Thermal spin pumping mediated by magnon in the semiclassical regime

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Abstract

We microscopically analyze thermal spin pumping mediated by magnons, at the interface between a ferromagnetic insulator and a non-magnetic metal, in the semiclassical regime. The generation of a spin current is discussed by calculating the thermal spin transfer torque, which breaks the spin conservation law for conduction electrons and operates the coherent magnon state. Inhomogeneous thermal fluctuations between conduction electrons and magnons induce a net spin current, which is pumped into the adjacent non-magnetic metal. The pumped spin current is proportional to the temperature difference. When the effective temperature of magnons is lower than that of conduction electrons, localized spins lose spin angular momentum by emitting magnons and conduction electrons flip from down to up by absorbing all the emitted momentum, and vice versa. Magnons at the zero mode cannot contribute to thermal spin pumping because they are eliminated by the spin-flip condition. Consequently thermal spin pumping does not cost any kinds of applied magnetic fields. We have discussed the distinction from the theory proposed by Xiao et al. [Phys. Rev. B, **81** (2010) 214418], Adachi et al. [Phys. Rev. B, **83** (2011) 094410], and Bender et al. [arXiv:1111.2382]. Supplement is available at this URL; http://dl.dropbox.com/u/5407955/SupplementTSP.pdf

1 Introduction

Recently spintronics has developed a new branch of physics called spin caloritronics [1, 2], which combines thermoelectrics with spintronics. Spin caloritronics has been attracting a special interest because of potential applications to green information and communication technologies [3]. The central theme is the utilization of thermal fluctuations as well as spin degrees of freedom in order to induce a (pure) spin current. Thus establishing methods for the generation of a

spin current by using thermal difference, without any kinds of applied magnetic fields, is a significant issue.

In the previous work [4], we have studied quantum spin pumping mediated by magnons under a time-dependent transverse magnetic field at the interface between a ferromagnetic insulator and a non-magnetic metal. There the ferromagnet act as a source of spin angular momentum; magnon battery named after the spin battery [5]. The applied time-dependent transverse magnetic field acts as a quantum fluctuation to induce a pumped net spin current under a thermal equilibrium condition. Spin angular momentum is exchanged between conduction electrons and localized spins via magnons accompanying the exchange interaction at the interface. The interface is defined as an effective area where the Fermi gas (conduction electrons) and the Bose gas (magnons) coexist to interact; the width of the interface is supposed to be of the order of the lattice constant [6]. In addition, the pumped net spin current has a resonance structure as a function of the angular frequency of the applied transverse field, which is useful to enhance the spin pumping effect induced by quantum fluctuations. Here it should be stressed that magnons accompanying the exchange interaction cannot contribute to spin pumping without quantum fluctuations. That is, quantum fluctuations (i.e. time-dependent transverse magnetic fields) are essential to quantum spin pumping mediated by magnons.

In this paper, we microscopically propose an alternative mechanism for the generation of the spin current without any kinds of applied magnetic fields (i.e. quantum fluctuations); thermal spin pumping [7]. Inhomogeneous thermal fluctuations, i.e. the temperature difference, between conduction electrons and magnons induce a net spin current, which is pumped into the adjacent non-magnetic metal. This method can be viewed as an alternative way for the local spin injection.

We assume the local equilibrium condition [8]; since the relaxation times in the localized spins (i.e. magnons) and conduction electrons subsystems are much shorter than the lattice relaxation time [9, 10, 11], the reservoirs become thermalized internally before they equilibrate with each other. Therefore we may assume that during the relaxation process, conduction electrons and magnons can be described by their effective local temperatures; $T_{\rm s}$ and $T_{\rm m}$ [7, 12]. According to Xiao et al. [13], the condition (i.e. temperature difference) can be generated by a temperature bias applied over the ferromagnetic film.

The theoretical setup [12] is almost the same with our previous work [4] except the point that applied magnetic fields are not essential; in particular, transverse magnetic fields are absent. We consider a ferromagnetic insulator and non-magnetic metal junction shown in Fig. 1 where conduction electrons couple with localized spins $\mathbf{S}(\mathbf{x},t)$, $\mathbf{x}=(x,y,z)\in\mathbf{R}^3$, at the interface;

$$\mathcal{H}_{\rm ex} = -2Ja_0^3 \int_{\mathbf{x}\in({\rm interface})} d\mathbf{x}\ \mathbf{S}(\mathbf{x},t)\cdot\mathbf{s}(\mathbf{x},t). \tag{1}$$

The exchange coupling constant reads $2J$, and the lattice constant of the ferromagnet is a_0. In this paper, we take $\hbar=1$ for convenience. The magnitude

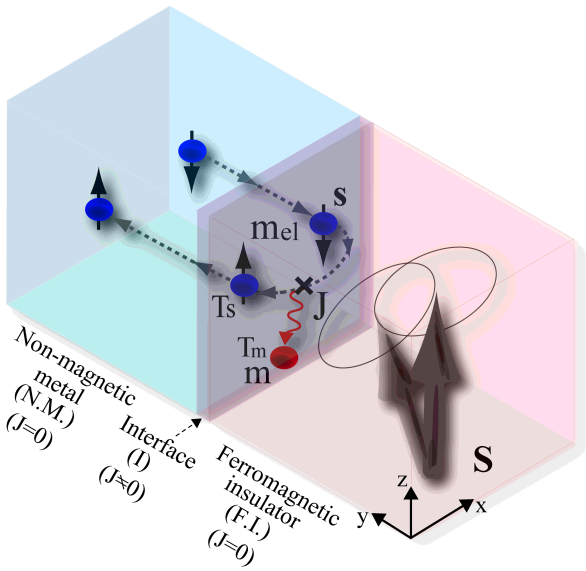

Figure 1: A schematic picture of thermal spin pumping mediated by magnons. Spheres represent magnons and those with arrows are conduction electrons. When the effective temperature of magnons ($T_{\rm m}$) is lower than that of conduction electrons ($T_{\rm s}$), localized spins lose spin angular momentum by emitting a magnon and conduction electrons flip from down to up by absorbing the momentum, and vice versa. The interface is defined as an effective area where the Fermi gas (conduction electrons) and the Bose gas (magnons) coexist to interact; $J \neq 0$. In addition, conduction electrons cannot enter the ferromagnet, which is an insulator.

of the interaction is supposed to be constant and we adopt the continuous limit also in the present study. Conduction electron spin variables are represented as

$$s^j = \sum_{\eta,\zeta=\uparrow,\downarrow} [c_\eta^\dagger (\sigma^j)_{\eta\zeta} c_\zeta]/2 \tag{2}$$

$$\equiv (c^\dagger \sigma^j c)/2, \tag{3}$$

where σ^j are the 2×2 Pauli matrices; $[\sigma^j, \sigma^k] = 2i\epsilon_{jkl}\sigma^l$, $(j, k, l = x, y, z)$. Operators $c^\dagger/c$ are creation/annihilation operators for conduction electrons, which satisfy the (fermionic) anticommutation relation; $\{c_\eta(\mathbf{x}, t), c_\zeta^\dagger(\mathbf{x}', t)\} = \delta_{\eta,\zeta}\delta(\mathbf{x} - \mathbf{x}')$.

We focus on the dynamics at the interface where spin angular momentum is exchanged between conduction electrons and the ferromagnet. We suppose the uniform magnetization and thus localized spin degrees of freedom can be

mapped into magnon ones via the Holstein-Primakoff transformation;

$$S^+(\mathbf{x},t) \equiv S^x(\mathbf{x},t) + iS^y(\mathbf{x},t) \tag{4}$$
$$= \sqrt{2\tilde{S}}a(\mathbf{x},t) + \mathcal{O}(\tilde{S}^{-1/2}), \tag{5}$$

$$S^-(\mathbf{x},t) \equiv S^x(\mathbf{x},t) + iS^y(\mathbf{x},t) \tag{6}$$
$$= \sqrt{2\tilde{S}}a^\dagger(\mathbf{x},t) + \mathcal{O}(\tilde{S}^{-1/2}), \tag{7}$$

$$S^z(\mathbf{x},t) = \tilde{S} - a^\dagger(\mathbf{x},t)a(\mathbf{x},t), \tag{8}$$

$\tilde{S} \equiv S/a_0^3$, where operators $a^\dagger/a$ are magnon creation/annihilation operators satisfying the (bosonic) commutation relation; $[a(\mathbf{x},t), a^\dagger(\mathbf{x}',t)] = \delta(\mathbf{x}-\mathbf{x}')$. Up to the $\mathcal{O}(S)$ terms, localized spins reduce to a free boson system. Consequently in the quadratic dispersion (i.e. long wavelength) approximation, the localized spin with the applied magnetic field along the quantization axis (z-axis) is described by the Hamiltonian $\mathcal{H}_{\rm mag}$;

$$\mathcal{H}_{\rm mag} = \int_{\mathbf{x}\in({\rm interface})} d\mathbf{x}\ a^\dagger(\mathbf{x},t)\Big(-\frac{\nabla^2}{2m} + B\Big)a(\mathbf{x},t), \tag{9}$$

and the Hamiltonian, $\mathcal{H}_{\rm ex}(\equiv \mathcal{H}^S_{\rm ex} + \mathcal{H}'_{\rm ex})$, can be rewritten as

$$\mathcal{H}^S_{\rm ex} = -JS\int_{\mathbf{x}\in({\rm interface})} d\mathbf{x}\ c^\dagger(\mathbf{x},t)\sigma^z c(\mathbf{x},t), \tag{10}$$

$$\mathcal{H}'_{\rm ex} = -Ja_0^3\sqrt{\frac{\tilde{S}}{2}}\int_{\mathbf{x}\in({\rm interface})} d\mathbf{x}[a^\dagger(\mathbf{x},t)c^\dagger(\mathbf{x},t)\sigma^+c(\mathbf{x},t) + a(\mathbf{x},t)c^\dagger(\mathbf{x},t)\sigma^-c(\mathbf{x},t)]. \tag{11}$$

The variable m represents the effective mass of a magnon. We have denoted a constant applied magnetic field along the quantization axis as B, which includes g-factor and Bohr magneton. Let us mention that though we formulate the thermal spin pumping theory with B for generalization, in this paper we finally take $B = 0$ in sec. 3 and discuss the thermal spin pumping effect in sec. 4.

