice holds unless the wage is extremely high or low.

Proposition 4. *Suppose that the borrowing constraint for any prospective entrepreneur is binding (i.e., $0 < \lambda z < [\alpha \bar{A}/(\delta + r)]^{1/(1-\alpha)}$). (a) If the wage is low enough for $w < \bar{w}(\lambda z)$, then there exist three types of individuals: (i) constrained entrepreneurs with $\phi \in [0, \bar{\phi}(\lambda z))$; (ii) unconstrained entrepreneurs with*

²⁸ $\min_{Q \in \mathcal{P}(P, \phi)} E^Q[\pi^C(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha, \lambda, z)]$ is defined not only for $\phi < \underline{\phi}$ but also for $\phi \geq \underline{\phi}$. Clearly, ϕ^C exists and is unique.

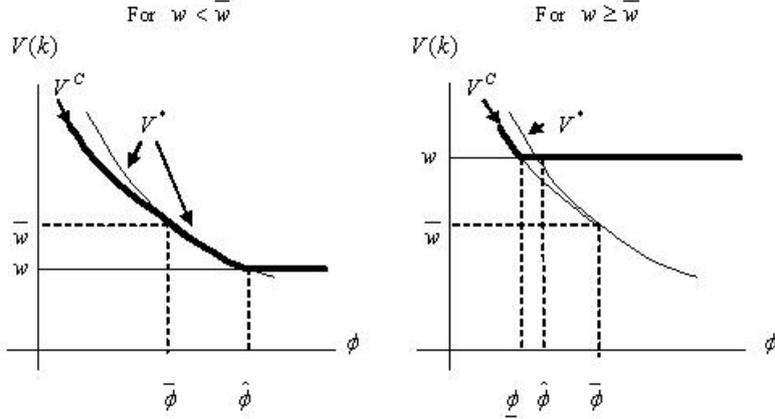


Figure 2: Occupational choice in the presence of borrowing constraints

$\phi \in [\bar{\phi}(\lambda z), \hat{\phi}]$; and (iii) wage workers with $\phi \geq \hat{\phi}$. (b) If the wage is high enough for $w \geq \bar{w}(\lambda z)$, then there exist three types of individuals: (i) constrained entrepreneurs with $\phi \in [0, \underline{\phi}(\lambda z)]$; (ii) constrained wage workers with $\phi \in [\underline{\phi}(\lambda z), \bar{\phi}(\lambda z)]$; and (iii) wage workers with $\phi \geq \bar{\phi}(\lambda z)$.

Figure 2 clarifies the statements in the proposition (bold lines depict the maximized multiprior expected payoff for each ϕ). It is immediately apparent that:

$$\frac{\partial \underline{\phi}(\sigma_\epsilon, \mu_\epsilon, \theta, \alpha, \lambda, z)}{\partial z} = -\frac{\lambda}{\sqrt{2}\sigma_\epsilon} \frac{(1-\alpha)(\delta+r)(\lambda z) - \alpha w}{(\lambda z)^{1+\alpha}} < 0.$$

This implies that an increase in the amount of beginning-of-period wealth makes inframarginal constrained entrepreneurs unconstrained. Note that if (and only if) $\hat{\phi} \leq \bar{\phi}$, the presence of borrowing constraints prevents individuals in the middle range of ambiguity aversion ($\phi \in [\underline{\phi}, \hat{\phi}]$) from becoming entrepreneurs. Hence, we summarize this argument, for comparison with Corollary 1.

Corollary 2. *In the presence of borrowing constraints, constrained wage workers (who would otherwise become entrepreneurs) emerge if and only if the wage is high enough for $w \geq \bar{w}(\lambda z)$.*

This is because if the wage is sufficiently low, constrained multiprior expected entrepreneurial income for any $\phi > 0$ still exceeds the wage; thus, occupational

choice is unaffected by borrowing constraints. Our results have implications for economic development and occupational choice. Iyigun and Owen (1999) provide empirical evidence that shows that in economies with higher per capita incomes, few individuals are employers relative to the number of individuals who work for others. If the market wage w rises as the economy develops, our model predicts a fall in the proportion of entrepreneurs in the labor force. Iyigun and Owen (1999) also provide empirical evidence that shows that the fraction of workers classified as managerial and professional workers increases with per capita income. By interpreting the constrained wage workers in the region of $[\underline{\phi}(\lambda z), \bar{\phi})$ as managerial and professional workers (who earn wages), we also predict the rise of such workers in the process of economic development.

