Population growth and north-south uneven development

By Hiroaki Sasaki

Graduate School of Economics, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan; e-mail: sasaki@econ.kyoto-u.ac.jp

Abstract

This paper develops a model of North-South trade and economic development. The model is consistent with two empirical facts: (1) the growth rate of income *per capita* differs across countries; and (2) the relationship between the growth of population and that of income *per capita* differs for developed and developing countries. We assume that the North and the South are characterized by increasing-returns-to-scale and decreasing-returns-to-scale technologies, respectively. Real national income grows at the same rate in both countries along the balanced growth path owing to a terms of trade effect. However, real income *per capita* grows at different rates because of population growth differentials. In developed countries, the correlation between population growth and income *per capita* growth can be positive or negative while in developing countries, the correlation is negative.

JEL classifications: F43, O11, O41.

1 Introduction

This paper investigates the differences in income *per capita* growth across countries and the relationship between population growth and the income *per capita* growth by combining a traditional North-South trade model with a modern endogenous growth model.[1]

Are differences in income across countries converging or diverging? Using Penn World Table 5 (PWT), Acemoglu and Ventura (2002) compare the world income distribution in 1960 and 1990, and observe that the world income distribution over the past 30 years has been relatively stable. Following their lead, Felbermayr (2007) also observes the transition

[1]Traditional North-South models here mean models that consider some asymmetries between developed and developing countries. See, for instance, Findlay (1980), Taylor (1981), Molana and Vines (1989), and Sarkar (2001). See also Chui *et al*. (2002) for a survey of new North-South models.

of the world income distribution from 1960 to 2000 by using PWT 6.0. He concludes that the period 1960–1980 shows convergence while the period 1980–2000 shows stability.

The relatively stable income distribution means that the growth rates of income *per capita* are about the same across countries. Based on this observation, the above works build growth models that explain these empirical facts. The model of Acemoglu and Ventura (2002) describes a world that consists of a continuum of countries with mass 1. Each country is endowed with a different, constant-returns-to-scale AK production function. In the integrated international equilibrium, countries with low productivity face a higher relative price for their export goods. This equalizes the returns to capital accumulation, and consequently, also growth rates. In other words, even if each country is endowed with a constant-returns-to-scale technology, international trade introduces de facto diminishing returns via the terms of trade effects, and consequently, all countries' growth rates are equalized. The model of Felbermayr (2007) describes a situation where the capital abundant North and the capital scarce South trade with each other. In his model, the trade pattern is endogenously determined and he analyses the situation where the North produces investment goods and the South produces consumption goods. The production technology of investment goods is AK and that of consumption goods is decreasing returns to scale. Along the balanced growth path (BGP), the Southern terms of trade are continuously improving such that even the decreasing-returns-to-scale South can grow at the same rate as the North.

To better understand the situation, we draw graphs similar to those in Acemoglu and Ventura (2002) and Felbermayr (2007) by using the data for 1980, 1990, and 2000 from Extended Penn World Tables 3.0 (EPWT).[2] In Figs 1 and 2, the horizontal axes measure the log of income *per capita* of each country relative to the world average in 1980 and the vertical axes measure the log of income *per capita* relative to the world average in 1990 and 2000.[3] The world average is a simple arithmetic mean. The 45° line is drawn in these figures. Countries on the 45° line are those that grew at the same rate as the world average over the periods 1980–1990 and 1980–2000. Countries above/below the 45° line are those that grew faster/slower than the world average. Comparing the findings for 1980 and those for 1990, we find that most countries are located near the 45° line. However, on comparing the observations for 1980 with those for 2000, we find that countries tend to deviate from the 45° line, which suggests that income distribution diverged with time.[4]

[2] EPWT 3.0 is a database which Adalmir Marquetti and Duncan Foley calculate from PWT 6.2. The period covered is 1963–2003. EPWT 3.0 is downloadable at http://homepage.newschool.edu/˜foleyd/epwt/. We choose 97 countries whose data for 1980, 1990, and 2000 are available. The list of 97 countries is presented in sections A-1 and A-2 of the Appendix, which is available online at the OUP website.

[3] We can obtain similar results even though we use income per employed worker, not income *per capita*.

[4] The standard deviation of log income *per capita* is 1.12 in 1980, 1.17 in 1990, and 1.23 in 2000. The

We calculate the growth rates of income *per capita* of countries in Figs 1 and 2. In Figs. 3 and 4, the horizontal axes measure the annual average growth rate of income *per capita* in each country over the period 1980–2000. Figure 3 shows the countries whose income *per capita* in 1980 is higher than the world average while Fig. 4 shows countries whose income *per capita* in 1980 is lower than the world average. It is immediately obvious that the growth rate of income *per capita* varies across countries (finding 1). The average in Fig. 3 is 1.72% and that in Fig. 4 is 1.05%. In addition, there are countries with negative growth rates in both Figs 3 and 4.

We present a non-scale growth model that can explain the differences in *per capita* income growth across countries. Ever since Jones (1995) challenged the scale effects of the endogenous growth model, non-scale growth models have gained attention in this field. In the scale-growth model, the growth rate of output *per capita* along the BGP depends positively on the size of the population, that is, the larger the size of the population, the faster the growth of the country. This, however, seems counterfactual. Jones (1995) attempts to remove the scale-effects and presents a non-scale growth model, in which the growth rate of output *per capita* depends positively on the rate of population growth, and not on the size of the population. That is, the higher the growth rate of population, the faster the growth of the country.[5]

The main reasons why we use the non-scale growth model are as follows: (1) we can obtain sustainable growth of income *per capita* even though population growth is strictly positive and (2) we do not need to impose knife-edge conditions on the parameters of the model. Acemoglu and Ventura (2002) and Felbermayr (2007) do not explicitly consider population growth in their models. We, on the other hand, explicitly consider positive population growth. In addition, their models belong to the AK class of models and as such, knife-edge conditions are imposed on the production functions.

However, non-scale growth models face the population puzzle problem. Referring to the empirical finding of Kuznets (1973), Goodfriend and McDermott (1995) point out that the correlation between population growth and *per capita* income growth is not as unambiguous as the non-scale growth model predicts. Goodfriend and McDermott (1995) state that the population-driven models of growth must confront the population puzzle.

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differences between our results and those of Felbermayr (2007) lie in the selection of countries. From PWT 6.0, Felbermayr (2007) chooses 28 countries which are classified as an open economy according to the Sachs-Warner index. In contrast, we simply choose 97 countries whose data are available. For evidence of divergence in income *per capita* across countries, see also Quah (1996), and Epstein *et al*. (2007).

