At Starting of an Advanced Research on Hydrodynamics for Geo-Sciences in Space-Planetary Problems

宇宙空間惑星の諸問題に関連した地球科学の研究をはじめるために

Shigehisa Nakamura

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Author: Shigehisa Nakamura

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Note to the Author's Autobiography:

1958 BSc, Kyoto University - (Geophysics-Geoelectro-Magnetics)
1960 MSc, Kyoto University - (Geophysics-Physical Oceanography)
1971 DEng, Kyoto University - (Civil Engineering-Coastal Engineering)
1963-1973 Researches and educations

Membership: American Geophysical Union, USA
- European Geo-Science Union, EU
- Royal meteorological Society, UK
- International Tsunami Society, Hawaii, USA
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INTRODUCTION

In the fields of geo-science, especially, of ocean science, it is essential to have an understanding of water motions on the planet earth.

The author would give a note to the scientists interested in ocean science and the those in the fields of various kinds of the researches closely related to the problems of the ocean.

By this time, a scientific works have been seen in these two hundred years.

It is well know that Ekman had observed a vertical current distribution by using an instrument which was made by Ekman himself. He observed result was the key to see the wind induced current in the sea surface layer in a form of so-called “Ekman spiral.”

Nansen was once trapped his survey ship named as “Fram” during his observation of the ocean water properties to see the water density is determined depending on the salinity and water temperature at the interested water depth. Pressure at the depth was also one of the important factors to determine the density of the water in the ocean, Nansen had found that there were generated an internal wave at the propeller driven for the survey ship. The energy of the propeller was transferred to generated an internal waves instead to move the ship steering forward.

In the northwestern Pacific, a western boundary intensified current was found in an old day. The current was known as “Kuroshio”. A similar western boundary flow has been found as “Gulf Stream “ in the area of the northwest Atlantic. Several model of the wind-induced water circulations. For this purpose, the scientists had introduced an application of the technique in the fields of hydrodynamics with a consideration of the earth’s rotation. Nevertheless, there has been left to be seen a singular point in the interested area.

After the time period for the wind induced water motion, it had been developed several models for thermohaline water circulation in the ocean. This led to consider abyssal water motion, which had been negligible for a long time.

Some developments of instrumentations, turbulence in the ocean had been observed by using submarines. Though, the observed result had been left to be understood in any scope of hydrodynamics.

By an application of satellite monitoring, it has been developed a global observation of the ocean surface. Energy radiation out of the ocean surface was realized as an effect of the ocean surface waves.

On the other hand, there has been promoted several experimental researches by using a water tank with a consideration of similarity for convenience.
REYNOLDS NUMBER

Hydrodynamics uses the equation of motions and the equation of continuity. Basically, the equations are nonlinear though the linearized equations have been used for research as a convenient expression.

Essentially, the equation for fluid motion is in a dynamics of Newton's formula, because the forcing effect is essentially same to the case of Newton Dynamics under an assumption of conservative system, for example, energy conservation, momentum conservation, and mass conservation.

As for the two dimensional problems of fluids, stream function and velocity potential are introduced to define, then, the equations for a fluid expressed in a Newtonian form can be rewritten in a Hamiltonian form, when the equations are described in a scheme of Lagrangean. In case of the conservative system, the Hamiltonian is convenient at any case for analysis.

In order to use the equations in hydrodynamics for various scaling cases in the ocean, a non-dimensional expression of the equations was considered. The first step was an introduction of Reynolds Number.

This parameter was use for mainly an uniform flow. A simple water flow in a conduit or an open channel. Some cases of infinite ocean, the boundary conditions must be considered for the sea surface and for the sea floor.

Usually, irrotational uniform flow is considered first though some specific case of rotational flow can be considered.

The density of the ocean water is a function of salinity, water temperature and pressure, generally, Though, the pressure can be simply considered without any factor of compressibility.

Essentially, a motion of an idealized flow can be easily expressed in a mathematical form for a linear problem, though a motion of a viscous flow must be usually written in a nonlinear form.

When a fluid is slipperly in a conduit or an open channel, an uniform pattern of the flow is seen. Nevertheless, non-uniform pattern of the flow is seen in the conduit or the open-cannel.