4 Concluding Remarks

In this paper, we analyze the effects of ambiguity on entrepreneurial choice. We show that ambiguity-averse individuals are less likely to become entrepreneurs. It is also shown that an increase in ambiguity unambiguously reduces the amount of investment. In the presence of borrowing constraints, the *less* ambiguity averse is the individual, the more likely is his or her investment to be constrained. More interestingly, we showed that constrained wage workers emerge if and only if the market wage is high enough. By interpreting these constrained wage workers, who exhibit an intermediate degree of ambiguity aversion, as managerial and professional workers, our model predicts the rise of such workers in the course of economic development.

Avenues for future research include the determination of consumption and savings decisions in a dynamic framework à la Schjerning (2006) and Buera (2009).²⁹

How are entrepreneurial choice and savings behavior related in the presence of am-

²⁹Schjerning (2006) and Buera (2009) construct a discrete-time and a continuous-time infinite-horizon model of entrepreneurial choice and savings behavior, respectively. See Adachi (2009) for a finite-horizon version of the dynamic model with many periods (considering one's life cycle). See also Malchow-Møller, Markusen and Skaksen (2010) for a dynamic analysis of the effects of labor market institutions on entrepreneurial choice.

biguity aversion? Numerical analysis is needed to answer this and other interesting questions. This is because the consumption–savings problem is not well defined in the context of risk-averse utility. However, it is hoped that the analytical results obtained in this paper under the assumption of risk neutrality can be used to generalize the formulation of utility. Although structural estimation of a model of entrepreneurial choice incorporating ambiguity aversion and risk neutrality would be useful, it would be more fruitful to estimate separately ambiguity aversion and risk aversion in a coherent framework.

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Appendix: The Characteristics of $\widehat{\phi}(w, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha)$

Verifying the sign of each parameter requires a direct application of the implicit function theorem. Let $H(\phi, \sigma_\epsilon, \mu_\epsilon, w, \theta, \alpha) \equiv V^*(\phi, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha) - w$. Then, we have:

$$\frac{\partial H}{\partial \phi} = -\sqrt{2}\sigma_\epsilon\theta \cdot \left(\frac{\alpha\bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta)}{\delta + r} \right)^{\alpha/(1-\alpha)} < 0,$$

$$\frac{\partial H}{\partial \theta} = (\mu_\epsilon - \sqrt{2}\sigma_\epsilon\theta) \cdot \left(\frac{\alpha\bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta)}{\delta + r} \right)^{\alpha/(1-\alpha)} > 0,$$

$$\frac{\partial H}{\partial \alpha} = \frac{\log\left(\frac{\alpha \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta)}{\delta + r}\right)}{(1 - \alpha)^2} \left[\left(\frac{\alpha \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta)}{\delta + r}\right)^{\alpha/(1-\alpha)} + (\delta + r) \left(\frac{\alpha \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta)}{\delta + r}\right)^{1/(1-\alpha)} \right] > 0,$$

$$\frac{\partial H}{\partial \mu_\epsilon} = \theta \cdot \left(\frac{\alpha \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta)}{\delta + r}\right)^{\alpha/(1-\alpha)} > 0,$$

$$\frac{\partial H}{\partial \sigma_\epsilon} = -\sqrt{2}\phi\theta \cdot \left(\frac{\alpha \bar{A}(\phi, \sigma_\epsilon, \mu_\epsilon, \theta)}{\delta + r}\right)^{\alpha/(1-\alpha)} < 0,$$

and

$$\frac{\partial H}{\partial w} = -1 < 0.$$

As an example, we have:

$$\begin{aligned} \frac{\partial \hat{\phi}(w, \sigma_\epsilon, \mu_\epsilon, \theta, \alpha)}{\partial \theta} &= -\frac{\partial H / \partial \theta}{\partial H / \partial \phi} \\ &= \frac{\mu_\epsilon - \sqrt{2}\sigma_\epsilon\theta}{\sqrt{2}\sigma_\epsilon\theta}. \end{aligned}$$