[5]For a systematic exposition as to scale effects and non-scale growth, see Jones (1999). For more sophisticated non-scale growth models, see also Kortum (1997), Dinopoulos and Thompson (1998), Peretto (1998), Segerstrom (1998), Young (1998), and Howitt (1999).

Let us return to Figs 3 and 4. The vertical axes represent the annual average growth rates of population over the period 1980–2000 (the sample average of Fig. 3 is 0.99% and that of Fig. 4 is 2.37%). While Fig. 4 shows a negative correlation between population growth and *per capita* income growth, Fig. 3 does not show a clear correlation between the two growth rates.[6] In this way, the relationship between population growth and *per capita* income growth differs for above-average and below-average countries (finding 2). Previous non-scale growth models do not consider this issue.

Hence, this paper presents a model of non-scale growth that is consistent with the above-mentioned two empirical findings. Existing non-scale growth models are built under a closed economy setting. However, for most countries around the world, the assumption that the economy is open would seem more realistic and a better starting point for analysis. Therefore, we need an open economy model to apply the obtained implications in the real world.[7] Further, it would be inappropriate if we apply the same specifications to the developed and developing countries given that they have different structures. Hence, we develop a model of North-South trade and economic development that takes into account the various asymmetries between the developed North and developing South. Using it, we examine how the growth rate of the population in each country affects the growth rates of income *per capita* in both countries.

Our model is based on Conway and Darity (1991), who develop a model of North-South trade and economic development along the lines of Kaldor's idea. They model a situation where the North is characterized by increasing returns to scale while the South is characterized by decreasing returns to scale. Then, they analyse the consequences of the asymmetry (in terms of the returns to scale) on the rate of capital accumulation in each country, the terms of trade, and so forth. In this paper, we extend Conway and Darity's model in some respects. The two main differences are as follows. First, following Dutt (1996), we assume that the Northern imports from the South are used for both consumption and intermediate input while the Southern imports from the North are used for both consumption and investment. From this assumption, the North-South interdependence arises because the Southern good is used as an input in the production of the Northern good. Second, we employ a dynamic optimization technique to solve the model. In Conway and Darity's

[6]We estimate $g_{y_i} = \beta_0 + \beta_1 n_i + \varepsilon_i$, where g_{y_i} denotes the growth rate of income *per capita*; n_i, the growth rate of population; and ε_i, an error term. For the above-average countries, we obtain $\hat{\beta}_0 = 0.02$, $\hat{\beta}_1 = -0.46$, and $R^2 = 0.076$ with a t-value for β_1 being $t = -1.63$, which cannot reject the null hypothesis of $\beta_1 = 0$ at the 1% level. For the below-average countries, we obtain $\hat{\beta}_0 = 0.04$, $\hat{\beta}_1 = -1.38$, and $R^2 = 0.238$ with a t-value for β_1 being $t = -4.37$, which rejects the null hypothesis of $\beta_1 = 0$ at the 1% level.

[7]Christiaans (2003) develops a small open economy non-scale growth model that considers imported intermediate goods and exogenous export demand.

model, wage income is entirely consumed and profit income is entirely invested.

As stated above, in Acemoglu and Ventura (2002) and Felbermayr (2007), the terms of trade effects play a crucial role in equating the growth of income *per capita* across countries. The terms of trade in our model change continuously. In contrast to Felbermayr (2007), however, the Southern terms of trade continuously improve in some cases and deteriorate in other cases. Opinions vary on the long-term trend of the terms of trade.[8] Therefore, we are unable to reach a single conclusion.

The remainder of the paper is organized as follows. Section 2 presents the basic framework of our model. Section 3 obtains the growth rates of endogenous variables on the BGP. Section 4 derives the growth rate of income *per capita* in each country and investigates the relationship between population growth and income growth. Section 5 analyses the transitional dynamics of the model by using numerical simulations. Section 6 concludes the paper.

2 The model

Consider a world that consists of North, a developed country, and South, a developing country. Each country completely specializes in the production of a single good, that is, the production pattern is fixed. We assume that the two goods are imperfect substitutes. The Northern good is used for consumption and investment in both countries. The Southern good, in contrast, is used for consumption in both countries and for intermediate input in the North. That is, we assume that the North exports a final consumption-cum-investment good to the South while the South exports a final consumption-cum-intermediate good to the North. This production structure covers the main characteristics of traditional North-South models. Note that in the North, the value of total production differs from the total value added due to the existence of imported intermediate input.

In this paper, we focus on a competitive equilibrium path. As will be explained below, in the North, there exist externalities arising from capital accumulation. Therefore, a competitive equilibrium path diverges from an optimal path in which a social planner internalizes externalities. However, growth rates on the BGP are equal in both cases.

[8] It is often suggested that the terms of trade deteriorate for developing countries in the long run (the Prebisch-Singer hypothesis). Felbermayr (2007), in contrast, discusses that developing countries' terms of trade have been improving. According to Fig. II of Acemoglu and Ventura (2002, p. 676), changes in the terms of trade across countries between 1965 and 1985 can be positive or negative. For further evidence of the terms of trade, see also Grilli and Yang (1988), Zanias (2005), Cashin and Pattillo (2006), and literature cited therein.

2.1 Firms

The North produces good N according to the following Cobb-Douglas production function:

$$X_N = B_N K_N^{1-\mu-\beta} L_N^{\mu} M^{\beta}, \quad 0 < \mu < 1,\ 0 < \beta < 1,\ \mu + \beta < 1, \tag{1}$$

where X_N denotes total production; B_N, a shift term; K_N, capital stock; $L_N = e^{n_N t}$, employment with n_N being the growth rate of the Northern population; and M, imported intermediate good. If B_N is regarded as an exogenous variable, Northern production is constant returns to scale. Externalities due to capital accumulation are as follows:

$$B_N = A_N K_N^{\theta}, \quad 0 < \theta < 1, \tag{2}$$

where A_N denotes the level of total factor productivity; and θ, the extent of externality. This specification captures the learning-by-doing effects based on Arrow (1962), who considers that capital accumulation creates new knowledge as a by-product. In what follows, we assume that $\theta \leq \mu$. This assumption means that in the North, the extent of externality is smaller than the output elasticity of labor. That is, the contribution of the externality to production is less than that of labor, which is reasonable. We assume Marshallian externality in the following analysis. Accordingly, profit maximizing firms regard B_N as exogenously given. Substituting equation (2) in equation (1), we can rewrite the production function as $X_N = A_N K_N^{1-\mu-\beta+\theta} L_N^{\mu} M^{\beta}$, which shows the increasing returns to scale in K_N, L_N, and M because $(1 - \mu - \beta + \theta) + \mu + \beta = 1 + \theta > 1$.

