Bloch-sphere representation of three-vertex geometric phases

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The properties of the geometric phases between three quantum states are investigated in a high-dimensional Hilbert space using the Majorana representation of symmetric quantum states. We found that the geometric phases between the three quantum states in an N-state quantum system can be represented by $N-1$ spherical triangles on the Bloch sphere. The parameter dependence of the geometric phase was analyzed based on this picture. We found that the geometric phase exhibits rich nonlinear behavior in a high-dimensional Hilbert space.

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I. INTRODUCTION

The geometric phase was first discovered by Berry for cyclic and adiabatic evolution in 1984 [1]. Subsequently, it was generalized to nonadiabatic evolution [2] and extended to noncyclic and nonunitary evolutions [3]. The early studies by Pancharatnam [4] on the interference of polarized light were seminal for the generalization of geometric phases. Pancharatnam proposed a method for comparing the phases of two differently polarized light beams. When two polarized light beams produce maximum constructive interference, they are regarded as being *in phase*. This means that two quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ are *in phase* when $\langle\psi_1|\psi_2\rangle$ is real and positive [5]. Pancharatnam also pointed out that a geometric phase manifests itself when three differently polarized states are successively compared using the *in-phase* relationship.

The generality of Pancharatnam's geometric phase is apparent in its kinematic definition. It is defined without reference to a Hamiltonian or a dynamic evolution. Mukunda and Simon further developed the kinematic approach for treating geometric phases [6]. In their studies they focus on the Bargmann invariant [7]. For given three states $|\psi_1\rangle$, $|\psi_2\rangle$, and $|\psi_3\rangle$, the three-vertex Bargmann invariant is defined as $\langle\psi_1|\psi_3\rangle\langle\psi_3|\psi_2\rangle\langle\psi_2|\psi_1\rangle$. We refer to the argument of the Bargmann invariant as a three-vertex geometric phase and define it as follows:

$$\gamma(\psi_1,\psi_2,\psi_3) \equiv \arg\langle\psi_1|\psi_3\rangle\langle\psi_3|\psi_2\rangle\langle\psi_2|\psi_1\rangle. \tag{1}$$

The above definition is equivalent to the quantum mechanical expression for Pancharatnam's geometric phase. The three-vertex geometric phase has been shown to equal the geometric phase of the geodesic triangle that has the three states as vertices [3].

As shown in Ref. [6], any pure-state geometric phase can be constructed as a sum of three-vertex geometric phases. Therefore, the three-vertex geometric phase serves as a primitive building block for geometric phases. An experiment has been performed to directly observe the three-vertex geometric phase using a kinematic setup [8].

Three-vertex geometric phases are also important in the field of quantum information since they are closely related to distinguishability of three quantum states [9–11].

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Despite its considerable importance, little is known about the properties of the three-vertex geometric phase of high-dimensional Hilbert spaces. In this paper we propose a way to represent three-vertex geometric phases for N-state systems on the Bloch sphere and we then investigate their characteristic behaviors in high-dimensional Hilbert spaces. For this purpose we use the Majorana representation of symmetric quantum states [12]. Some theoretical studies have investigated geometric phases in N-dimensional Hilbert space based on the Majorana representation [13,14].

This paper is organized as follows. In Sec. II we demonstrate how the three-vertex geometric phase can be represented on the Bloch sphere with the Majorana representation. In Sec. III we review a measurement method for the three-vertex geometric phase in a quantum eraser. In Sec. IV we consider a three-state system as an example and illustrate how the parameter dependence of the three-vertex geometric phase can be visualized on the Bloch sphere. We also show the nonlinear behavior of the geometric phase in a high-dimensional Hilbert space. Section V summarizes the findings of this study.