References

- Adachi, T., 2009. A life-cycle model of entrepreneurial choice: understanding entry into and exit from self-employment. Unpublished manuscript.
- Ahn, D., Choi, S., Gale, D., Kariv, S., 2011. Estimating ambiguity aversion in a portfolio choice experiment. Unpublished manuscript.
- Arrow, K. J., Hurwicz, L., 1972. An optimality criterion for decision-making under ignorance. In: Carter, C.F., Ford, J.L. (Eds.), *Uncertainty and expectations in economics: Essays in honour of G. L. S. Shackle*. Basil Blackwell, Oxford, UK, 1-11.
- Asano, T., 2006. Portfolio inertia under ambiguity. *Mathematical Social Sciences* 52, 223-232.
- , 2010a. Portfolio inertia and epsilon-contaminations. *Theory and Decision* 68, 341-365.
- , 2010b. Optimal tax policy and foreign direct investment under ambiguity. *Journal of Macroeconomics* 32, 185-200.
- , 2010c. Precautionary principle and the optimal timing of environmental policy under ambiguity. *Environmental and Resource Economics* 47, 173-196.
- , Okudaira, H., Sasaki, M., 2011. An experimental test of a search model under Knightian uncertainty. Unpublished manuscript.
- Buera, F.J., 2009. A dynamic model of entrepreneurship with borrowing constraints: Theory and evidence. *Annals of Finance* 5, 443-464.
- Cao, H.H., Wang, T., Zhang, H.H., 2005. Model uncertainty, limited market participation, and asset prices. *Review of Financial Studies* 18, 1219-1251.
- Cressy, R., 1999. The Evans and Jovanovic equivalence theorem and credit rationing: Another look. *Small Business Economics* 12, 295-297.
- , 2000. Credit rationing or entrepreneurial risk aversion? An alternative explanation for the Evans and Jovanovic finding. *Economics Letters* 66, 235-240.

Diamantaras, D., Gilles, R.P., 2011. Ambiguity, social opinion and the use of common property resources. *Journal of Economic Behavior & Organization* 80, 210-222.

Dunn, T., Holtz-Eakin, D., 2000. Financial capital, human capital, and the transition to self-employment: Evidence from intergenerational links. *Journal of Labor Economics* 18, 282-305.

Eichberger, J., Kelsey, D., Schipper, B.C., 2008. Ambiguity and social interaction. *Oxford Economic Papers* 61, 355-379.

Engle-Warnick, J.C., Escobal, J., Laszlo, S.C., 2011. Ambiguity aversion and portfolio choice in small-scale Peruvian farming. *B.E. Journal of Economic Analysis & Policy* 11 (Topics), Article 68.

Epstein, L.G., Miao, J., 2003. A two-person dynamic equilibrium under ambiguity. *Journal of Economic Dynamics and Control* 27, 1253-1288.

—, Wang, T., 1994. Intertemporal asset pricing under Knightian uncertainty. *Econometrica* 62, 283-322.

Evans, D.S., Jovanovic, B., 1989. An estimated model of entrepreneurial choice under liquidity constraints. *Journal of Political Economy* 97, 808-827.

Ghirardato, P., Marinacci, M., 2002. Ambiguity made precise: A comparative foundation. *Journal of Economic Theory* 102, 251-289.

—, Maccheroni, F., Marinacci, M., 2004. Differentiating ambiguity and ambiguity attitude. *Journal of Economic Theory* 118, 133-173.

Gilboa, I., 2009. *Theory of decision under uncertainty*. Cambridge University Press.

—, Schmeidler, D. 1989. Maxmin expected utility with non-unique priors. *Journal of Mathematical Economics* 18, 141-153.

Hansen, L.P., Sargent, T.J., 2008. *Robustness*. Princeton University Press.