The South produces good S according to the following constant-returns-to-scale Cobb-Douglas production function:

$$Y_S = A_S K_S^{1-a-b} L_S^{a} T^{b}, \quad 0 < a < 1,\ 0 < b < 1,\ a + b < 1, \tag{3}$$

where Y_S denotes output; K_S, capital stock; $L_S = e^{n_S t}$, employment with n_S being the growth rate of the Southern population; and T, land input. Suppose that the supply of land is fixed. Then, we can normalize land input to $T = 1$. From this, equation (3) can be rewritten as $Y_S = A_S K_S^{1-a-b} L_S^{a}$, which shows the decreasing returns to scale in K_S and L_S.

Let $p \equiv p_S/p_N$ be the Southern terms of trade relative to the North with the N good being the numeraire. Then, the profits of the firms in the North and South are given by $\Pi_N = X_N - r_N K_N - w_N L_N - pM$ and $\Pi_S = pY_S - r_S K_S - w_S L_S - qT$, respectively, where w_i denotes the wage in country i; r_i, rental rate of capital; and q, rental rate of land: all prices are measured in terms of the N good, and accordingly, the profits are also expressed in terms of the N good. From profit maximizing conditions, we obtain the following relations: $w_N = \mu X_N/L_N$, $p = \beta X_N/M$, and $r_N = (1 - \mu - \beta)X_N/K_N$ for the North; and $w_S = paY_S/L_S$, $r_S = p(1 - a - b)Y_S/K_S$, and $q = pbY_S/T$ for the South. Note that in the above derivation, K_N^{θ} is treated as an exogenous variable by the firms in the North.

2.2 Consumers

A representative household in each country chooses the flow of consumption to maximize the present discounted value of lifetime utility. We assume that instantaneous utility is given by the log-linear function.

$$U_N = \int_0^{\infty} [\gamma \ln(C_N^S/L_N) + (1-\gamma)\ln(C_N^N/L_N)] \exp[-(\rho_N - n_N)t]\, dt, \quad \rho_N > n_N, \tag{4}$$

$$U_S = \int_0^{\infty} [\gamma \ln(C_S^S/L_S) + (1-\gamma)\ln(C_S^N/L_S)] \exp[-(\rho_S - n_S)t)\, dt, \quad \rho_S > n_S, \tag{5}$$

where C_N^S, for instance, denotes the consumption of the S good in the North, γ is a parameter governing an expenditure share for the S good, and ρ_i is the rate of time preference, which can differ with country. Here, we define real aggregate consumption *per capita* as $c_N \equiv C_N/L_N = (C_N^S/L_N)^{\gamma}(C_N^N/L_N)^{1-\gamma}$ and $c_S \equiv C_S/L_S = (C_S^S/L_S)^{\gamma}(C_S^N/L_S)^{1-\gamma}$.

The budget constraints are as follows.

$$(K_N\dot{/}L_N) = (r_N - n_N)(K_N/L_N) + w_N - (C_N^N/L_N) - p(C_N^S/L_N), \tag{6}$$

$$(K_S\dot{/}L_S) = (r_S - n_S)(K_S/L_S) + w_S + q(T/L_S) - (C_S^N/L_S) - p(C_S^S/L_S). \tag{7}$$

A dot over a variable denotes the time derivative of the variable, for example, $(K_N\dot{/}L_N) = d(K_N/L_N)/dt$. For simplicity, t is omitted unless needed. Since the total income in the North is given by $w_N L_N + r_N K_N = (1-\beta)X_N$ and that in the South is given by $w_S L_S + r_S K_S + qT = pY_S$, the respective equations of motion for capital stock lead to

$$\dot{K}_N = (1-\beta)X_N - C_N^N - pC_N^S, \tag{8}$$

$$\dot{K}_S = pY_S - C_S^N - pC_S^S. \tag{9}$$

By formulating current-value Hamiltonian functions with equations (4), (5), (6), and (7), we can derive the first-order necessary conditions as follows.

$$\frac{\gamma}{(C_N^S/L_N)} - p\lambda_N = 0, \tag{10}$$

$$\frac{1-\gamma}{(C_N^N/L_N)} - \lambda_N = 0, \tag{11}$$

$$\lambda_N(r_N - n_N) = (\rho_N - n_N)\lambda_N - \dot{\lambda}_N, \tag{12}$$

$$\frac{\gamma}{(C_S^S/L_S)} - p\lambda_S = 0, \tag{13}$$

$$\frac{1-\gamma}{(C_S^N/L_S)} - \lambda_S = 0, \tag{14}$$

$$\lambda_S(r_S - n_S) = (\rho_S - n_S)\lambda_S - \dot{\lambda}_S, \tag{15}$$

where λ_i are the costate variables. In addition, we need the transversality conditions:

$$\lim_{t\to+\infty} \lambda_i(t)[K_i(t)/L_i(t)] \exp[-(\rho_i - n_i)t] = \lim_{t\to+\infty} \lambda_i(t)K_i(t) \exp(-\rho_i t) = 0, \quad i = N, S. \tag{16}$$

From equations (12) and (15), we have $\dot{\lambda}_i(t)/\lambda_i(t) = \rho_i - r_i$, and consequently, $\lambda_i(t) = \lambda_i(0) \exp\left[\int_0^t \{\rho_i - r_i(\tau)\}\, d\tau\right]$, which is substituted in equation (16).

$$\lim_{t\to+\infty} \lambda_i(0)K_i(t) \exp\left[-\int_0^t r_i(\tau)\, d\tau\right] = 0. \tag{17}$$

2.3 Market clearing conditions

Let us describe the market clearing conditions for both goods. Taking into account the fact that investment in the South depends entirely on the imports from the North and the intermediate input in the North depends entirely on the imports from the South, we can write the market clearing conditions for both goods as follows:

$$X_N = C_N^N + C_S^N + I_N + I_S, \tag{18}$$

$$pY_S = \frac{\gamma}{1-\gamma}(C_N^N + C_S^N) + pM, \tag{19}$$

where equation (19) is a rewritten form of $Y_S = C_N^S + C_S^S + M$, and will be used later. The trade balance condition is given by $pC_N^S + pM = C_S^N + I_S$.