II. BLOCH SPHERE REPRESENTATION OF THREE-VERTEX GEOMETRIC PHASES

An N-dimensional Hilbert space $\mathcal{H}_N$ is isomorphic to a symmetrized $(N-1)$-qubit Hilbert space:

$$\mathcal{H}_N \simeq \mathcal{S}\big(\mathcal{H}_2^{\otimes N-1}\big), \tag{2}$$

where $\mathcal{S}$ denotes the projection operator onto the permutation-symmetric subspace. We can construct an isomorphism map from $\mathcal{H}_N$ to $\mathcal{S}(\mathcal{H}_2^{\otimes N-1})$ by identifying the ith basis state of $\mathcal{H}_N$ to the $(N-1)$-qubit symmetric Dicke state with $i-1$ excitations. For example, for $N=3$, the three basis states $|1\rangle$, $|2\rangle$, and $|3\rangle$ in $\mathcal{H}_3$ are identified with the states $|00\rangle$, $(|01\rangle+|10\rangle)/\sqrt{2}$, and $|11\rangle$ in $\mathcal{S}(\mathcal{H}_2\otimes\mathcal{H}_2)$. In the following discussion we identify the N-dimensional Hilbert space with the symmetrized $(N-1)$-qubit Hilbert space.

According to Majorana [12], an arbitrary symmetric state $|\Psi\rangle$ can be represented by the symmetrization of product states:

$$|\Psi\rangle = K\cdot\mathcal{S}(|\psi^{(1)}\rangle\cdots|\psi^{(N-1)}\rangle), \tag{3}$$

where $|\psi^{(i)}\rangle$ $(i=1,\ldots,N-1)$ are qubit states and K is a normalization factor. Equation (3) is called the Majorana representation. We can depict the state as an unordered set

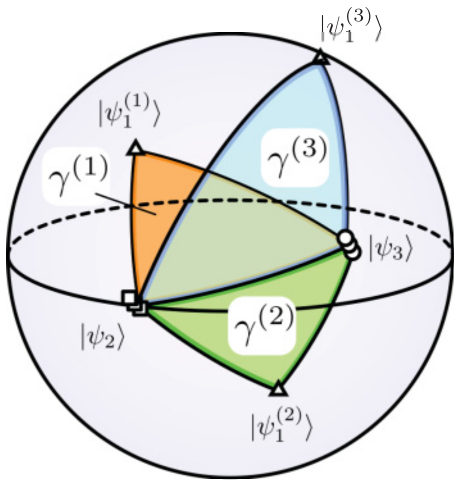

FIG. 1. (Color online) Bloch sphere representation of the three-vertex geometric phase of an N-state system. This figure shows $N = 4$ case. The three-vertex geometric phase of a four-state system is represented as the sum of three geometric phases for a two-state system.

of $N - 1$ points on the Bloch sphere; these points are called Majorana points.

To characterize the geometric phase of the N-state system, we first consider the following special set of three states:

$$|\Psi_1\rangle = K \cdot \mathcal{S}\big(\big|\psi_1^{(1)}\big\rangle \cdots \big|\psi_1^{(N-1)}\big\rangle\big), \tag{4}$$

$$|\Psi_2\rangle = |\psi_2\rangle \cdots |\psi_2\rangle, \tag{5}$$

$$|\Psi_3\rangle = |\psi_3\rangle \cdots |\psi_3\rangle. \tag{6}$$

The first state $|\Psi_1\rangle$ is an arbitrary symmetric state and the other two, $|\Psi_2\rangle$ and $|\Psi_3\rangle$, are chosen from symmetric product states. The three-vertex geometric phase of these three states is expressed by

$$\begin{aligned}\gamma(\Psi_1,\Psi_2,\Psi_3) &= \arg\langle\Psi_1|\Psi_3\rangle\langle\Psi_3|\Psi_2\rangle\langle\Psi_2|\Psi_1\rangle \\ &= \sum_{i=1}^{N-1} \arg\big\langle\psi_1^{(i)}\big|\psi_3\big\rangle\langle\psi_3|\psi_2\rangle\big\langle\psi_2\big|\psi_1^{(i)}\big\rangle \\ &= \sum_{i=1}^{N-1} \gamma^{(i)},\end{aligned} \tag{7}$$

where $\gamma^{(i)} \equiv \gamma(\psi_1^{(i)},\psi_2,\psi_3)$. Equation (7) implies that the geometric phase $\gamma(\Psi_1,\Psi_2,\Psi_3)$ of an N-state system can be expressed as the sum of the $N-1$ geometric phases $\gamma^{(i)}$ of a two-state system. For the two-state system, the three-vertex geometric phase $\gamma^{(i)}$ is proportional to the solid angle $\Omega^{(i)}$ of the geodesic triangle with three vertices $|\psi_1^{(i)}\rangle$, $|\psi_2\rangle$, and $|\psi_3\rangle$; that is, $\gamma^{(i)} = -\Omega^{(i)}/2$ [4]. Therefore, we can regard the geometric phase of the N-state system as $N-1$ spherical triangles on the Bloch sphere, as shown in Fig. 1.