The total Hamiltonian of the system (interface), $\mathcal{H}$, is given as

$$\mathcal{H} = \mathcal{H}_{\rm mag} + \mathcal{H}'_{\rm ex} + \mathcal{H}_{\rm el},\ \text{where} \tag{12}$$

$$\mathcal{H}_{\rm el} = \int_{\mathbf{x}\in({\rm interface})} d\mathbf{x}\ c^\dagger(\mathbf{x},t)\Big[-\frac{\nabla^2}{2m_{\rm el}} - (JS + \frac{B}{2})\sigma^z\Big]c(\mathbf{x},t). \tag{13}$$

The variable $m_{\rm el}$ denotes the effective mass of a conduction electron. Eq. (64) shows that

$$JS \tag{14}$$

acts as an effective magnetic field.

The dynamics at the interface is described by the Hamiltonian $\mathcal{H}'_{\rm ex}$; eq. (63) shows that localized spins at the interface lose spin angular momentum by emitting a magnon and a conduction electron flips from down to up by absorbing the spin angular momentum (see Fig. 1), and vice versa. This Hamiltonian $\mathcal{H}'_{\rm ex}$, which describes the interchange of spin angular momentum between localized spins and conduction electrons, is essential to spin pumping. Therefore we clarify the contribution of magnons accompanying this exchange interaction to spin pumping. This is the main purpose of this paper. Here it should be noted that we treat localized spins as not classical variables [13] but magnon degrees of freedom. As the result, we can microscopically capture the (non-equilibrium) spin-flip dynamics on the basis of the rigorous quantum mechanical theory.

This paper is structured as follows. First, through the Heisenberg equation of motion, the thermal spin transfer torque which breaks the spin conservation law for conduction electrons is defined in sec. 2. Second, we evaluate it through the Schwinger-Keldysh formalism at finite temperature in sec. 3. Last we discuss why thermal spin pumping does not cost any applied magnetic fields in sec. 4, with pointing out the distinction from the farseeing work by Adachi et al. [12].

2 Thermal spin transfer torque

2.1 Definition

The thermal spin transfer torque (TSTT) [12, 14, 15], $\mathcal{T}_{\rm s}^z$, is defined as the term which breaks the spin conservation law for conduction electrons;

$$\dot{\rho}_{\rm s}^z + \nabla \cdot \mathbf{j}_{\rm s}^z = \mathcal{T}_{\rm s}^z, \tag{15}$$

where the dot denotes the time derivative, $\mathbf{j}_{\rm s}$ is the spin current density[16], and $\rho_{\rm s}^z$ represents the z-component of the spin density. We here have defined the spin density of the system as the expectation value (estimated for the total Hamiltonian, $\mathcal{H}$);

$$\rho_{\rm s}^z \equiv \langle c^\dagger \sigma^z c/2 \rangle. \tag{16}$$

In this paper, we focus on the z-component of the TSTT.

Through the Heisenberg equation of motion, the z-component of the TSTT is defined as

$$\mathcal{T}_{\rm s}^z = iJa_0^3\sqrt{\frac{\tilde{S}}{2}}\langle a^\dagger(\mathbf{x},t)c^\dagger(\mathbf{x},t)\sigma^+c(\mathbf{x},t) - a(\mathbf{x},t)c^\dagger(\mathbf{x},t)\sigma^-c(\mathbf{x},t)\rangle. \tag{17}$$

This term arises from $\mathcal{H}'_{\rm ex}$, which consist of electron spin-flip operators;

$$\mathcal{T}_{\rm s}^z = [\rho_{\rm s}^z, \mathcal{H}'_{\rm ex}]/i. \tag{18}$$

Thus, eq. (15) shows that the TSTT ($\mathcal{T}_{\rm s}^z > 0$) can be understood as the number density of conduction electrons which flip from down to up per a unit of time [8], and vice versa. In addition, the TSTT operates the coherent magnon state [17].

2.2 Pumped net spin current

In this subsection, we clarify the relation between the TSTT and the pumped net spin current. As discussed in the last subsection, the spin conservation law for conduction electrons is broken due to the interaction $\mathcal{H}'_{\rm ex}$;

$$\dot{\rho}^z_{\rm s} + \nabla \cdot \mathbf{j}^z_{\rm s} = \mathcal{T}^z_{\rm s}. \tag{19}$$

Thus one cannot simply view the time derivative of the spin density for conduction electrons, $\dot{\rho}^z_{\rm s}$, as the spin current density.

In respect to Planck's constant (we here partially recover $\hbar$), the time derivative of the spin density and the TSTT satisfy the relation [18, 19];

$$\frac{\dot{\rho}^z_{\rm s}}{\mathcal{T}^z_{\rm s}} = \mathcal{O}(\hbar). \tag{20}$$

Therefore $\dot{\rho}^z_{\rm s}$ is negligible in comparison with $\mathcal{T}^z_{\rm s}$ at the semiclassical regime, where our interest lies. As the result, the spin continuity equation, eq. (19), becomes

$$\mathcal{T}^z_{\rm s} = \nabla \cdot \mathbf{j}^z_{\rm s}. \tag{21}$$

Then by integrating over the interface, we can evaluate the pumped net spin current, $\int \mathbf{j}^z_{\rm s} \cdot d\mathbf{S}_{\rm interface}$;

$$\int_{\mathbf{x}\in(\rm interface)} d\mathbf{x}\ \mathcal{T}^z_{\rm s} = \int_{\mathbf{x}\in(\rm interface)} d\mathbf{x}\ \nabla \cdot \mathbf{j}^z_{\rm s} \tag{22}$$

$$= \int \mathbf{j}^z_{\rm s} \cdot d\mathbf{S}_{\rm interface}. \tag{23}$$

In addition, conduction electrons cannot enter the ferromagnet, which is an insulator [20]. Thus the net spin current pumped into the non-magnetic metal can be calculated by integrating the TSTT over the interface, eq. (23).

From now on, we focus on $\mathcal{T}^z_{\rm s}$ and qualitatively clarify the behavior of the thermal spin pumping effect mediated by magnons, at room temperature in the semiclassical regime, in sections 3 and 4.

2.2.1 The spin continuity equation for the whole system

It will be useful to point out that the spin conservation law for localized spins (i.e. magnons) is also broken. The magnon continuity equation for localized spins [21] reads

$$\dot{\rho}^z_{\rm m} + \nabla \cdot \mathbf{j}^z_{\rm m} = \mathcal{T}^z_{\rm m}, \tag{24}$$

where $\mathbf{j}_{\rm m}$ is the magnon current density, and $\rho^z_{\rm m}$ represents the z-component of the magnon density. We have defined the magnon density of the system also as the expectation value (estimated for the total Hamiltonian, $\mathcal{H}$);

$$\rho^z_{\rm m} \equiv \langle a^\dagger a \rangle. \tag{25}$$

In addition, we call $\mathcal{T}_{\rm m}^z$ the magnon source term [21], which breaks the magnon conservation law. This term arises also from $\mathcal{H}'_{\rm ex}$;

$$\mathcal{T}_{\rm m}^z = [\rho_{\rm m}^z, \mathcal{H}'_{\rm ex}]/i. \tag{26}$$

Through the Heisenberg equation of motion, the magnon source term can be determined and it satisfies the relation;

$$\mathcal{T}_{\rm m}^z = \mathcal{T}_{\rm s}^z. \tag{27}$$

Then the z-component of the spin continuity equation for the total system (i.e. conduction electrons and magnons) becomes

$$\dot{\rho}_{\rm total}^z + \nabla \cdot \mathbf{j}_{\rm total}^z = 0, \tag{28}$$

where the density of the total spin angular momentum, $\rho_{\rm total}^z$, is defined as

$$\rho_{\rm total}^z \equiv \rho_{\rm s}^z - \rho_{\rm m}^z, \tag{29}$$

and consequently the z-component of the total spin current density, $\mathbf{j}_{\rm total}^z$, becomes

$$\mathbf{j}_{\rm total}^z = \mathbf{j}_{\rm s}^z - \mathbf{j}_{\rm m}^z \tag{30}$$

(note that, $S^z = \tilde{S} - a^\dagger a$, via the Holstein-Primakoff transformation in sec. 1). The spin continuity equation for the whole system, eq. (28), means that though each spin conservation law for electrons and magnons is broken (see eqs. (19) and (24)), the total spin angular momentum is, of course, conserved [8].

2.2.2 The work by Bender et al.

Last, let us mention a recent preprint [22] by Bender et al., where the authors consider a similar problem. We have chosen a different definition of the pumped spin current, for reasons now explained.