3 Balanced growth path

This section derives the BGP. The BGP in the present paper is the situation where all variables grow at constant rates, which are not necessarily the same. From equations (8) and (9), we obtain two equations of motion for capital stock. Using equations (11), (12), (14), and (15), we obtain two Euler equations for consumption. These four equations are presented below.

$$\frac{\dot{K}_N}{K_N} = (1-\beta)\frac{X_N}{K_N} - \frac{1}{1-\gamma}\frac{C_N^N}{K_N}, \tag{20}$$

$$\frac{\dot{K}_S}{K_S} = \frac{pY_S}{K_S} - \frac{1}{1-\gamma}\frac{C_S^N}{K_S}, \tag{21}$$

$$\frac{\dot{C}_N^N}{C_N^N} = (1-\mu-\beta)\frac{X_N}{K_N} - \rho_N + n_N, \tag{22}$$

$$\frac{\dot{C}_S^N}{C_S^N} = (1-a-b)\frac{pY_S}{K_S} - \rho_S + n_S. \tag{23}$$

With these equations, let us derive the BGP growth rates of variables. In what follows, we denote the growth rate of variable z as $g_z \equiv \dot{z}(t)/z(t)$.

To begin with, for consumption to grow at a constant rate, we need $g_{X_N} = g_{K_N}$ and $g_p + g_{Y_S} = g_{K_S}$ from equations (22) and (23). When $g_{X_N} = g_{K_N}$, the output-capital ratio in the North, X_N/K_N, becomes constant. Substituting $M = \beta X_N/p$ in the production function (1), we can rewrite the output-capital ratio as follows:

$$\frac{X_N}{K_N} = A_N \beta^{\frac{\beta}{1-\beta}} K_N^{\frac{\theta-\mu}{1-\beta}} L_N^{\frac{\mu}{1-\beta}} p^{-\frac{\beta}{1-\beta}}. \tag{24}$$

The right-hand side of the equation (24) will be constant along the BGP, that is, the rate of change in the right-hand side will be zero. Therefore, along the BGP, the following relation is obtained.

$$g_p = \frac{\theta-\mu}{\beta} g_{K_N} + \frac{\mu}{\beta} n_N. \tag{25}$$

Next, for capital stock to grow at a constant rate, we require $g_{C_N^N} = g_{K_N}$ and $g_{C_S^N} = g_{K_S}$ from equations (20) and (21), respectively, provided that the output-capital ratios in both countries are constant. In this case, C_N^N and C_S^N must grow at the same rate; otherwise, the world consumption for the N good will not grow at a constant rate.[9] From this observation, we have $g_{C_N^N} = g_{C_S^N}$, so that $g_{K_N} = g_{K_S}$, that is, capital stocks in both countries grow at the same rate along the BGP. Applying $g_{K_S} = g_{K_N} = g_{X_N}$ to $g_p + g_{Y_S} = g_{K_S}$, we obtain $g_p + g_{Y_S} = g_{X_N} = g_{K_N}$ along the BGP.

Substituting $g_{Y_S} = (1-a-b)g_{K_S} + an_S = (1-a-b)g_{K_N} + an_S$ from the Southern production function and equation (25) in $g_p + g_{Y_S} = g_{K_N}$, we obtain g_{K_N}:

$$g_{K_N}^* = g_{K_S}^* = \phi n_N + \psi n_S, \quad \text{where } \phi \equiv \frac{\mu}{\beta(a+b)+(\mu-\theta)} > 0,\ \psi \equiv \frac{\beta a}{\beta(a+b)+(\mu-\theta)} > 0. \tag{26}$$

An asterisk ($*$) denotes the BGP value of a variable. These growth rates depend on the growth rates of the population and the parameters of the production functions. Since $\theta \le \mu$, we have $\phi > 0$ and $\psi > 0$, that is, the growth rate of capital stock along the BGP is positive and increasing in both n_N and n_S.

Substituting equation (26) in equation (25), we find the growth rate of the terms of trade along the BGP:

$$g_p^* = \delta n_N + \varepsilon n_S, \quad \text{where } \delta \equiv \frac{\mu(a+b)}{\beta(a+b)+(\mu-\theta)} > 0,\ \varepsilon \equiv -\frac{a(\mu-\theta)}{\beta(a+b)+(\mu-\theta)} < 0. \tag{27}$$

[9]For X_N to grow at a constant rate, it is necessary that the right-hand side of equation (18) grows at a constant rate. Calculating the growth rate of the right-hand side of equation (18) and letting the resultant expression be constant, we find that world consumption for the N good ($C_N^N + C_S^N$) should grow at a constant rate. For details, see section A-3 of the Appendix, which is available online at the OUP website.

Since $\theta \leq \mu$, we have $\delta > 0$ and $\varepsilon \leq 0$, from which we obtain the following proposition:

Proposition 1. *The rate of change in the terms of trade along the BGP can be positive or negative. In either case, the terms of trade change continuously along the BGP. Moreover, an increase in the Northern (Southern) population growth improves the growth rate of the Southern (Northern) terms of trade.*

The Southern terms of trade continuously improve if the following condition holds:

$$\frac{n_N}{n_S} > \frac{a(\mu - \theta)}{\mu(a + b)}. \tag{28}$$

Given the right-hand side, this condition is likely to hold as the Northern population growth increases and the Southern population growth decreases. Given the left-hand side, this condition is likely to hold as the extent of the externality of the North, θ, increases.[10]

Finally, let us demonstrate that the transversality condition (17) holds. As shown above, $g^*_{K_N} = g^*_{K_S} = \phi n_N + \psi n_S$ holds along the BGP. After some calculations, we obtain the BGP rental rates of capital in both countries: $r^*_N = \rho_N - n_N + \phi n_N + \psi n_S$ and $r^*_S = \rho_S - n_S + \phi n_N + \psi n_S$.[11] From these observations, we obtain $r^*_N - g^*_{K_N} = \rho_N - n_N > 0$ and $r^*_S - g^*_{K_S} = \rho_S - n_S > 0$. Applying these results to equation (17), we can see that the transversality condition holds.