The validity of the above geometric description for $\gamma(\Psi_1,\Psi_2,\Psi_3)$ depends critically on the particular choice of the three states. However, an arbitrary set of three states in the N-state system can always be transformed into the set of forms shown in Eqs. (4)–(6) by a unitary transformation. We assume that $|\Phi_1\rangle$, $|\Phi_2\rangle$, and $|\Phi_3\rangle$ are an arbitrary set of three states. There exists a pair of symmetric product states, $|\Psi_2\rangle$ and $|\Psi_3\rangle$, such that $\langle\Psi_2|\Psi_3\rangle = \langle\Phi_2|\Phi_3\rangle$ since the inner product of a pair of symmetric product states can attain an arbitrary complex number with a modulus less than or equal to 1. Then we can always find a unitary operator $\hat{U}$ such that $|\Psi_2\rangle = \hat{U}|\Phi_2\rangle$, $|\Psi_3\rangle = \hat{U}|\Phi_3\rangle$. Finally, $|\Psi_1\rangle$ is chosen to satisfy $|\Psi_1\rangle = \hat{U}|\Phi_1\rangle$.

As geometric phases are invariant under unitary transformations,

$$\gamma(\Phi_1,\Phi_2,\Phi_3) = \gamma(\Psi_1,\Psi_2,\Psi_3). \tag{8}$$

Therefore, we can represent an arbitrary three-vertex geometric phase on the Bloch sphere by applying a proper unitary transformation.

III. GEOMETRIC PHASE IN QUANTUM ERASERS

In this section we briefly describe a method for measuring the three-vertex geometric phases in a quantum eraser [15,16].

We consider a particle interferometer with internal degrees of freedom, as shown in Fig. 2. Assuming that the particle has an initial state of $|\Psi_1\rangle$, the state of the composite system after the first beam splitter can be written as

$$|\Phi_{\rm i}\rangle\!\rangle = |\Psi_1\rangle|+\rangle, \tag{9}$$

where $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ is a superposition of the upper path state $|0\rangle$ and the lower path state $|1\rangle$. In the lower path the internal state is transformed to a state $|\Psi_2\rangle$ and we have the following intermediate state of the composite system:

$$|\Phi_{\rm m}\rangle\!\rangle = \frac{1}{\sqrt{2}}(|\Psi_1\rangle|0\rangle + |\Psi_2\rangle|1\rangle). \tag{10}$$

We then project the internal state onto $|\Psi_3\rangle$ and obtain the final state

$$|\Phi_{\rm f}\rangle\!\rangle = K|\Psi_3\rangle(\langle\Psi_3|\Psi_1\rangle|0\rangle + \langle\Psi_3|\Psi_2\rangle|1\rangle), \tag{11}$$

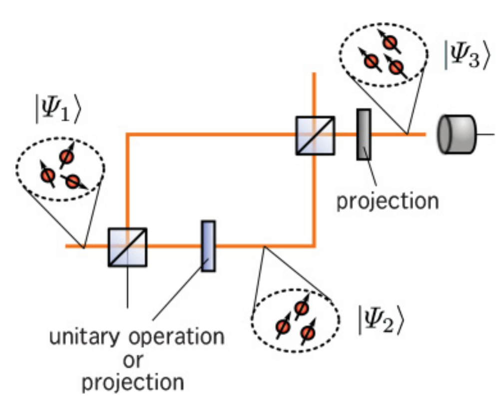

FIG. 2. (Color online) Scheme for measuring three-vertex geometric phases in a quantum eraser. We first prepare an internal state to be $|\Psi_1\rangle$ and split the beam into two. The internal state of the lower beam is then transformed into state $|\Psi_2\rangle$. Finally, the two beams are combined and the interference pattern is observed with or without a projector to the internal state. We can extract the three-vertex geometric phase from the shift of the interference fringes.