Though they have simply recognized the time derivative of the spin density for localized spins,

$$\dot{\rho}_{\rm m}^z, \tag{31}$$

as the spin current,[1] it reads

$$\dot{\rho}_{\rm m}^z \overset{\text{eq.(28)}}{=} \dot{\rho}_{\rm s}^z + \nabla \cdot (\mathbf{j}_{\rm s}^z - \mathbf{j}_{\rm m}^z). \tag{32}$$

Thus it is clear that even when the total spin angular momentum is conserved (eq. (28)), $\dot{\rho}_{\rm m}^z$ is not directly related to the spin current itself, $\mathbf{j}_{\rm s}^z$. That is, $\dot{\rho}_{\rm m}^z$ includes other contributions arising from $\dot{\rho}_{\rm s}^z$ and $\mathbf{j}_{\rm m}^z$ as well as $\mathbf{j}_{\rm s}^z$. Therefore the definition of the pumped spin current by Bender et al. [22] is, in any regime, inadequate to their and our case; the mixture of the Bose (magnon) gas and Fermi (conduction electron) one.

That is why, we have adopted different definition of the pumped spin current, eq. (23), and evaluate the TSTT.

[1] Note that we have adopted our notation. The variable dS_L^z/dt in Ref. [22] corresponds to $\dot{\rho}_{\rm m}^z$.

3 Schwinger-Keldysh formalism

The interface is, in general, a weak coupling regime [23]; the exchange interaction, J, is supposed to be smaller than the Fermi energy and the exchange interaction among ferromagnets. Thus $\mathcal{H}'_{\rm ex}$ can be treated as a perturbative term.

Through the standard procedure of the Schwinger-Keldysh (or non-equilibrium) Green's function [24, 25, 26], the Langreth method [27, 28, 29], the TSTT can be evaluated as

$$\begin{aligned}\mathcal{T}_{\rm s}^z &= 2iJ^2a_0^3S\int\frac{d\mathbf{k}_1}{(2\pi)^3}\int\frac{d\mathbf{k}_2}{(2\pi)^3}\int\frac{d\omega_1}{2\pi}\int\frac{d\omega_2}{2\pi}\\ &\times [\mathcal{G}^{>}_{\uparrow,\mathbf{k}_2,\omega_2}\mathrm{G}^{>}_{\mathbf{k}_1,\omega_1}\mathcal{G}^{<}_{\downarrow,\mathbf{k}_1+\mathbf{k}_2,\omega_1+\omega_2}-\mathcal{G}^{<}_{\uparrow,\mathbf{k}_2,\omega_2}\mathrm{G}^{<}_{\mathbf{k}_1,\omega_1}\mathcal{G}^{>}_{\downarrow,\mathbf{k}_1+\mathbf{k}_2,\omega_1+\omega_2}]+\mathcal{O}(B^3) \qquad (33)\end{aligned}$$

The variable $\mathcal{G}^{<(>)}$ is the fermionic lesser (greater) Green's function, and $\mathrm{G}^{<(>)}$ is the bosonic one. We here have taken the extended time defined on the Keldysh contour [26, 27, 28], c, on the forward path $\mathrm{c}_{\rightarrow}$; $c=\mathrm{c}_{\rightarrow}+\mathrm{c}_{\leftarrow}$. Even when the time is located on the backward path $\mathrm{c}_{\leftarrow}$, the result of the calculation does not change because each Green's function is not independent; $\mathcal{G}^{\rm r}-\mathcal{G}^{\rm a}=\mathcal{G}^{>}-\mathcal{G}^{<}$, where $\mathcal{G}^{\rm r(a)}$ represents the retarded (advanced) Green's function [21]. This relation comes into effect also for the bosonic case [26].

Each Green's function reads as follows [25];

$$\mathrm{G}^{<}_{\mathbf{k},\omega} = -2\pi i f_{\rm B}(\omega)\delta(\omega-\omega_{\mathbf{k}}), \qquad (34)$$

$$\mathrm{G}^{>}_{\mathbf{k},\omega} = -2\pi i[1+f_{\rm B}(\omega)]\delta(\omega-\omega_{\mathbf{k}}), \qquad (35)$$

$$\mathcal{G}^{<}_{\sigma,\mathbf{k},\omega} = 2\pi i f_{\rm F}(\omega)\delta(\omega-\omega_{\sigma,\mathbf{k}}), \qquad (36)$$

$$\mathcal{G}^{>}_{\sigma,\mathbf{k},\omega} = -2\pi i[1-f_{\rm F}(\omega)]\delta(\omega-\omega_{\sigma,\mathbf{k}}), \qquad (37)$$

where the variables $f_{\rm B}(\omega)$ and $f_{\rm F}(\omega)$ are the Bose distribution function and the Fermi one. The energy dispersion relation reads $\omega_{\mathbf{k}}\equiv Dk^2+B$ and $\omega_{\sigma,\mathbf{k}}\equiv Fk^2-(JS+B/2)\sigma-\mu$, where $D\equiv 1/(2m)$, $F\equiv 1/(2m_{\rm el})$, $\sigma=+1,-1(=\uparrow,\downarrow)$, and μ denotes the chemical potential; $\mu(T)=\epsilon_{\rm F}-(\pi k_{\rm B}T)^2/(12\epsilon_{\rm F})+\mathcal{O}(T^4)$. The variable $\epsilon_{\rm F}$ represents the Fermi energy.

Consequently, eq.(75) can be rewritten as

$$\begin{aligned}\mathcal{T}_{\rm s}^z &= 4\pi J^2a_0^3S\int\frac{d\mathbf{k}_1}{(2\pi)^3}\int\frac{d\mathbf{k}_2}{(2\pi)^3}\int d\omega_1\int d\omega_2\\ &\times \delta(\omega_1-\omega_{\mathbf{k}_1})\delta(\omega_2-\omega_{\uparrow,\mathbf{k}_2})\delta(\omega_1+\omega_2-\omega_{\downarrow,\mathbf{k}_1+\mathbf{k}_2})\\ &\times \Big\{[1+f_{\rm B}(\omega_1)]f_{\rm F}(\omega_1+\omega_2)[1-f_{\rm F}(\omega_2)]-f_{\rm B}(\omega_1)f_{\rm F}(\omega_2)[1-f_{\rm F}(\omega_1+\omega_1)]\Big\} \qquad (38)\\ &= 4\pi J^2a_0^3S\int\frac{d\mathbf{k}_1}{(2\pi)^3}\int\frac{d\mathbf{k}_2}{(2\pi)^3}\delta(\omega_{\mathbf{k}_1}+\omega_{\uparrow,\mathbf{k}_2}-\omega_{\downarrow,\mathbf{k}_1+\mathbf{k}_2})\\ &\times \Big\{f_{\rm F}(\omega_{\mathbf{k}_1}+\omega_{\uparrow,\mathbf{k}_2})[1-f_{\rm F}(\omega_{\uparrow,\mathbf{k}_2})]+f_{\rm B}(\omega_{\mathbf{k}_1})f_{\rm F}(\omega_{\mathbf{k}_1}+\omega_{\uparrow,\mathbf{k}_2})-f_{\rm B}(\omega_{\mathbf{k}_1})f_{\rm F}(\omega_{\uparrow,\mathbf{k}_2})\Big\} \qquad (39)\end{aligned}$$

Spin-flip condition

The delta function in eq. (77) represents the condition for spin-flip between conduction electrons and magnons. The modes (i.e. $\mathbf{k}_1$ and $\mathbf{k}_2$) which do not satisfy this condition cannot contribute to thermal spin pumping.

The delta function reads

$$\delta(\omega_{\mathbf{k}_1} + \omega_{\uparrow,\mathbf{k}_2} - \omega_{\downarrow,\mathbf{k}_1+\mathbf{k}_2}) = \delta\Big((D-F)k_1^2 - 2F\mathbf{k}_1\cdot\mathbf{k}_2 - 2JS\Big) \tag{40}$$

$$= \frac{1}{2Fk_1k_2}\delta\Big(\cos\theta - \frac{(D-F)k_1^2 - 2JS}{2Fk_1k_2}\Big), \tag{41}$$

where $\cos\theta \equiv \mathbf{k}_1\cdot\mathbf{k}_2/(k_1k_2)$. Eq. (41) holds true on the condition; $k_1 \neq 0, k_2 \neq 0$, and $F \neq 0$. This condition can be justified because the zero-mode for conduction electrons ($k_2 = 0$) originally cannot contribute to spin pumping which is the low energy dynamics; in order to excite the zero-mode so as to become relevant to spin pumping, it costs vast energy which amounts to the Fermi energy. Such a (relatively high energy) dynamics is out of the system we focus on, $\mathcal{H}$. In addition, when the zero mode for magnons ($k_1 = 0$) is substituted into eq. (40), it gives zero because of the finite effective magnetic fields $JS(\neq 0)$. Thus the zero-mode of magnons also originally cannot contribute to spin pumping and are eliminated. Then we are allowed to calculate eq. (77) on the condition; $k_1 \neq 0$ and $k_2 \neq 0$.