Harada, N., Kijima, K., 2005. Consumption-leisure preference structure: A new explanation of the Evans-Jovanovic results for entrepreneurial choice. *Small*

Business Economics 24, 187-191.

Holtz-Eakin, D., Joulfaian, D., Rosen, H.S., 1994a. Entrepreneurial decisions and liquidity constraints. *RAND Journal of Economics* 25, 334-347.

—, —, —, 1994b. Sticking it out: entrepreneurial survival and liquidity constraints. *Journal of Political Economy* 102, 53-75.

Hurst, E., Lusardi, A., 2004. Liquidity constraints, household wealth, and entrepreneurship. *Journal of Political Economy* 112, 319-347.

Iyigun, M., Owen, A.L., 1999. Entrepreneurs, professionals, and growth. *Journal of Economic Growth* 4, 213-232.

Jaffray, J.-Y., 1989. Linear utility theory for belief functions *Operations Research Letters* 8, 107-112.

Kelsey, D., Spanjers, W., 2004. Ambiguity in partnerships. *Economic Journal* 114, 528-546.

Kihlstrom, R.E., Laffont, J.-J., 1979. A general equilibrium entrepreneurial theory of firm formation based on risk aversion. *Journal of Political Economy* 87, 719-748.

Knight, F.H., 1921. Risk, uncertainty and profit. Houghton Mifflin Co.

Kogan, L., Wang, T., 2002. A simple theory of asset pricing under model uncertainty. Unpublished manuscript..

Lucas, R. E., Jr., 1978. On the size distribution of business firms. *Bell Journal of Economics* 9, 508-523.

Maccheroni, F., Marinacci, M., Rustichini, A., 2006. Ambiguity aversion, robustness, and the variational representation of preferences. *Econometrica* 74, 1447-1498.

—, —, —, Taboga, M., 2009. Portfolio selection with monotone mean-variance preferences. *Mathematical Finance* 19, 487-521.

Malchow-Møller, N., Markusen, J.R., Skaksen, J.R., 2010. Labor market institutions, learning and self-employment. *Small Business Economics* 35, 36-52.

- Markowitz, H. M., 1952. Portfolio selection. *Journal of Finance* 7, 77-91.
- Mas-Colell, A., Whinston, M.D., Green, J.R. 1995. *Microeconomic theory*. Oxford University Press.
- Miao, J., 2004. A note on consumption and savings under Knightian uncertainty. *Annals of Economics and Finance* 5, 299-311.
- , Wang, N., 2011. Risk, uncertainty, and option exercise. *Journal of Economic Dynamics and Control* 35, 442-461.
- Nishimura, K.G., Ozaki, H., 2004. Search and Knightian uncertainty. *Journal of Economic Theory* 119, 299-333.
- , —, 2007. Irreversible investment and Knightian uncertainty. *Journal of Economic Theory* 136, 668-694.
- Rigotti, L., Ryan, M., Vaithianathan, R., 2008. Tolerance of ambiguity and entrepreneurial innovation. Unpublished manuscript.
- , —, —, 2011. Optimism and firm formation. *Economic Theory* 47, 1-38.
- Savage, L.J., 1954. *The Foundation of statistics* (2nd revised edition). Dover Publications.
- Schjerning, B., 2006. A dynamic model for occupational choice, savings and formation of entrepreneurial human capital under liquidity constraints and income uncertainty. Unpublished manuscript.
- Schmeidler, D., 1989. Subjective probability and expected utility without additivity. *Econometrica* 57, 571-587.
- Schultz, T.W., 1980. Investment in entrepreneurial ability. *Scandinavian Journal of Economics* 82, 437-448.
- Takii, K., 2003. Prediction ability. *Review of Economic Dynamics* 6, 80-98.
- Tobin, J., 1958. Liquidity preference as behavior towards risk. *Review of Economic Studies* 25, 65-86.
- van Praag, C.M., Cramer, J.S., 2001. The roots of entrepreneurship and labour demand: individual ability and low risk aversion. *Economica* 68, 45-62.

Wakker, P.P., 2010. Prospect theory. Cambridge University Press.