where $K = (|\langle\Psi_3|\Psi_1\rangle|^2 + |\langle\Psi_3|\Psi_2\rangle|^2)^{-1/2}$ is the normalization factor. The final measurement for the path state is represented by the projector

$$\hat{P}(\delta) = \hat{1} \otimes |\delta\rangle\langle\delta|, \tag{12}$$

$$|\delta\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\delta}|1\rangle), \tag{13}$$

where δ is the phase difference between the upper and lower paths. The output probability $P(\delta)$ is given by

$$\begin{aligned} P(\delta) &= \langle\langle\Phi_f|\hat{P}(\delta)|\Phi_f\rangle\rangle \\ &= \tfrac{1}{2}(1 + V\cos\varphi), \end{aligned} \tag{14}$$

where

$$V = \frac{2|\langle\Psi_1|\Psi_3\rangle\langle\Psi_3|\Psi_2\rangle|}{|\langle\Psi_3|\Psi_1\rangle|^2 + |\langle\Psi_3|\Psi_2\rangle|^2}, \tag{15}$$

$$\varphi = \arg\langle\Psi_1|\Psi_3\rangle\langle\Psi_3|\Psi_2\rangle - \delta. \tag{16}$$

We change the parameter δ and thereby measure the interference fringes. We can extract the constructive interference point δ_f by setting $\varphi = 0$ as

$$\delta_f = \arg\langle\Psi_1|\Psi_3\rangle\langle\Psi_3|\Psi_2\rangle. \tag{17}$$

Similarly, we can obtain the constructive interference point δ_m for the case without final projection of the internal state as

$$\delta_m = \arg\langle\Psi_1|\Psi_2\rangle. \tag{18}$$

As a result, we can extract the geometric phase γ of the three states as the difference of δ_f and δ_m:

$$\gamma = \delta_f - \delta_m = \arg\langle\Psi_1|\Psi_3\rangle\langle\Psi_3|\Psi_2\rangle\langle\Psi_2|\Psi_1\rangle. \tag{19}$$

This method enables us to measure the three-vertex geometric phase using only state preparation, projection, and interference.

IV. PARAMETER DEPENDENCE OF THE THREE-VERTEX GEOMETRIC PHASE

To investigate the characteristic behavior of the three-vertex geometric phase, we consider a three-state system as an example.

In a three-state system, the Majorana representation that corresponds to Eqs. (4)–(6) is given by

$$|\Psi_1\rangle = K\left(\left|\psi_1^{(1)}\right\rangle\left|\psi_1^{(2)}\right\rangle + \left|\psi_1^{(2)}\right\rangle\left|\psi_1^{(1)}\right\rangle\right), \tag{20}$$

$$|\Psi_2\rangle = |\psi_2\rangle|\psi_2\rangle, \tag{21}$$

$$|\Psi_3\rangle = |\psi_3\rangle|\psi_3\rangle. \tag{22}$$

The normalization factor is calculated as $K = [2(1 + |\langle\psi_1^{(1)}|\psi_1^{(2)}\rangle|^2)]^{-1/2}$. We consider the case where each qubit state is given as follows:

$$\left|\psi_1^{(1)}\right\rangle = \frac{1}{\sqrt{2}}(e^{-i\phi/2}|0\rangle + e^{i\phi/2}|1\rangle), \tag{23}$$

$$\left|\psi_1^{(2)}\right\rangle = \frac{1}{\sqrt{2}}(e^{i\phi/2}|0\rangle + e^{-i\phi/2}|1\rangle), \tag{24}$$