4 Population growth and per capita income growth

We now focus on the BGP growth rate of real income *per capita*, $g^*_{y_i}$ $(i = N, S)$. For this purpose, we have to obtain real national income, which in turn requires an appropriate definition of the consumption price index.[12]

Let p_C denote the price index that is consistent with the expenditure minimizing problem. Then, the price index is given by $p_C = \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)} p_N^{1-\gamma} p_S^{\gamma} = \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)} p^{\gamma}$. Since both countries face the common relative price, p, and their preferences are the same, the price index is also the same for both countries. Note that the relation $g_{p_C} = \gamma g_p$ holds between the price index and the terms of trade.

[10]When $\theta = \mu$, we have $g^*_p = \mu n_N / \beta > 0$: the Southern terms of trade improve continuously along the BGP.

[11]Along the BGP, $g_{C^N_N} = \phi n_N + \psi n_S$ holds. Substituting this expression in equation (22), we obtain $r_N = (1 - \mu - \beta) X_N / K_N = \rho_N - n_N + \phi n_N + \psi n_S$. We can obtain r_S in a similar way.

[12]Temple (2005) points out that in calculating real national income in an open economy setting, there is an important distinction between a GDP price index and a cost-of-living index because the structure of consumption and that of production can be different. Greenwood *et al.* (1997) also use a consumption deflator to divide nominal GDP. However, this method is not necessarily optimal. For this issue, see Whelan (2003), who derives the real GDP growth consistent with real chain aggregated data such as the U.S. National Income and Product Accounts.

From the above analysis, we get that the nominal national incomes in the North and South are given by $X_N - pM$ and pY_S, respectively. Both are measured in terms of the N good. Accordingly, real national incomes in the North and South are given by $(1-\beta)X_N/p_C$ and pY_S/p_C, respectively. As such, the growth rate of real national income in the North is $g_{X_N} - \gamma g_p$ and that in the South is $g_{Y_S} + (1-\gamma)g_p$. Recalling that $g_p + g_{Y_S} = g_{X_N}$ along the BGP, we observe that the real national income growth rates are identical.

Note that the growth rate of real national income is equal to that of aggregate consumption. As stated above, aggregate consumption is defined as $C_N \equiv (C_N^N)^{1-\gamma}(C_N^S)^\gamma$ in the North and $C_S \equiv (C_S^N)^{1-\gamma}(C_S^S)^\gamma$ in the South. Let E_N and E_S be the nominal expenditure in the North and South, respectively. Then, we have $E_N = p_C C_N$ and $E_S = p_C C_S$. From the Euler equations, we have $g_{E_N} = g_{K_N}$ and $g_{E_S} = g_{K_S}$ along the BGP. Since $g_{K_N} = g_{K_S}$, we obtain $g_{E_N} = g_{E_S}$, which in turn leads to $g_{C_N} = g_{C_S} = g_{K_N} - \gamma g_p$. Since $g_{K_N} = g_{X_N}$, real aggregate consumption grows at the same rate as real national income in the steady-state.

$$g^*_{\mathrm{NI},N} = g^*_{\mathrm{NI},S} = g^*_{C_N} = g^*_{C_S} = \underbrace{\frac{\mu[1-\gamma(a+b)]}{\beta(a+b)+(\mu-\theta)}}_{+} n_N + \underbrace{\frac{a[\beta+\gamma(\mu-\theta)]}{\beta(a+b)+(\mu-\theta)}}_{+} n_S, \tag{29}$$

where $g^*_{\mathrm{NI},i}$ is the growth rate of real national income. Since $\theta \le \mu$, the growth rate of real national income is positive. Moreover, the growth rate of real national income is increasing in both n_N and n_S. The result that all countries grow at the same rate even if their production structures are different is the same as that obtained in Acemoglu and Ventura (2002) and Felbermayr (2007).

Let us explain this result in detail. As stated above, we find that $g^*_p + g^*_{Y_S} = g^*_{X_N}$ along the BGP. This means that the value of output in terms of the N good grows at the same rate in both countries. Rearranging this equation, we have $g^*_p = g^*_{X_N} - g^*_{Y_S}$, which shows that the rate of change in the terms of trade is equal to the difference between the Northern output growth and Southern output growth. Since $g^*_p \gtrless 0 \iff g^*_{X_N} \gtrless g^*_{Y_S}$, the country with the higher output growth worsens its terms of trade. Further, the capital stock grows at the same rate in the two countries along the BGP; however, the outputs grow at different rates because of the differences in the returns to scale and population growth. Nevertheless, the real national income growth rates are identical. These growth rates are given by $g_{\mathrm{NI},N} = g_{X_N} - \gamma g_p$ and $g_{\mathrm{NI},S} = g_{Y_S} + (1-\gamma)g_p$, respectively, and are not identical in general. However, along the BGP, the terms of trade change such that the two growth rates are equalized, that is, the terms of trade effect equalizes both countries' real national income growth rates.

Because the real national income growth rate is identical in both countries, the growth rates of real income *per capita* in both countries differ when the growth rates of population in both countries differ. Let us suppose that population grows at a different rate in each country. Then, the growth rates of incomes *per capita* along the BGP are given by

$$g_{y_N}^* = \underbrace{\frac{\theta - (a+b)(\beta + \gamma\mu)}{\beta(a+b) + (\mu - \theta)}}_{+/-} n_N + \underbrace{\frac{a[\beta + \gamma(\mu - \theta)]}{\beta(a+b) + (\mu - \theta)}}_{+} n_S, \tag{30}$$

$$g_{y_S}^* = \underbrace{\frac{\mu[1 - \gamma(a+b)]}{\beta(a+b) + (\mu - \theta)}}_{+} n_N - \underbrace{\frac{(\mu - \theta)(1 - a\gamma) + \beta b}{\beta(a+b) + (\mu - \theta)}}_{+} n_S. \tag{31}$$

If $n_N < n_S$, then $g_{y_N}^* > g_{y_S}^*$. As discussed above, in each country, the growth rate of real income *per capita* is equal to the growth rate of consumption *per capita*.