Consequently by using eq. (41), the TSTT (eq. (77)) can be rewritten as

$$\frac{4\pi^3 DF^2}{a_0^3 S\epsilon_{\rm F}^4}\mathcal{T}_{\rm s}^z = \int_{\searrow 0}^{\infty} d\bar{k}_1 \int_{\searrow 0}^{\infty} d\bar{k}_2 \bar{\mathcal{T}}_{\rm s}^z(\bar{k}_1,\bar{k}_2) \tag{42}$$

$$\equiv \int_{\searrow 0}^{\infty} d\bar{k}_1 \bar{\mathcal{T}}_{\rm s}^z(\bar{k}_1), \tag{43}$$

$$\equiv \bar{\mathcal{T}}_{\rm s}^z, \tag{44}$$

where

$$\begin{aligned}\bar{\mathcal{T}}_{\rm s}^z(\bar{k}_1,\bar{k}_2) &\equiv \bar{J}^2\int_{-1}^{1} d\zeta\ \delta\Big(\zeta - \frac{(1-\frac{F}{D})\bar{k}_1^2 - 2\bar{J}S}{2\sqrt{\frac{F}{D}}\bar{k}_1\bar{k}_2}\Big)\cdot\bar{k}_1\bar{k}_2 \\ &\times \Bigg\{ -\frac{1}{{\rm e}^{(\bar{k}_1^2+\bar{B})/\bar{T}_{\rm m}}-1}\cdot\frac{1}{{\rm e}^{(\bar{k}_2^2-\bar{J}S-\bar{B}/2-1+\pi^2\bar{T}_{\rm s}^2/12)/\bar{T}_{\rm s}}+1} \\ &+ \Big[1 - \frac{1}{{\rm e}^{(\bar{k}_2^2-\bar{J}S-\bar{B}/2-1+\pi^2\bar{T}_{\rm s}^2/12)/\bar{T}_{\rm s}}+1} + \frac{1}{{\rm e}^{(\bar{k}_1^2+\bar{B})/\bar{T}_{\rm m}}-1}\Big] \\ &\times \frac{1}{{\rm e}^{(\bar{k}_1^2+\bar{k}_2^2-\bar{J}S+\bar{B}/2-1+\pi^2\bar{T}_{\rm s}^2/12)/\bar{T}_{\rm s}}+1}\Bigg\}. \end{aligned} \tag{45}$$

We here have defined a variable, $\zeta \equiv \cos\theta$, and have introduced dimensionless variables; $\bar{k}_1 \equiv \sqrt{D/\epsilon_{\rm F}}k_1, \bar{k}_2 \equiv \sqrt{F/\epsilon_{\rm F}}k_2, \bar{B} \equiv B/\epsilon_{\rm F},\ \bar{J} \equiv J/\epsilon_{\rm F}, \bar{T}_{\rm m(s)} \equiv T_{\rm m(s)}/T_{\rm F} \equiv k_{\rm B}T_{\rm m(s)}/\epsilon_{\rm F}$, where $k_{\rm B}$ denotes the Boltzmann constant. The variable $T_{\rm m(s)}$ is the effective local temperature of magnons (conduction electrons)

[7, 12, 13], and

$$\bar{\mathcal{T}}_{\rm s}^{z}(\bar{k}_1, \bar{k}_2) \tag{46}$$

represents the dimensionless TSTT in the wavenumber space for magnons and conduction electrons;

$$\bar{\mathcal{T}}_{\rm s}^{z}(\bar{k}_1) \tag{47}$$

denotes the dimensionless TSTT in the wavenumber space for magnons, $\bar{k}_1$, after integrating over the wavenumber space for conduction electrons, $\bar{k}_2$. Both quantities, $\bar{\mathcal{T}}_{\rm s}^{z}(\bar{k}_1, \bar{k}_2)$ and $\bar{\mathcal{T}}_{\rm s}^{z}(\bar{k}_1)$, describe the exchange interaction (J) and the temperature ($T_{\rm m(s)}$) dependence of the TSTT.

We set each parameter, as a typical case, as follows [13, 20, 30]; $\epsilon_{\rm F} = 5.6$ eV, $B/\epsilon_{\rm F} = 0$, $F = 4$ eV Å^2, $D = 0.3$ eV Å^2, $S = 1/2$. Here it should be noted that we do not apply magnetic fields along the quantization axis;

$$B = 0. \tag{48}$$

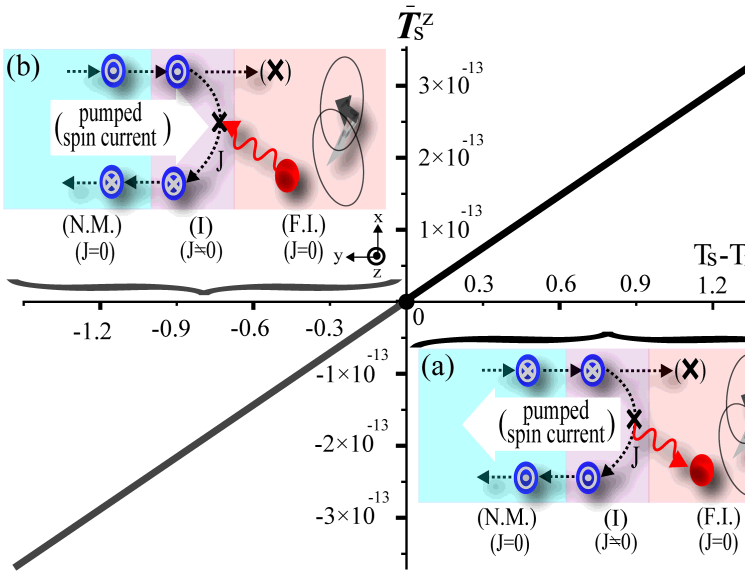

Figure 2: The temperature difference dependence of the dimensionless TSTT, $\bar{\mathcal{T}}_{\rm s}^{z}$, and the corresponding schematic pictures. Each parameter reads $\bar{J} = 0.002$ and $T_{\rm s} = 300$ K. When the effective temperature of magnons is lower than that of conduction electrons, localized spins at the interface lose spin angular momentum by emitting magnons and conduction electrons flip from down to up by absorbing the momentum (a), and vice versa (b).

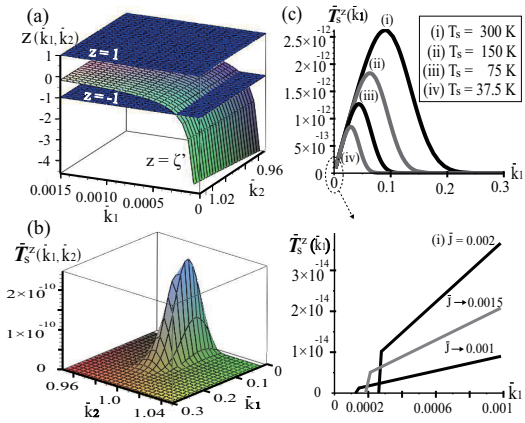

Figure 3: (a) The spin-flip condition via magnons; $z(\bar{k}_1, \bar{k}_2) = \zeta' \equiv [(1 - F/D)\bar{k}_1^2 - 2\bar{J}S](2\sqrt{F/D}\bar{k}_1\bar{k}_2)^{-1}$, where $\bar{J} = 0.002$. Magnons at (near) the zero-mode cannot contribute to thermal spin pumping because they do not satisfy the spin-flip condition, eq. (40). (b) The TSTT in the wavenumber space for conduction electrons and magnons, $\bar{\mathcal{T}}_{\rm s}^z(\bar{k}_1, \bar{k}_2)$. Each parameter reads $\bar{J} = 0.002$, $T_{\rm s} = 300$ K, and $T_{\rm s} - T_{\rm m} = 1.2$ K. A sharp peak exists on the Fermi wavenumber. (c) The TSTT in the wavenumber space for magnons, $\bar{\mathcal{T}}_{\rm s}^z(\bar{k}_1)$; the condition is the same with (b). The higher the effective magnon temperature becomes, the longer wavenumber of magnons becomes relevant to thermal spin pumping.

4 Thermal spin pumping effect

Fig. 2 shows that under the thermal equilibrium condition where temperature difference does not exist between ferromagnet and non-magnetic metal, spin currents cannot be pumped because of the balance between thermal fluctuations in ferromagnet and those in non-magnetic metal [7, 12, 13]. In addition, it can be concluded that the pumped spin current is proportional to the temperature difference between the magnon and conduction electron temperatures (i.e. $T_{\rm s} - T_{\rm m}$); when the effective temperature of magnons is lower than that of conduction electrons (see Fig. 2 (a)), localized spins at the interface lose spin angular momentum by emitting magnons and conduction electrons flip from down to up by absorbing all the emitted momentum [8], and vice versa (see Fig. 2 (b)). This result exhibits the good agreement with the work by Xiao et al. [13]; they have reached this result by combining the spin pumping theory proposed by Tserkovnyak et al. [31] with the Landau-Lifshitz-Gilbert equation.

Figs. 3 (a) and (c) show that magnons at (near) the zero-mode cannot contribute to thermal spin pumping because they do not satisfy the spin-flip condition between conduction electrons and magnons, due to the finite effective magnetic field JS. (see eqs. (40), (113), and Fig. 3 (a)).

The distinction from the work by Xiao et al. and Adachi et al.

Let us mention that we have set $B = 0$. That is, a spin current can be generated via the thermal spin pumping effect without any applied magnetic fields. This point cannot be obtained by Xiao et al. [13]. The pumped spin current is proportional to the temperature difference between the magnon and conduction electron temperatures; inhomogeneous thermal fluctuations induce a net spin current [7]. This is the main difference from the quantum spin pumping effect [4].