In fact, the ocean water has several contents solved and suspended. Especially, the contents of the salinity and water temperature should be taken into account of consideration at densimetric problem. The ocean water motion is governed by the density distribution in the ocean. Adding to this, diffusion processes of the dissolved materials or the suspended material should be considered. Especially, population of zoo planktons in the water, an active diffusion could be preferable for consideration.
RICHARDSON NUMBER

Successively, various dynamical parameters has been introduced. One of the typical examples must be introduction of so-called "Richardson Number".

Especially, in case of turbulence and rotation of waters in oceans and seas, it is necessary to consider Richard Number.

It is generally taken that Reynolds Number is an index of turbulence. An equivalent eddy friction coefficient may introduced. It looks to be corresponding to viscosity.

As for Richardson Number, turbulence around a discontinuity of the stratified water layers should be considered.

With a dimensional consideration, parameters density $\rho$ as a function of the depth $z$, make us to introduce following relations by using the dimension of mass $M$, length $L$, and time $t$, as $M, L, t$ respectively. Then,

$$[\rho] = ML^{-3}, \quad [g] = Lt^{-2},$$

$$[\partial \rho / \partial z] = ML^{-4}, \quad [\partial v / \partial z] = t^{-1}.$$

The dimensionless relationship is written as Richardson Number, i.e.,

$$Ri = [g \partial \rho / \partial z] / (\rho (\partial v / \partial z)^2).$$

When, $(\partial \rho / \partial z) < 0$, then, stratification is unstable. Under this condition, turbulence can easily appear.

As for a locally isotropic turbulence, Kolmogorov' minus 5/3 law is found, i.e.,

$$S(k) = c_1 \epsilon^{2/3} k^{-5/3},$$

where, the dimension is for energy spectrum of turbulence $[S] = L^3 T^{-2}$, the rate of kinetic energy decay down the cascade of eddies $[\epsilon] = L^2 T^{-3}$, and the wave number $[k] = 1/L$. The notation $c_1$ is an universal constant (empirical value is 1.4).

Then, the expression for the eddy coefficient for momentum $A(1/L)$ can be derived as that,

$$A(1/L) = c_2 \epsilon^{2/3} k^{-5/3},$$

Where $c_2 = 0.1$. This is well known law of geophysical law (for "4/3 law")

It was found by A. Obukhov (1941), and, also found empirically by L. F. Richardson in 1926 [cf. David Tolmazin, 1985].
ROTATING RING WATER TANK

Swinny was studied under Richardson first.

His work was on water motion in a rotating ring water tank on a circular disc turning. What Swinny in 2012 told us was that the water motion in the tank rotating slowly no significant flow trend was seen except a turbulent like random motion.

Increasing of the rotating speed, the water motion is looked as if it were forming a magical complicated pattern. It was hard to describe in a simple mathematical form.

At a certain speed of the rotation, the several straying eyes of eddies were found, when Richardson had said to Swinny that the water motion is in a stage of chaos.

A little change of the rotating speed, then, four or five cat's eyes were seen to be in a stable state. What of his talking about his indoor experiment was appeared quite similar to a butterfly pattern in a phase space for a meteorological process which was a result reduced numerically during mathematical process to solve a simultaneous equations under a specific condition by Shukuro Manabe.

Swinny had demonstrated this cat's eye formation process in a moving picture.

A specific speed of the rotation was sensitively selected to obtain the most table cat's eye pattern in the water flow with a minor reversal flow along the inner wall of the ring water tank.

As far as the author concern, the above cat's eye pattern must be kept to be stable when two reversal minor flows along the inner and outer wall appeared to balance to the cat's eye pattern in shearing the water flow energy.

This pattern formation is very interesting in a scope of hydrodynamics. At the same time, this process might appear somewhere on the ocean surface if the available satellites properly monitored on the ocean surface.

In this rotating water tank, it was seen that a stable pattern formation of four or five cat's eyes patterns. There might be a transitional process must be seen though the experiment was concentrated Swinny's interest to a steadily maintained cats'eyes pattern. Otherwise, he might be considered to demonstrate several typical cases for the other scientists willingly.