To begin with, from equation (30), we obtain the following proposition with regard to the Northern growth rate of income *per capita* along the BGP:

Proposition 2. *The Northern growth rate of real income per capita (i) can be positive or negative, (ii) can be increasing or decreasing in the Northern population growth, (iii) is increasing in the Southern population growth, and (iv) can be positive even if the Northern population growth is zero.*

The coefficient of n_N in $g_{y_N}^*$ can be positive or negative. As Fig. 3 shows, the correlation between population growth and income *per capita* growth is ambiguous in developed countries. Our result is consistent with the empirical fact. Note, however, that in our model, the North does not necessarily correspond to the countries in Fig. 3. Thus, we should interpret the result with care. The same applies to the South. The condition that the coefficient becomes positive is given by $\theta > (a+b)(\beta + \gamma\mu)$. Given the other parameters, the coefficient is likely to be positive as the extent of the externality in the North increases. Since the coefficient of n_S is positive, the Northern growth rate of income *per capita* is always positive when the coefficient of n_N is positive. However, when the coefficient of n_N is negative, the Northern growth rate of income *per capita* can be negative.[13] As the Northern population growth increases and the Southern population growth decreases, the Northern growth rate of income *per capita* is likely to be negative.

Next, from equation (31), we obtain the following proposition with regard to the Southern growth rate of income *per capita* along the BGP:

Proposition 3. *The Southern growth rate of real income per capita (i) can be positive or negative, (ii) is decreasing in the Southern population growth, (iii) is increasing in the Northern population growth, and (iv) can be positive even if the Southern population growth is zero.*

[13]This can occur when the extent of the Northern externality is small. If, for example, $\theta = 0$, that is, the Northern production is constant returns to scale, we have $g_{y_N}^* < 0$ when $n_N/n_S > a/(a+b)$, which is exacltly the same condition as that for $g_p^* > 0$. From this, it follows that we obtain $g_{y_N}^* < 0$ when the extent of the Northern externality is very small and the Northern terms of trade are worsening.

Whereas the coefficient of n_N is positive, the coefficient of n_S is negative. Thus, $g^*_{y_S}$ can be positive depending on which effect dominates. The condition that $g^*_{y_S}$ is positive is given by

$$\frac{n_N}{n_S} > \underbrace{\frac{(\mu-\theta)(1-a\gamma)+\beta b}{\mu[1-\gamma(a+b)]}}_{\geq 0}. \tag{32}$$

This condition is likely to hold as the Northern population growth increases, Southern population growth decreases, and the extent of externality of the North increases. Let us recall equation (28), that is, the condition that the Southern terms of trade continue to improve. We can confirm that the right-hand side of equation (32) is larger than the right-hand side of equation (28). This means that we have $g^*_p > 0$ whenever $g^*_{y_S} > 0$, i.e., $g^*_p > 0$ is a necessary condition for $g^*_{y_S} > 0$. Therefore, the Southern growth rate of income *per capita* cannot be positive unless the Southern terms of trade continuously improve.

The result that a country that is at a disadvantage in terms of production can attain sustainable growth through the terms of trade effect is also obtained in Felbermayr (2007) and Álvarez-Albelo and Perera-Tallo (2008). Felbermayr (2007) shows that even if the South specializes in a technologically stagnant sector, it can achieve sustainable growth owing to endogenous, continuous improvements in the term-of-trade. Álvarez-Albelo and Perera-Tallo (2008) also reach a similar result by using a different two-country endogenous growth model.

Note that n_N/n_S appears on the left-hand side in both equations (28) and (32). This implies that the size of relative population growth is important for the terms of trade and economic development. Chamon and Kremer (2009) also point out the importance of relative population growth.[14] Population growth in developing countries is considered to be a problem. However, population growth in developing countries is rapidly declining and hence may not be an obstacle to development. Nevertheless, if the population growth in developed countries declines more rapidly than that in developing countries, the size of relative population growth also declines. This decline will be an obstacle for developing countries.

Finally, table 1 shows the seven possible combinations of g^*_p, $g^*_{y_N}$, $g^*_{y_S}$, and $\partial g^*_{y_N}/\partial n_N$. These take either positive or negative values, and as such, formally, the combinations of these values will be $2^4 = 16$. However, because some combinations are ruled out by the model, we obtain seven combinations listed in table 1.[15]

[14]In Chamon and Kremer's model, population growth in developing countries creates negative externalities in other countries, while population growth in developed countries creates a positive externality for the rest of the world.

[15]For details, see sections A-4 and A-5 of the Appendix, which is available online at the OUP website.

5 Transitional dynamics

This section analyses the transitional dynamics of our two-country model. To ensure that the foregoing analysis of the BGP is relevant, it is necessary that there be a path that converges to the BGP. In addition, if the convergence process requires time, it is important to know the behavior of each endogenous variable along the transitional dynamics. Moreover, even a parameter that does not affect the BGP does affect the transitional dynamics.

We use numerical simulations to examine the transitional dynamics. In this section, we consider the following situation. Suppose that two countries are in their steady states initially. Further, suppose that some parameters of the model change for any reason and new steady states emerge. We investigate the transitional dynamics from initial steady states to new steady states.[16] In this respect, Trimborn *et al.* (2008) develop a MATLAB code that is very useful for analyzing transitional dynamics.[17] This program enables us to easily compute the transitional dynamics from the pre-shock initial steady states to the post-shock new steady states.

We compute the time path of the relative income *per capita* (y_S/y_N), time path of the growth rate of real income *per capita* (g_{y_i}), and in some cases, time path of the terms of trade (p).[18] For this purpose, we consider the following four scenarios:

Scenario 1 the dynamics associated with the South, starting with the capital that is one-half of its steady-state value, and with the North, which is already in its steady state;

Scenario 2 the dynamics associated with a fall in the population growth rate by one-tenth in both countries;

Scenario 3 the dynamics associated with a rise in the Northern TFP level by one-and-a-half times;

Scenario 4 the dynamics associated with a rise in the Southern TFP level by one-and-a-half times.

[16] An analysis of transitional dynamics along these lines is conducted by Eicher and Turnovsky (2001), who use a two-sector R&D-based non-scale growth model.

[17] Manuel Bichsel provides a *Mathematica* version of the code, which is used in this paper. The MATLAB code by Trimborn *et al.* (2008), and the *Mathematica* code by Manuel Bichsel are both downloadable at http://www.relaxation.uni-siegen.de.

[18] We also compute the relative consumption (c_S/c_N) and the growth rate of consumption *per capita* (g_{c_i}). The results are similar to those of y_S/y_N and g_{y_i}. For details, see section A-9 of the Appendix, which is available online at the OUP website.