$$|\psi_2\rangle = \cos(\theta/2)|+\rangle - \sin(\theta/2)|-\rangle, \tag{25}$$

$$|\psi_3\rangle = \cos(\theta/2)|+\rangle + \sin(\theta/2)|-\rangle. \tag{26}$$

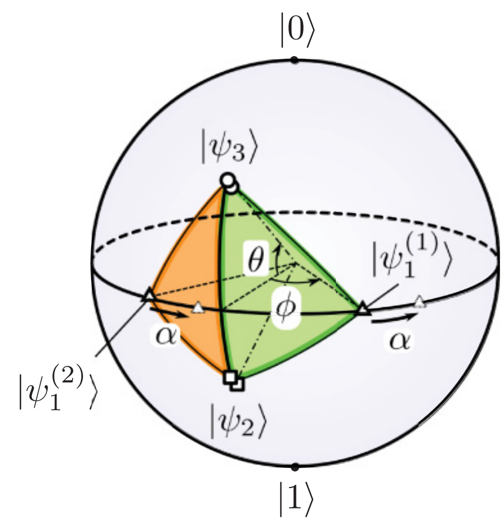

FIG. 3. (Color online) Variation of the Majorana points. The parameter θ represents the half angle between the states $|\psi_2\rangle$ and $|\psi_3\rangle$. The parameter ϕ represents the half angle between the states $|\psi_1^{(1)}(\alpha)\rangle$ and $|\psi_1^{(2)}(\alpha)\rangle$. The two states $|\psi_1^{(1)}(\alpha)\rangle$ and $|\psi_1^{(2)}(\alpha)\rangle$ are rotated on the Bloch sphere in accordance with the parameter α.

where $|0\rangle$ and $|1\rangle$ are the orthonormal basis states and $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. The Majorana points for Eqs. (23)–(26) are depicted in Fig. 3.

We introduce a parameter α for the state $|\Psi_1\rangle$ by applying a unitary operator $\hat{U}(\alpha)$ to each qubit:

$$|\Psi_1(\alpha)\rangle = \hat{U}(\alpha) \otimes \hat{U}(\alpha)|\Psi_1\rangle, \tag{27}$$

$$\hat{U}(\alpha) = \exp(-i\alpha\hat{\sigma}_z/2), \tag{28}$$

where $\hat{\sigma}_z = |0\rangle\langle 0| - |1\rangle\langle 1|$. Some calculation leads to

$$|\Psi_1(\alpha)\rangle = K\left[\left|\psi_1^{(1)}(\alpha)\right\rangle\left|\psi_1^{(2)}(\alpha)\right\rangle + \left|\psi_1^{(2)}(\alpha)\right\rangle\left|\psi_1^{(1)}(\alpha)\right\rangle\right], \tag{29}$$

$$\left|\psi_1^{(1)}(\alpha)\right\rangle = \frac{1}{\sqrt{2}}(e^{-i(\phi+\alpha)/2}|0\rangle + e^{i(\phi+\alpha)/2}|1\rangle), \tag{30}$$

$$\left|\psi_1^{(2)}(\alpha)\right\rangle = \frac{1}{\sqrt{2}}(e^{i(\phi-\alpha)/2}|0\rangle + e^{-i(\phi-\alpha)/2}|1\rangle). \tag{31}$$

As shown in Eq. (7), the three-vertex geometric phase γ for the three-state system ($N = 3$) is the sum of the geometric phases for two qubits:

$$\gamma(\alpha) = \gamma^{(1)}(\alpha) + \gamma^{(2)}(\alpha). \tag{32}$$

Assuming $-\pi/2 < \theta < \pi/2$, the geometric phase of each qubit can be expressed as

$$\gamma^{(1)}(\alpha) = 2\tan^{-1}\left(\tan\frac{\theta}{2}\tan\frac{\phi+\alpha}{2}\right), \tag{33}$$

$$\gamma^{(2)}(\alpha) = -2\tan^{-1}\left(\tan\frac{\theta}{2}\tan\frac{\phi-\alpha}{2}\right). \tag{34}$$

Figure 4 shows the variation of the three-vertex geometric phase. The parameter ϕ is kept constant at $\pi/4$ while the geometric phase γ is varied between 0 and 4π. The three-vertex geometric phase varies extremely nonlinearly at points $\alpha = 3\pi/4$ and $5\pi/4$. This is closely related to the nonlinear behavior of the three-vertex geometric phases of two-state systems [15–19]. The nonlinearity in the two-state systems