Last we should discuss the distinction from the important work by Adachi et al. [12], with emphasizing that they have already studied thermal spin pumping via magnons before our study. They have pointed out that the approach by using the stochastic Landau-Lifshitz-Gilbert equation coupled with the Bloch equation is equivalent to the one by the Schwinger-Keldysh formalism (i.e. linear-response theory) in the classical regime where quantum fluctuations are negligible. This fact has already been confirmed also by the numerical calculation [32]. Though they have studied the thermal spin pumping effect mediated by magnons via the Schwinger-Keldysh formalism by the same procedure with our work, unfortunately we have doubts the validity of their calculation; with reflecting the statistical properties, the Keldysh Green's function [26] for fermions ($\mathcal{G}^{\mathrm{K}}$) should be[25]

$$\mathcal{G}^{\mathrm{K}}_{\mathbf{k},\omega}(\equiv \mathcal{G}^{<}_{\mathbf{k},\omega} + \mathcal{G}^{>}_{\mathbf{k},\omega}) = 2i\ \mathrm{Im}\mathcal{G}^{\mathrm{r}}_{\mathbf{k},\omega}\tanh(\beta\omega/2), \tag{49}$$

not $2i\ \mathrm{Im}\mathcal{G}^{\mathrm{r}}_{\mathbf{k},\omega}\coth(\beta\omega/2)$ [12]. The variable $\mathcal{G}^{\mathrm{r}}_{\mathbf{k},\omega}$ denotes the fermionic retarded Green's function and β is defined as $\beta \equiv 1/(k_{\mathrm{B}}T)$. That is, the fermionic Keldysh Green's function is different from the bosonic one. In addition, we would like to mention that though they have taken a classical approximation, we have discussed the thermal spin pumping effect in the semiclassical regime. Moreover, we stress that thermal spin pumping does not cost any applied magnetic field, magnetic fields along the quantization axis nor transverse magnetic fields, because magnons at the zero-mode are eliminated because of the spin-flip condition, eq. (40).

5 Summary and discussion

We have qualitatively studied thermal spin pumping mediated by magnons in the semiclassical regime. Pumped spin currents are proportional to the temperature difference between conduction electrons and magnons. That is, inhomogeneous thermal fluctuations induce a net spin current; when the effective temperature of magnons is lower than that of conduction electrons, localized spins lose spin angular momentum by emitting magnons and conduction electrons flip from down to up by absorbing the momentum, and vice versa. Thermal spin pumping has the advantage that it does not cost any kinds of applied magnetic fields because magnons at the zero mode are eliminated due to the spin-flip con-

dition. This fact will be useful for potential applications to green information and communication technologies; spin currents can avoid Joule heating.

Though the behavior of the thermal spin pumping effect mediated by magnons can be qualitatively captured by calculating the TSTT, we recognize that the theoretical estimation for the width of the interface, so called proximity effects, is essential for the quantitative understanding. In addition, we are also interested in the contribution of phonons and that of magnons under a spatially nonuniform magnetization to spin pumping.

6 Acknowledgements

We would like to thank K. Totsuka for stimulating the study and useful comments. We are also grateful to T. Takahashi and Y. Korai for fruitful discussion. We are supported by the Grant-in-Aid for the Global COE Program "The Next Generation of Physics, Spun from Universality and Emergence" from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan.

Supplement is available at this URL;
http://dl.dropbox.com/u/5407955/SupplementTSP.pdf

A Review of Our Formalism for Thermal Spin Pumping[33]

Here, let us briefly arrange and summarize our formalism[33] based on the spin continuity equation for conduction electrons.

• Spin density for conduction electrons

Spin variables of conduction electrons, s^j, are represented by creation/annihilation operators satisfying the (fermionic) anticommutation relation; $\{c_\eta(\mathbf{x},t), c^\dagger_\zeta(\mathbf{x}',t)\} = \delta_{\eta,\zeta}\delta(\mathbf{x}-\mathbf{x}')$.

$$s^j = \sum_{\eta,\zeta=\uparrow,\downarrow} [c^\dagger_\eta (\sigma^j)_{\eta\zeta} c_\zeta]/2, \tag{50}$$

$$\equiv (c^\dagger \sigma^j c)/2. \tag{51}$$

Then, the spin density for conduction electrons is defined as

$$\rho^z_{\rm s} \equiv c^\dagger \sigma^z c/2. \tag{52}$$

•Hamiltonian

The total Hamiltonian $\mathcal{H}$ of the system (i.e. interface; see Fig. 4) reads

$$\mathcal{H} = \mathcal{H}_{\rm mag} + \mathcal{H}'_{\rm ex} + \mathcal{H}_{\rm el}, \text{ where} \tag{53}$$

$$\mathcal{H}_{\rm el} = \int_{\mathbf{x}\in(\text{interface})} d\mathbf{x}\; c^\dagger(\mathbf{x},t)\Big[-\frac{\nabla^2}{2m_{\rm el}} - (JS+\frac{B}{2})\sigma^z\Big]c(\mathbf{x},t), \tag{54}$$

$$\mathcal{H}'_{\rm ex} = -Ja_0^3\sqrt{\frac{\tilde{S}}{2}}\int_{\mathbf{x}\in(\text{interface})} d\mathbf{x}[a^\dagger(\mathbf{x},t)c^\dagger(\mathbf{x},t)\sigma^+c(\mathbf{x},t) + a(\mathbf{x},t)c^\dagger(\mathbf{x},t)\sigma^-c(\mathbf{x},t)], \tag{55}$$

$$\mathcal{H}_{\rm mag} = \int_{\mathbf{x}\in(\text{interface})} d\mathbf{x}\; a^\dagger(\mathbf{x},t)\Big(-\frac{\nabla^2}{2m}+B\Big)a(\mathbf{x},t). \tag{56}$$

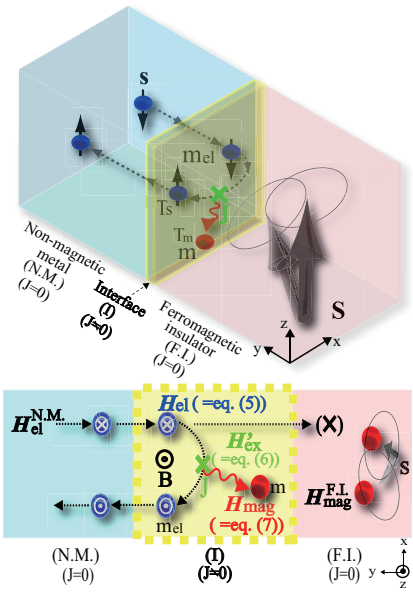

Figure 4: The interface is defined as an effective area where the Fermi gas (conduction electrons) and the Bose gas (magnons) coexist to interact; $J \neq 0$. In addition, conduction electrons cannot enter the ferromagnet, which is an insulator. We have focused on the dynamics at the interface (i.e. the yellow quadrilateral), $\mathcal{H} = \mathcal{H}_{\rm mag} + \mathcal{H}'_{\rm ex} + \mathcal{H}_{\rm el}$ (see eqs. (53)-(56)), where spin angular momentum is exchanged between conduction electrons and the ferromagnet. [The lower illustration] the Hamiltonian of the (free-) conduction electrons in the non-magnetic metal, $\mathcal{H}_{\rm el}^{\rm N.M.}$, and that of (free-) magnons in the ferromagnetic insulator, $\mathcal{H}_{\rm mag}^{\rm F.I.}$, are given as follows; $\mathcal{H}_{\rm el}^{\rm N.M.} = \int_{\mathbf{x}'\in(\rm N.M.)} d\mathbf{x}'\; c^\dagger(\mathbf{x}')(-\nabla^2/2m_{\rm el})c(\mathbf{x}')$, $\mathcal{H}_{\rm mag}^{\rm F.I.} = \int_{\mathbf{x}''\in(\rm F.I.)} d\mathbf{x}''\; a^\dagger(\mathbf{x}'')(-\nabla^2/2m)a(\mathbf{x}'')$.

•Heisenberg equation of motion

The time-development of the spin density for conduction electrons, $\dot{\rho}_{\rm s}^z$, can be determined explicitly and uniquely via the Heisenberg equation of motion;

$$\begin{aligned}
\dot{\rho}_{\rm s}^z &= [\rho_{\rm s}^z, \mathcal{H}]/i && (57)\\
&= [\rho_{\rm s}^z, \mathcal{H}_{\rm el} + \mathcal{H}'_{\rm ex} + \mathcal{H}_{\rm mag}]/i && (58)\\
&= [\rho_{\rm s}^z, \mathcal{H}_{\rm el} + \mathcal{H}'_{\rm ex}]/i && (59)\\
&= [\rho_{\rm s}^z, \mathcal{H}_{\rm el}]/i + [\rho_{\rm s}^z, \mathcal{H}'_{\rm ex}]/i. && (60)
\end{aligned}$$

Therefore, eq. (60) can be rewritten as

$$\begin{aligned}
\dot{\rho}_{\rm s}^z &= [\rho_{\rm s}^z, \mathcal{H}_{\rm el}]/i + [\rho_{\rm s}^z, \mathcal{H}'_{\rm ex}]/i, && (61)\\
&= -\nabla\cdot\mathbf{j}_{\rm s}^z + \mathcal{T}_{\rm s}^z, \text{ where} && (62)\\
-\nabla\cdot\mathbf{j}_{\rm s}^z &:= [\rho_{\rm s}^z, \mathcal{H}_{\rm el}]/i, && (63)\\
\mathcal{T}_{\rm s}^z &:= [\rho_{\rm s}^z, \mathcal{H}'_{\rm ex}]/i. && (64)
\end{aligned}$$

Then, the spin current density[28, 34] and the spin transfer torque can be determined explicitly and uniquely, via eqs. (63) and (64), as[2]

$$\begin{aligned}
j_{{\rm s}_\mu}^z &= -\frac{i}{4m_{\rm el}} c^\dagger \overleftrightarrow{\nabla}_\mu \sigma^z c := -\frac{i}{4m_{\rm el}} c^\dagger (\overrightarrow{\nabla}_\mu - \overleftarrow{\nabla}_\mu)\sigma^z c && (65)\\
\mathcal{T}_{\rm s}^z &= iJa_0^3\sqrt{\frac{\tilde{S}}{2}}[a^\dagger(\mathbf{x},t)c^\dagger(\mathbf{x},t)\sigma^+ c(\mathbf{x},t) - a(\mathbf{x},t)c^\dagger(\mathbf{x},t)\sigma^- c(\mathbf{x},t)]. && (66)
\end{aligned}$$

Finally, the spin continuity equation for conduction electrons, eq. (62), reads

$$\dot{\rho}_{\rm s}^z + \nabla\cdot\mathbf{j}_{\rm s}^z = \mathcal{T}_{\rm s}^z, \qquad (67)$$

where the spin current density and the spin transfer torque are represented by eqs. (65) and (66).