5.1 Dynamical system of scale-adjusted variables

In our model, K_N, K_S, C_N^N, and C_S^N increase continuously along the BGP, and consequently, we cannot investigate the dynamic stability of the system without modifications. Therefore, we introduce the following scale-adjusted variables while considering the BGP growth rates of the above variables and the terms of trade: $\pi \equiv p/(L_N^\delta L_S^\varepsilon)$, $k_N \equiv K_N/(L_N^\phi L_S^\psi)$, $k_S \equiv K_S/(L_N^\phi L_S^\psi)$, $c_N^N \equiv C_N^N/(L_N^\phi L_S^\psi)$, and $c_S^N \equiv C_S^N/(L_N^\phi L_S^\psi)$.

Using these variables, we can rewrite our model as follows:

$$\dot{k}_N = k_N\left[A_N(1-\beta)\beta^{\frac{\beta}{1-\beta}}\pi^{-\frac{\beta}{1-\beta}}k_N^{\frac{\theta-\mu}{1-\beta}} - \frac{1}{1-\gamma}\frac{c_N^N}{k_N} - \phi n_N - \psi n_S\right], \tag{33}$$

$$\dot{k}_S = k_S\left[A_S\pi k_S^{-a-b} - \frac{1}{1-\gamma}\frac{c_S^N}{k_S} - \phi n_N - \psi n_S\right], \tag{34}$$

$$\dot{c}_N^N = c_N^N\left[A_N(1-\mu-\beta)\beta^{\frac{\beta}{1-\beta}}\pi^{-\frac{\beta}{1-\beta}}k_N^{\frac{\theta-\mu}{1-\beta}} - \rho_N - \phi n_N - \psi n_S + n_N\right], \tag{35}$$

$$\dot{c}_S^N = c_S^N\left[A_S(1-a-b)\pi k_S^{-a-b} - \rho_S - \phi n_N - \psi n_S + n_S\right], \tag{36}$$

$$A_S\pi k_S^{1-a-b} - \frac{\gamma}{1-\gamma}(c_N^N + c_S^N) = A_N\beta^{\frac{1}{1-\beta}}\pi^{-\frac{\beta}{1-\beta}}k_N^{\frac{1-\beta+\theta-\mu}{1-\beta}} \implies \pi = \pi(\underset{+}{k_N}, \underset{-}{k_S}, \underset{+}{c_N^N}, \underset{+}{c_S^N}), \tag{37}$$

where the sign below the variables denotes the sign of the corresponding partial derivative of the variable. Equations (33)–(36) are rewritten forms of equations (20)–(23), while equation (37) is a rewritten form of (19). The endogenous variables are k_N, k_S, c_N^N, c_S^N, and π; π is implicitly determined through equation (37).[19]

First, by using the above system, we calculate the dynamics of k_N, k_S, c_N^N, c_S^N, and π. Next, by using the values obtained now, we compute the time paths of y_S/y_N, g_{y_i}, and p.[20]

Table 2 summarizes the parameters used in the simulation.[21] With these parameters, we have Case 1 in table 1.[22] In particular, note that $g_{y_N}^* \simeq 0.013$ and $g_{y_S}^* \simeq 0.003$ along the BGP.

[19]To observe how π and each partial derivative are determined, see sections A-6 and A-7 of the Appendix, which is available online at the OUP website.

[20]For the expressions of y_S/y_N, g_{y_i}, and p, see section A-8 of the Appendix, which is available online at the OUP website.

[21]In this setting, the Northern wage share is $\mu/(1-\beta) = 2/3$ and the Northern profit share is $1/3$. We set the extent of the Northern externality to $\theta = 0.3$, which is based on the simulation of Graham and Temple (2006), who set the extent of increasing returns in non-agricultural production to 0.05–0.50. In the South, the wage share (a), land share (b), and profit share ($1 - a - b$) are 0.4, 0.2, and 0.4, respectively. These values are also based on the study of Graham and Temple (2006). For the rates of time preference, we set 0.03 in the North and 0.04 in the South. Although these values seems rather high, the effective discount rate is given by $\rho_i - n_i$, which leads to 0.01 in both countries.

[22]We also conduct another simulation corresponding to Case 3 in the BGP. See section A-10 of the Appendix, which is available online at the OUP website.

5.2 Results of numerical simulations

5.2.1 Scenario 1

Figures 5 and 6 correspond to the case where the South starts with k_S that is one-half of its steady-state value. The horizontal axes represent time, whereas the vertical axes represent y_S/y_N and g_{y_i}. For comparison, we present the benchmark case where the South starts with its steady-state value. Note that even in the benchmark case, y_S/y_N decreases with time because $g^*_{y_N} > g^*_{y_S}$ in the long run.

As Fig. 5 shows, y_S/y_N initially decreases slightly as compared to the benchmark, but it gradually returns to the benchmark. As Fig. 6 shows, the growth rates of both countries initially increase compared to each benchmark value, but converge to each benchmark with time. In Scenario 1, we suppose that initially, the North is in a steady state. However, as shown by this simulation, first, both the North and the South grow rapidly. Such a growth linkage exists in our two-country model.

5.2.2 Scenario 2

Figures 7 and 8 correspond to the case where both n_N and n_S simultaneously decrease by one-tenth. As Fig. 7 shows, the time path of y_S/y_N becomes flatter than that of the benchmark. Besides, y_S/y_N initially exceeds unity, that is, the South temporarily becomes richer than the North. As Fig. 8 shows, initially, g_{y_N} falls and g_{y_S} rises, as compared to each benchmark value. They gradually converge to each post-shock BGP value, 0.0013 and 0.0003.

As discussed in section 4, a fall in the growth rate of the population in developing countries has a positive effect on their *per capita* income growth. However, in reality, we observe a fall in the growth rate of the population in developed countries, which has a negative effect on their *per capita* income growth. Therefore, as Chamon and Kremer (2009) point out, we should consider the degree of relative population growth. In Scenario 2, the population growth rates of both countries simultaneously decrease by one-tenth, and consequently, the degree of relative population growth is the same as that of the benchmark case. In this case, the *per capita* income growth along the BGP of both countries decreases by one-tenth, and thus, the growth rate of relative income along the BGP is equal to that of the benchmark case. However, along the transitional dynamics, the decline in the relative income becomes far more moderate than that in the benchmark. This means that a fall in the Southern population growth is beneficial in reducing income disparity even if the Northern population growth declines.

5.2.3 Scenario 3

Figures 9, 10, and 11 correspond to the case where the Northern TFP level rises. Figure 9 shows that the time path of y_S/y_N slightly declines compared to the benchmark (although the figure does not clearly show the decline). In this case, the absolute level of income *per capita* in each country increases through time relative to the benchmark.[23] That is, both countries are rendered better off in terms of the absolute income level when the North experiences technical progress. As Fig. 10 shows, at first, g_{y_i} rises as compared to the benchmark, and gradually converges to the benchmark. As Fig. 11 shows, the Southern terms of trade improve on the whole.