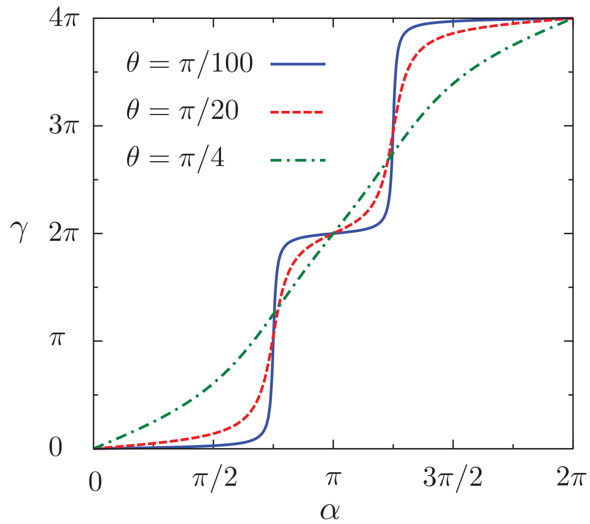

FIG. 4. (Color online) Variation of three-vertex geometric phase. The angle ϕ is kept constant at $\pi/4$. There are two singular points at $\alpha = 3\pi/4$ and $5\pi/4$. As the angle θ decreases, the slopes at these singular points increases.

is due to the geometry of the Bloch sphere and it originates from the drastic change in the geodesic arcs surrounding the spherical triangle. The Majorana representation implies that we can understand the nonlinear behavior of the geometric phase of three-state systems in a similar manner to that of two-state systems. The three-state system has two corresponding spherical triangles. The variation of the three-vertex geometric phase can be interpreted as the nonlinear behavior of these triangles. When $\alpha = 3\pi/4$ and $5\pi/4$, the points corresponding to $|\psi_1^{(1)}(\alpha)\rangle$ and $|\psi_1^{(2)}(\alpha)\rangle$ move around the back side of the Bloch sphere and the areas of the spherical triangles change rapidly.

While the behavior of the geometric phase in this example is related to the geometric phase of two-state systems, it differs qualitatively from the geometric phase of two identical qubit states [20,21], which is simply twice the geometric phase of a one qubit state. It is possible to flexibly control the geometric phase of a three-state system by considering the arrangement of Majorana points.

These results can be extended to N-state systems in a straightforward manner. The variation of the three-vertex geometric phase in an N-state system can be decomposed into the contributions of the $N-1$ three-vertex geometric phases in a two-state system. Although three-vertex geometric phases of N-state systems have more complex and interesting behaviors than those of two state systems, they can be understood simply as a sum of the $N-1$ three-vertex geometric phases in a two-state system.

Finally, we note that a three-state system can be realized by the polarization of two photons in the same spatiotemporal mode, a biphotonic qutrit [22]. While it is difficult to implement general transformations on biphotonic qutrits [23,24], it has been demonstrated that the preparation of biphotonic qutrits in arbitrary states can be realized simply by using nonlinear crystals and wave plates [25–27]. As shown in Eqs. (20)–(22), the measurement of three-vertex geometric phases only requires the preparation of biphotonic qutrits in arbitrary states and the projection onto symmetric product states, which can be implemented with linear polarizers. Therefore, we can observe the nonlinear behavior described in this section by directly applying the measurement method in Sec. III to a biphotonic qutrit system, which is similar to the setup used in Ref. [21].

V. SUMMARY

We presented a way to represent three-vertex geometric phases on the Bloch sphere. Our method is based on the Majorana representation and the three-vertex geometric phase is represented as a set of $N-1$ spherical triangles connecting the corresponding Majorana points. We considered a three-state system as an example and examined the parameter dependence of the three-vertex geometric phase. We showed that the three-vertex geometric phase exhibits interesting nonlinear behavior in a high-dimensional Hilbert space. The characteristic behavior of the three-vertex geometric phase in the N-state system can be well understood by decomposing it into the $N-1$ three-vertex geometric phases of a two-state system.

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