A.1 Local spin current term $j_{{\rm s}_\mu}^z$

Here let us stress that the (local) spin current density $j_{{\rm s}_\mu}^z$, in fact, arises from $\mathcal{H}_{\rm el}$; see eq. (63). In addition, the information about the energy dispersion relation of conduction electrons has been included into $\mathcal{H}_{\rm el}$;

$$\begin{aligned}
\mathcal{H}_{\rm el} &= \int_{\mathbf{x}\in(\text{interface})} d\mathbf{x}\; c^\dagger(\mathbf{x})\Big[-\frac{\nabla^2}{2m_{\rm el}} - (JS + \frac{B}{2})\sigma\Big]c(\mathbf{x}), && (68)\\
&= \sum_{\mathbf{k}}\Big[\frac{\mathbf{k}^2}{2m_{\rm el}} - (JS + \frac{B}{2})\sigma\Big]c_{\mathbf{k}}^\dagger c_{\mathbf{k}}, && (69)\\
&= \sum_{\mathbf{k}} \tilde{\omega}_{\sigma,\mathbf{k}} c_{\mathbf{k}}^\dagger c_{\mathbf{k}}, \text{ where} && (70)\\
\tilde{\omega}_{\sigma,\mathbf{k}} &:= \frac{\mathbf{k}^2}{2m_{\rm el}} - (JS + \frac{B}{2})\sigma. && (71)
\end{aligned}$$

[2] $\mu = x, y, z$.

Then, the energy dispersion relation reads $\omega_{\sigma,\mathbf{k}} \equiv Fk^2 - (JS + B/2)\sigma - \mu$, where $F \equiv 1/(2m_{\rm el})$, $\sigma = +1, -1 (=\uparrow, \downarrow)$, and μ denotes the chemical potential.

Moreover, it should be noted that the Hamiltonian $\mathcal{H}_{\rm el}$ also acts as the non-perturbative term on the perturbative calculation. Therefore if one neglects this Hamiltonian $\mathcal{H}_{\rm el}$, one cannot execute the calculation (see also Appendix D.1 and D.1.1); though the spin-flip mediated by magnons is generated by the Hamiltonian $\mathcal{H}'_{\rm ex}$, one needs the information about the dispersion of spin-flipped conduction electrons and that of magnons generating the spin-flip, which have been included into $\mathcal{H}_{\rm el}$ and $\mathcal{H}_{\rm mag}$, to execute the calculation. That is, the dynamics,

$$\mathcal{H}_{\rm el}, \tag{72}$$

and the accompanying local spin current density,

$$j^z_{{\rm s}_\mu}, \tag{73}$$

are essential to the theoretical (rigorous) description of the interface, where thermal spin pumping occurs.

In conclusion, the spin continuity equation for conduction electrons is not (see also Appendix D.1 and D.1.1)

$$\dot{\rho}^z_{\rm s} = [\rho^z_{\rm s}, \mathcal{H}'_{\rm ex}]/i \tag{74}$$

$$\Leftrightarrow \dot{\rho}^z_{\rm s} = \mathcal{T}^z_{\rm s}, \tag{75}$$

but

$$\dot{\rho}^z_{\rm s} = [\rho^z_{\rm s}, \mathcal{H}_{\rm el} + \mathcal{H}'_{\rm ex}]/i \tag{76}$$

$$\Leftrightarrow \dot{\rho}^z_{\rm s} + \nabla \cdot \mathbf{j}^z_{\rm s} = \mathcal{T}^z_{\rm s}. \tag{77}$$

One should discuss on the basis of eq. (77). If one neglects $\mathcal{H}_{\rm el}$ and adopts $\dot{\rho}^z_{\rm s} = \mathcal{T}^z_{\rm s}$ (i.e. eq. (75)) as spin continuity equation for conduction electrons, it corresponds to the condition;

$$\tilde{\omega}_{\sigma,\mathbf{k}} = 0. \tag{78}$$

Roughly speaking, it corresponds to the condition;

$$1/m_{\rm el} = 0 \Leftrightarrow m_{\rm el} \to \infty, \text{ and } \sigma = 0. \tag{79}$$

That is, it describes the extremely heavy spinless electrons, which is out of our purpose.

A.2 Conduction electron spin density $\dot{\rho}^z_{\rm s}$ and magnon density $\dot{\rho}^z_{\rm m}$

From the viewpoint of the Schwinger-Keldysh formalism, we could not believe the relation; $\langle \dot{\rho}^z_{\rm s} \rangle = -\langle \dot{\rho}^z_{\rm m} \rangle$, or $\mid \langle \dot{\rho}^z_{\rm s} \rangle \mid = \mid \langle \dot{\rho}^z_{\rm m} \rangle \mid$.

The direct calculation based on our formalism gives[3]

$$\begin{aligned}
\langle\dot{\rho}_{\rm s}^{z}\rangle &= \partial_t\Big\langle {\rm T_c} - \frac{\left(Ja_0^3\sqrt{\frac{\bar{S}}{2}}\right)^2}{2}\int d^3x'\int d^3x''\int d\tau'\int d\tau'' \\
&\cdot\ [a(x',\tau')c_{\downarrow}^{\dagger}(x',\tau')\sigma^{-}c_{\uparrow}(x',\tau')a^{\dagger}(x'',\tau'')c_{\uparrow}^{\dagger}(x'',\tau'')\sigma^{+}c_{\downarrow}(x'',\tau'') \\
&+\ a^{\dagger}(x',\tau')c_{\uparrow}^{\dagger}(x',\tau')\sigma^{+}c_{\downarrow}(x',\tau')a(x'',\tau'')c_{\downarrow}^{\dagger}(x'',\tau'')\sigma^{-}c_{\uparrow}(x'',\tau'')] \\
&\cdot\ c_{\sigma}^{\dagger}(x,\tau)\sigma c_{\sigma}(x,\tau)\Big\rangle + \mathcal{O}(J^3), \qquad (80)
\end{aligned}$$

and

$$\begin{aligned}
\langle\dot{\rho}_{\rm m}^{z}\rangle &= \partial_t\Big\langle {\rm T_c} - \frac{\left(Ja_0^3\sqrt{\frac{\bar{S}}{2}}\right)^2}{2}\int d^3x'\int d^3x''\int d\tau'\int d\tau'' \\
&\cdot\ [a(x',\tau')c_{\downarrow}^{\dagger}(x',\tau')\sigma^{-}c_{\uparrow}(x',\tau')a^{\dagger}(x'',\tau'')c_{\uparrow}^{\dagger}(x'',\tau'')\sigma^{+}c_{\downarrow}(x'',\tau'') \\
&+\ a^{\dagger}(x',\tau')c_{\uparrow}^{\dagger}(x',\tau')\sigma^{+}c_{\downarrow}(x',\tau')a(x'',\tau'')c_{\downarrow}^{\dagger}(x'',\tau'')\sigma^{-}c_{\uparrow}(x'',\tau'')] \\
&\cdot\ a^{\dagger}(x,\tau)a(x,\tau)\Big\rangle + \mathcal{O}(J^3). \qquad (81)
\end{aligned}$$

Though we have to complete the calculation (i.e. eqs. (80) and (81)) on the basis of the standard procedure of the Schwinger-Keldysh formalism, Wick's theorem and Langreth method[28, 4],[4] roughly speaking, they (i.e. eqs. (80) and (81)) represent[5]

$$\langle\dot{\rho}_{\rm s}^{z}\rangle \propto {\rm G}\mathcal{G}^3. \qquad (82)$$

$$\langle\dot{\rho}_{\rm m}^{z}\rangle \propto {\rm G}^2\mathcal{G}^2. \qquad (83)$$

Then, we consider that[6]

$$\langle\dot{\rho}_{\rm s}^{z}\rangle \neq -\langle\dot{\rho}_{\rm m}^{z}\rangle, \quad \text{or} \quad |\ \langle\dot{\rho}_{\rm s}^{z}\rangle\ | \neq |\ \langle\dot{\rho}_{\rm m}^{z}\rangle\ |\ . \qquad (84)$$

B Pumped Net Spin Current[33]

Let us arrange and summarize the result of our calculation for the spin/magnon density and the torque/source[7] term on the basis of the Schwinger-Keldysh formalism.[33, 4, 35]

[3]Note that $\langle\dot{\rho}_{\rm s}^{z}\rangle = \partial_t\langle\rho_{\rm s}^{z}\rangle$; $\because\ \rho_{\rm s}^{z}(t+\Delta t) = \rho_{\rm s}^{z}(t)+[\Delta\rho_{\rm s}^{z}/(\Delta t)]\Delta t+\mathcal{O}((\Delta t)^2) \Rightarrow \langle\rho_{\rm s}^{z}(t+\Delta t)\rangle = \langle\rho_{\rm s}^{z}(t)\rangle+\langle[\Delta\rho_{\rm s}^{z}/(\Delta t)]\rangle\Delta t+\mathcal{O}((\Delta t)^2) \Leftrightarrow \langle\rho_{\rm s}^{z}(t+\Delta t)\rangle-\langle\rho_{\rm s}^{z}(t)\rangle = \langle[\Delta\rho_{\rm s}^{z}/(\Delta t)]\rangle\Delta t+\mathcal{O}((\Delta t)^2) \Leftrightarrow [\langle\rho_{\rm s}^{z}(t+\Delta t)\rangle - \langle\rho_{\rm s}^{z}(t)\rangle]/(\Delta t) = \langle[\Delta\rho_{\rm s}^{z}/(\Delta t)]\rangle \stackrel{\Delta t\to 0}{\longrightarrow} \partial_t\langle\rho_{\rm s}^{z}\rangle = \langle\dot{\rho}_{\rm s}^{z}\rangle_{\blacksquare}$.