In the field of astronomy, the zeroth mode of the magnetic field of the sun was shown by Stix. This concerns to the magnetic main field of the sun. With some arrangement of one parameter, Stix had demonstrated the poloidal and toroidal modes of the sun for the eleven years cycle variation. So that, theoretical solution was obtained referring to a spherical co-ordinate in order to reduced the solution in a form of a set of spherical functions. Stix has left to notice a nonlinear solution could be obtained under a specific condition after his postulated form of a simultaneous equation. This simultaneous equation looks quite similar to that reduced by Manabe.
INTERNAL WAVE IN A WATER TANK

As for internal waves in the ocean, it must be our first experience by Fritchov Nansen in the Arctic Sea on board of the survey ship “Fram”. After his operation, he had a strange reaction of the ship when the ship was started to drive its propeller in order move to the other position. The propeller’s energy was transferred to generate internal waves except the ship promotion effect for driving forward.

Swinney in University of Texas at Austin had undertaken an experiment of the internal tide waves in a rectangular water tank of open channel type. He had made an equivalent ocean water artificially to fill the water into the tank which was arranged to rotate at a selected frequency around the vertical axis centered to be a pivot.

Swinney noes as that tidal flow over the continental shelf break and ocean bottom topography generates internal gravity waves, or so-called “internal tides” that account for a significant part of the energy budget of the oceans. Thence, the internal tides play a role in determining climate.

Following to note as Swinney, internal tides propagate at an angle \( \theta \) with respect to the horizontal, where (neglecting Coriolis effects) \( \sin \theta = \omega / N \) by using a notation \( \omega \) as tidal frequency and a notation of \( N \) for Brunt-Väisälä buoyancy frequency.

Swinney’s research group had a plan to undertake their laboratory experiments in order to reveal that intense resonant boundary current occur when the angle \( \theta \) matches the local topographic slope angle. This resonant condition is approximated well along the continental slope of the oceans, which may explain the reason why the continental slope angles are typically only a few degrees, much less than the angle of repose. To determine how the local buoyancy frequency \( N(z) \) varies with depth \( z \), they had examined data from the world ocean circulation experiment as WOCE.

An analysis of the more than 18000 WOCE data sets for temperature(\( z \)) and salinity(\( z \)) revealed the existence at many locations of “turning depth”, that is, depths below which the M2 semi-diurnal internal tides cannot propagate.

Below a turning depth (typically 4·5 km), an internal tide becomes exponentially damped and hence does not reflect and scatter from the ocean floor as is often assumed.

Then, they could also present results from numerical simulations of the Navier-Stokes equations and laboratory experiments on internal wave reflection, propagation, and tunneling for tidal flow over topography that lies below a turning depth. While experiments and models yield insights into internal wave dynamics, many important phenomena such as wave breaking and mixing remain poorly understood.
KELVIN-HELMHOLTZ INSTABILITY

In a case of the oscillations of the common boundary of two superposed liquids which are otherwise unlimited.

This boundary in the ocean is an interface where internal waves generated by some disturbances. It is possible to take it to be a formation of a stable internal wave on the boundary when the amplitudes of the disturbances are small enough.

The problem for case of the disturbances should be considered as a nonlinear problem even in the gravity field.

When the this boundary is initially to be horizontal, and when the density $\rho'$ in the upper layer is related to the density $\rho$ in the lower layer as $\rho < \rho'$, the undisturbed equilibrium arrangement is then unstable.

As for damping of sound in narrow channels, it had been solved and described already in 1926. Nevertheless, it is not well understood by the beginners in the related fields in sciences.

As for viscosity, it is also discussed already even for a case of oscillating sphere.

As for the cases of nonlinear processes, it is necessary to consider the second order serious limits the application of many preceding linear cases to fluid possessed of the first order of mobility. Unless the velocities, or the linear dimensions involved, be very small the actual motion in such case, so far as it admits of being observed, is found to be very different from that represented by our formulae. For example, when a solid of 'easy' shape moves through a liquid, an irregular eddying motion may be produced in a layer of the fluid next to the solid, and a trail of eddies left behind, whilst the motion at a distance laterally may be comparatively smooth and uniform.