[4]The lesser Green's function corresponds to the Fermi/Bose distribution function, which includes the information about the effective local temperature; $T_{\rm s(m)}(t)$.[10]

[5]The variable G represents the bosonic Keldysh Green's function, and $\mathcal{G}$ denotes the fermionic one.

[6]Note that $\langle\mathcal{T}_{\rm s}^{z}\rangle = \langle\mathcal{T}_{\rm m}^{z}\rangle \propto {\rm G}\mathcal{G}^2$.[33] Then, $\langle\mathcal{T}_{\rm s}^{z}\rangle \neq \langle\dot{\rho}_{\rm s}^{z}\rangle$.

[7]See Appendix D and D.1 in advance.

•Spin continuity equation for conduction electrons

$$\dot{\rho}_{\rm s}^z + \nabla \cdot \mathbf{j}_{\rm s}^z = \mathcal{T}_{\rm s}^z. \tag{85}$$

•Magnon continuity equation

$$\dot{\rho}_{\rm m}^z + \nabla \cdot \mathbf{j}_{\rm m}^z = \mathcal{T}_{\rm m}^z. \tag{86}$$

Roughly speaking,[8]

$$\langle \mathcal{T}_{\rm s}^z \rangle = \langle \mathcal{T}_{\rm m}^z \rangle \quad \propto \quad {\rm G}\mathcal{G}^2. \tag{87}$$
$$\langle \dot{\rho}_{\rm s}^z \rangle \quad \propto \quad {\rm G}\mathcal{G}^3. \tag{88}$$
$$\langle \dot{\rho}_{\rm m}^z \rangle \quad \propto \quad {\rm G}^2\mathcal{G}^2. \tag{89}$$

Then,[9]

$$\langle \dot{\rho}_{\rm s}^z \rangle \quad \neq \quad \langle \dot{\rho}_{\rm m}^z \rangle. \tag{90}$$
$$\langle \dot{\rho}_{\rm s}^z \rangle \quad \neq \quad \langle \mathcal{T}_{\rm s}^z \rangle. \tag{91}$$
$$\langle \dot{\rho}_{\rm m}^z \rangle \quad \neq \quad \langle \mathcal{T}_{\rm m}^z \rangle. \tag{92}$$

B.1 Edge current; Fig. 5

We[33] have regarded the exchange interaction between conduction electrons and magnons, J, as a constant parameter; $J \in$ (const.) $\Rightarrow \nabla {\rm J} = 0$. Then, the *direct* calculation of the spin current density, $\mathbf{j}_{\rm s}^z$, gives[10]

$$\langle \mathbf{j}_{\rm s}^z \rangle \quad = \quad \langle -\frac{i}{4m_{\rm el}} c^\dagger \overleftrightarrow{\nabla}_\mu \sigma^z c \rangle = 0. \tag{93}$$
$$\Rightarrow \langle \nabla \cdot \mathbf{j}_{\rm s}^z \rangle \quad = \quad \nabla \cdot \langle \mathbf{j}_{\rm s}^z \rangle = 0. \tag{94}$$

On the other hand, eqs. (85)-(92), which corresponds to the *indirect* calculation of the current density, gives

$$(\text{eqs.(85) and (91)}) \Rightarrow \langle \nabla \cdot \mathbf{j}_{\rm s}^z \rangle \quad \neq \quad 0. \tag{95}$$
$$\Rightarrow \mathbf{j}_{\rm s}^z \quad \neq \quad 0 \tag{96}$$

[8]The variable G represents the bosonic Keldysh Green's function, and $\mathcal{G}$ denotes the fermionic one.

[9]Our calculation on *quantum spin pumping*[4, 35] gives $\langle \dot{\rho}_{\rm s}^z \rangle \propto J^2\Gamma^2$, and $\langle \mathcal{T}_{\rm s}^z \rangle \propto J\Gamma^2$. Then, $\langle \dot{\rho}_{\rm s}^z \rangle \neq \langle \mathcal{T}_{\rm s}^z \rangle$.

[10]See eq. (65).

We understand these results as follows;[11]

$$\text{(eq.(93))} \quad \Rightarrow \quad \langle \mathbf{j}_{\mathrm{s}}^{z}(\mathbf{x}) \rangle \mid_{\mathbf{x} \in \{V_{\mathrm{interface}} \text{ except the edge}\}} = 0. \tag{97}$$

$$\text{(eq.(96))} \quad \Rightarrow \quad \langle \mathbf{j}_{\mathrm{s}}^{z}(\mathbf{x}) \rangle \mid_{\mathbf{x} \in (\mathrm{edge})} \neq 0. \tag{98}$$

Therefore[33] (see Fig. 5)

$$\begin{aligned}
\text{(pumped net spin current)} &= \int dV_{\mathrm{interface}} \, \nabla \cdot \langle \mathbf{j}_{\mathrm{s}}^{z}(\mathbf{x}) \rangle && (99)\\
&= \int \langle \mathbf{j}_{\mathrm{s}}^{z}(\mathbf{x}) \rangle \cdot d\mathbf{S}_{\mathrm{interface}} && (100)\\
&= \int \langle \mathbf{j}_{\mathrm{s}}^{z}(\mathbf{x}) \rangle \mid_{\mathbf{x} \in (\mathrm{edge})} \cdot d\mathbf{S}_{\mathrm{edge}} && (101)\\
&\neq 0. && (102)
\end{aligned}$$

This is the conclusion of our formalism.[33]

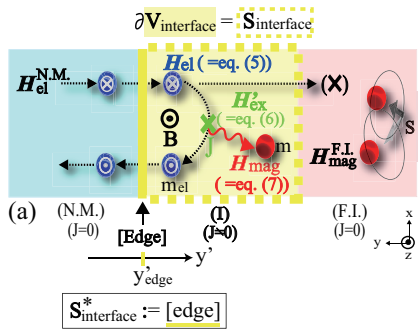

Figure 5: (a) A schematic picture of the edge.

C A Need for Further Progress

C.1 How to theoretically model the interface

C.1.1 Gradient of the exchange interaction; Fig. 6

If one supposes the magnitude of the exchange interaction between conduction electrons and magnons as not constant (see Fig. 6 (b-i)),[33] but

$$J(\mathbf{x}), \quad \text{i.e.} \tag{103}$$

$$\nabla J(\mathbf{x}) \neq 0, \tag{104}$$

[11] Note that the exchange interaction between conduction electrons and magnons can be roughly represented in the region, the interface and the non-magnetic metal, as $J\theta(y' - y'_{\mathrm{edge}})$; see Fig. 5. The y'-derivative becomes $J\delta(y' - y'_{\mathrm{edge}})(\neq 0$ on the edge).

one would obtain, by the *direct* calculation, a finite local spin current density, $\mathbf{j}_{\mathrm{s}}^{z}(\neq 0)$,[12] in the interface (i.e. the yellow quadrilateral, $V_{\mathrm{interface}}$) as well as the edge; see Fig. 6 (b-ii).

We recognize that to calculate the local spin current density, $\mathbf{j}_{\mathrm{s}}^{z}$, on the condition of eq. (104) (i.e. Fig. 6 (b-ii)) is a significant theoretical issue to be tackled in the near future.

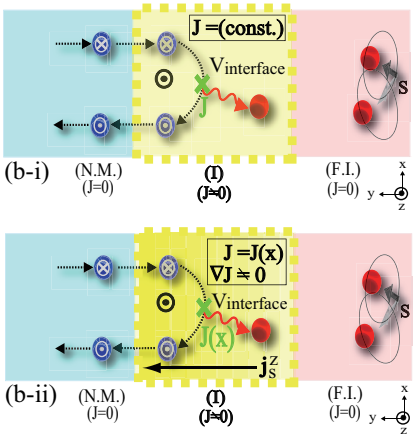

Figure 6: (b-i) Our case; $\nabla J = 0$.[33] (b-ii) $\nabla J(\mathbf{x}) \neq 0 \Rightarrow \mathbf{j}_{\mathrm{s}}^{z} \neq 0$ in the interface, $V_{\mathrm{interface}}$.

C.1.2 2-dim or 3-dim; Fig. 7

Last, let us mention the reason why we have theoretically modeled the interface, which is defined as an effective area where the Fermi gas (conduction electrons) and the Bose gas (magnons) coexist to interact; $J \neq 0$, as not 2-dim surface (see Fig. 7 b-(iii)), but 3-dim region (see Fig. 7 b-(i)).

We consider that any materials should be treated as not 2-dim objects, but 3-dim ones in principle.[13] If it is too hard to analysis, one should execute coarse graining appropriately[14] to lower the dimension.

In addition, we suspect that the magnon description for ferromagnetic localized spins might be prohibited by the Mermin-Wagner theorem if we treat the

[12] $\mathbf{j}_{\mathrm{s}}^{z} \propto \nabla J(\mathbf{x})$.

[13] Though the theoretical (detailed) study on the origin of the exchange interaction between conduction electrons and magnons at the interface is urgent, we consider it is generated by the quantum effect, i.e. the overlap/superposition of each wave function, which spreads in 3-dimension. Then, we have theoretically modeled the interface as the 3-dim region.

[14] In our case, though the width of the interface may be roughly supposed to be of the order of the lattice constant, we cannot know the accurate length at this stage; T. Oka in Tokyo University has advised us to take proximity effects into account. We consider that if the width of the interface is far shorter than the decay length of the spin current, the interface may be treated as 2-dim surface.

interface as a 2-dim surface;[15] the theorem prohibits the spontaneous (continuous) symmetry breaking at finite temperature in lower systems ($d \leq 2$).

Then, we have viewed the interface as 3-dim region; Fig. 7 b-(i).

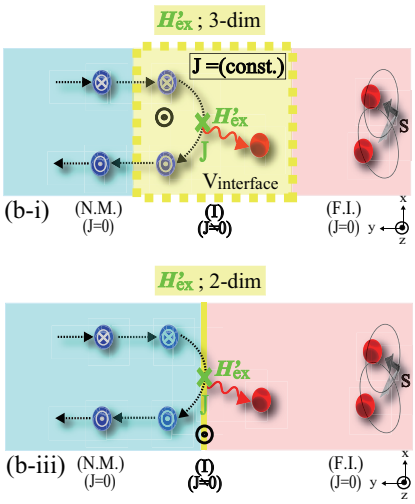

Figure 7: (b-iii) A schematic picture of the 2-dim interface.

D Magnon Continuity Equation

The magnon density of the system (interface) is defined as

$$\rho_{\rm m}^z \equiv a^\dagger a. \tag{105}$$

The time-development of the magnon density can be explicitly determined via Heisenberg equation of motion;

$$\begin{aligned}
\dot{\rho}_{\rm m}^z &= [\rho_{\rm m}^z, \mathcal{H}]/i && (106)\\
&= [\rho_{\rm m}^z, \mathcal{H}_{\rm el} + \mathcal{H}'_{\rm ex} + \mathcal{H}_{\rm mag}]/i && (107)\\
&= [\rho_{\rm m}^z, \mathcal{H}_{\rm mag} + \mathcal{H}'_{\rm ex}]/i && (108)\\
&= [\rho_{\rm m}^z, \mathcal{H}_{\rm mag}]/i + [\rho_{\rm s}^z, \mathcal{H}'_{\rm ex}]/i. && (109)
\end{aligned}$$

Therefore, eq. (109) can be rewritten as

$$\begin{aligned}
\dot{\rho}_{\rm m}^z &= [\rho_{\rm m}^z, \mathcal{H}_{\rm mag}]/i + [\rho_{\rm m}^z, \mathcal{H}'_{\rm ex}]/i, && (110)\\
&= -\nabla \cdot \mathbf{j}_{\rm m}^z + \mathcal{T}_{\rm m}^z, \text{ where} && (111)\\
-\nabla \cdot \mathbf{j}_{\rm m}^z &:= [\rho_{\rm m}^z, \mathcal{H}_{\rm mag}]/i, && (112)\\
\mathcal{T}_{\rm m}^z &:= [\rho_{\rm m}^z, \mathcal{H}'_{\rm ex}]/i. && (113)
\end{aligned}$$

[15] Adachi et al.[12] have adopted the magnon description in 2-dimension. On this point, we cannot conclude at this stage.

Then, the magnon current density[21] and the magnon source term can be determined explicitly and uniquely, via eqs. (112) and (113), as

$$j^z_{\mathrm{m}_\mu} = \frac{1}{m}\mathrm{Re}[i(\partial_\mu a^\dagger)a] \tag{114}$$

$$\mathcal{T}^z_{\mathrm{m}} = iJa_0^3\sqrt{\frac{\tilde{S}}{2}}[a^\dagger(\mathbf{x},t)c^\dagger(\mathbf{x},t)\sigma^+c(\mathbf{x},t) - a(\mathbf{x},t)c^\dagger(\mathbf{x},t)\sigma^-c(\mathbf{x},t)] \tag{115}$$

$$= \mathcal{T}^z_{\mathrm{s}}. \tag{116}$$

Finally, the magnon continuity equation, eq. (111), reads

$$\dot{\rho}^z_{\mathrm{m}} + \nabla\cdot\mathbf{j}^z_{\mathrm{m}} = \mathcal{T}^z_{\mathrm{m}}, \tag{117}$$

where the magnon current density and the magnon source term are represented by eqs. (114) and (115).

D.1 Dispersion relation of magnons

Here let us stress that the (local) magnon current density $j^z_{\mathrm{m}_\mu}$, in fact, arises from $\mathcal{H}_{\mathrm{mag}}$; see eq. (112). In addition, the information about the energy dispersion relation of magnons has been included into $\mathcal{H}_{\mathrm{mag}}$;

$$\mathcal{H}_{\mathrm{mag}} = \int_{\mathbf{x}\in(\mathrm{interface})} d\mathbf{x}\; a^\dagger(\mathbf{x})\Big(-\frac{\nabla^2}{2m}+B\Big)a(\mathbf{x}), \tag{118}$$

$$= \sum_{\mathbf{k}}\Big(\frac{\mathbf{k}^2}{2m}+B\Big)a^\dagger_{\mathbf{k}}a_{\mathbf{k}}, \tag{119}$$

$$= \sum_{\mathbf{k}}\omega_{\mathbf{k}}a^\dagger_{\mathbf{k}}a_{\mathbf{k}}, \text{ where} \tag{120}$$

$$\omega_{\mathbf{k}} := \frac{\mathbf{k}^2}{2m}+B. \tag{121}$$

The energy dispersion relation reads $\omega_{\mathbf{k}} \equiv Dk^2 + B$ where $D \equiv 1/(2m)$.

In conclusion, the magnon continuity equation is not

$$\dot{\rho}^z_{\mathrm{m}} = [\rho^z_{\mathrm{m}}, \mathcal{H}'_{\mathrm{ex}}]/i \tag{122}$$

$$\Leftrightarrow \dot{\rho}^z_{\mathrm{m}} = \mathcal{T}^z_{\mathrm{m}}, \tag{123}$$

but

$$\dot{\rho}^z_{\mathrm{m}} = [\rho^z_{\mathrm{m}}, \mathcal{H}_{\mathrm{mag}} + \mathcal{H}'_{\mathrm{ex}}]/i \tag{124}$$

$$\Leftrightarrow \dot{\rho}^z_{\mathrm{m}} + \nabla\cdot\mathbf{j}^z_{\mathrm{m}} = \mathcal{T}^z_{\mathrm{m}}. \tag{125}$$

One should discuss on the basis of eq. (125). If one neglects $\mathcal{H}_{\mathrm{mag}}$ and adopts $\dot{\rho}^z_{\mathrm{m}} = \mathcal{T}^z_{\mathrm{m}}$ (i.e. eq. (115)) as magnon continuity equation, it corresponds to the condition;

$$\omega_{\mathbf{k}} = 0. \tag{126}$$

Roughly speaking, it corresponds to the condition;

$$1/m = 0 \Leftrightarrow m \to \infty \text{ and } B = 0. \tag{127}$$

That is, it describes the extremely heavy magnons, which is out of our/your aim.

D.1.1 Ferromagnetic Heisenberg model and magnon picture

Note that the free-magnon picture, eq. (56), has been originally derived[16] from the ferromagnetic Heisenberg (F.H.) model via Holstein-Primakoff transformation (HP tr.) up to the lowest order (see eqs. (4)-(8) in our manuscript[33]);

$$\mathcal{H}_{\text{F.H.}} = -J_{\text{F.H.}} \sum_{\langle i,j\rangle \in (\text{interface})} \mathbf{S}_i \cdot \mathbf{S}_j, \quad \Big(J_{\text{F.H}} \in (\text{const.})\Big) \tag{128}$$

$$\overset{\text{HP tr.}}{\longrightarrow} \mathcal{H}_{\text{mag}} = \int_{\mathbf{x}\in(\text{interface})} d\mathbf{x}\; a^\dagger(\mathbf{x},t)\Big(-\frac{\nabla^2}{2m} + B\Big)a(\mathbf{x},t). \tag{129}$$

The exchange interaction, $J_{\text{F.H}}$, between localized spins and the effective mass of magnons, m, satisfy the relation;

$$2J_{\text{F.H}} S a_0^2 = \frac{1}{2m}, \tag{130}$$

where a_0 denotes the lattice constant of the (ferromagnetic) localized spins. Then[17]

$$m \to \infty \Leftrightarrow J_{\text{F.H}} = 0. \tag{131}$$

Therefore also from this viewpoint, one should take $\mathcal{H}_{\text{mag}}$ into account and adopt the magnon continuity equation described by eq. (125).

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