The case of flow through a pipe of circular section was made the subject of a careful experimental study by Reynolds. Since then, it has been seen a surprisingly advance research by this time. Hydrodynamics applied to the problems in the ocean and un the atmosphere has had extended to apply to the problems in the planets of the solar system.

Magneto-hydrodynamics has been already applied for the problems in the planets and their satellites in the solar system of the space.

Problems in an interface of a couple of the magnetospheres are applied to see the signals informing the processes which have been never seen in the last age.

Now, it should be careful to apply the techniques developed on the planet earth without any consideration of different process quite similar to what found on the earth. Consider that so-called "Kelvin-Helmholtz instability" of a specific planet or its satellites can be seen only from the dataset obtained on the spacecraft orbit.
TURNING WAVES

Lastly, the author would introduce an experiment for turning waves.

This looks as if it were to be in relation to a similar process neighbor the beach.

Outline of this "turning waves" is introduced as follows.

For convenience, Swinney's experiment in his water tank shaped to be a rectangular box without a cap.

Top view of the tank looks to be a square for convenience. Consider a straight shelf between a model deep and a model shore stretched along a side of the tank.

When a wave train is generated at the left corner of the opposite side of the tank to hit the center part of the model shore, the wave turns its direction at the point where the gap between the model deep and the model shore. The wave induces a wave on the shore forming a straight crest normal to the shoreline at the center part of the model shore, and travels to the right along the straight shore line parallel to the tank side wall.

Swinney tells us that the wave observed on the continental shelf off the east coast of the United States behaves quite similar to the wave which was seen in his experiment in the tank. His interest was not in reflected wave but observed wave turn on the edge between the continental shelf and deep in the northwest Atlantic.

The author, here, would note another case similar to what noted above.

The author reminds a quite similar experiment in the project of civil engineering in the function of the United States. The engineers had strongly interested the fact of the wave does not get to the shoreline to form a reflected wave. On the shore, a wave with a crest normal to the shoreline was observed to travelling. It cannot be taken as any reflected wave. Then, the engineers named it as "edge wave" which is trapped on the shore though no dynamical mechanism has been left to be solved later.

Then, the author has to introduce that a similar case was in the author's experiment. The process of wave propagation is essentially quite similar to what noted above. The author had ever presented what he had seen during his experiment in the water basin. Related references are introduced as follows, for example,


At the end of this note, the author tends to raise his notice to promote an advanced research on nonlinear process on the planet earth in the solar system.
MAGNETOHYDRODYNAMICS

The IPY (International Polar Year) has brought a key to start of research in Geomagnetism not only in Geophysics.

One of the significant contributions reduced out of the observed data during the IPY must be a finding of "Solar Quiet Day's Geomagnetic Variations", later noted as that "Sq" of geomagnetic field. This work was reduced in an application of a simple technique of a harmonic analysis.

Aurora Oval Boreares was first observed and analyzed to give a key to the solar wind effect to the geomagnetic sheath. In 2012, an extensive work is promoted to tell the shape of a geomagnetic sheath tail in a consideration of plasma dynamics.

Magneto-hydrodynamics (MHD) has been developed and applied to the problems in the specific geomagnetic field. In the year of 2012, this MHD is applied for what recorded at monitoring on the spacecraft's orbit. This orbit tells us a dimensional problem at our learning on the space or the planets in the solar system, for example.

Nevertheless, the natural processes even in the space is three dimensional.

Researches on ionospheres had given us an important key for the age of space science. Techniques for rocket have been applied well for launching the artificial satellites.

Satellites help us to inform all of natural process as well as the human activity. As for this fact, the scientists already know well.

Space crafts have informed about what can be observed as well as about what can be seen the planets of the solar system.

With the steps in advances of researches in the various kinds of fields, it has been for the scientists in geo-science to find what processes can be monitored.

Looking at the recent reports, the author feels it to be considered that the scientists are tending to consider the information in a manner of those on the earth. Then, the author has to wait for many kinds of findings after analyzing the monitored data from the spacecraft operating on their orbital motion.

Nevertheless, the author has to wait for some notes about what nonlinear process is found during the monitoring of the planet earth and of the space inside or outside of the solar system.

Now, the author has to notice here that he is considering no conclusion should be given in this notice because, the author's aim is to give the scientists a key to find their gates for their advanced research.