5.2.4 Scenario 4

Figures 12, 13, and 14 correspond to the case where the Southern TFP level rises. Figure 12 shows that the time path of y_S/y_N slightly declines, as in Scenario 3. Moreover, in this case, the absolute level of income *per capita* in each country increases through time, as compared to the benchmark. Hence, both countries are rendered better off, as in Scenario 3. As Fig. 13 shows, g_{y_i} rises initially compared to the benchmark, and gradually converges to the benchmark. As Fig. 14 shows, compared to the benchmark, the Southern terms of trade on the whole deteriorate.

From Scenarios 3 and 4, we observe that the South is rendered slightly worse off in terms of relative income, as compared to the benchmark, regardless of whether the North or the South experiences technical progress. However, both countries are rendered better off in terms of absolute income. The terms of trade of a country that experiences technical progress deteriorate. When technical progress occurs in the South (North), the Southern (Northern) terms of trade deteriorate compared to the benchmark, but the absolute income level rises: thus, the deterioration of the terms of trade does not necessarily worsen the income level.

6 Concluding remarks

In this paper, we have developed a two-country non-scale growth model that considers a North-South trade situation. Our results are consistent with two empirical findings: the growth rate of income *per capita* differs across countries and the relationship between population growth and *per capita* income growth differs for the developed and developing countries.

[23]The time path of the absolute level of income *per capita* in each country is given in section A-9 of the Appendix, which is available online at the OUP website.

Although the production structure is different in each country—the North is characterized by an increasing-returns-to scale technology while the South is characterized by a decreasing-returns-to-scale technology—real national income in both countries increases at the same rate because of a continuous change in the terms of trade. Hence, the differences in population growth lead to differences in the *per capita* income growth in each country. In our model, even the decreasing-returns South can experience a positive growth of income *per capita* if the continuous improvement in the Southern terms of trade is larger.

The correlation between population growth and *per capita* income growth can be positive or negative in the North, while it is always negative in the South. Therefore, empirical analysis taking both developed and developing countries together will yield an ambiguous correlation between the two growth rates.

It is true that the South can experience sustainable growth in *per capita* income depending on conditions. Even in this case, however, the growth rate of income *per capita* in the South is necessarily lower than that in the North if the growth rate of population in the South is higher than that in the North. For this reason, if the initial level of income *per capita* in the South is lower than that in the North, the South cannot catch up with the North in terms of income level. Therefore, uneven development still remains even if the Southern terms of trade continue to improve.

Supplementary material

Supplementary material (the Appendix) is available online at the OUP website.

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Figures and Tables

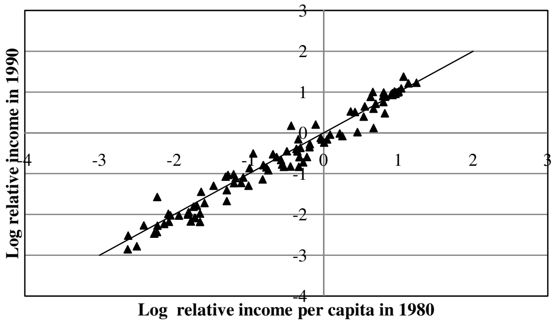

Figure 1: Income *per capita* in 1980 and 1990 relative to the world average. Source: EPWT 3.0

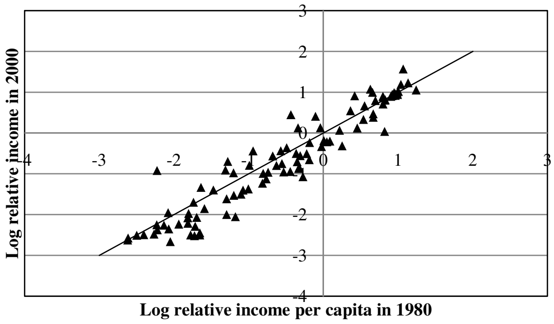

Figure 2: Income *per capita* in 1980 and 2000 relative to the world average. Source: EPWT 3.0

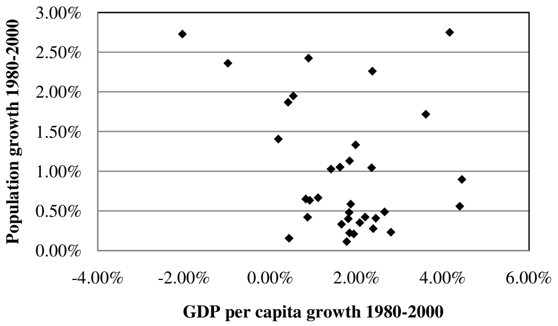

Figure 3: Population growth and income *per capita* growth 1980–2000 above the world average. Source: EPWT 3.0

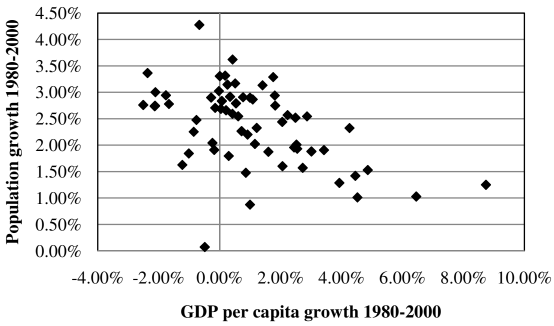

Figure 4: Population growth and income *per capita* growth 1980–2000 below the world average. Source: EPWT 3.0

Table 1: Seven possible combinations of g_p^*, $g_{y_N}^*$, $g_{y_S}^*$, and $\partial g_{y_N}^*/\partial n_N$

	g_p^*	$g_{y_N}^*$	$g_{y_S}^*$	$\partial g_{y_N}^*/\partial n_N$
Case 1	+	+	+	+
Case 2	+	+	+	−
Case 3	+	+	−	+
Case 4	+	+	−	−
Case 5	+	−	−	−
Case 6	−	+	−	+
Case 7	−	+	−	−

Table 2: Lists of parameters

	A_N	A_S	μ	β	θ	a	b	n_N	n_S	ρ_N	ρ_S	γ
Benchmark	1	1	0.5	0.25	0.3	0.4	0.2	0.02	0.03	0.03	0.04	0.4

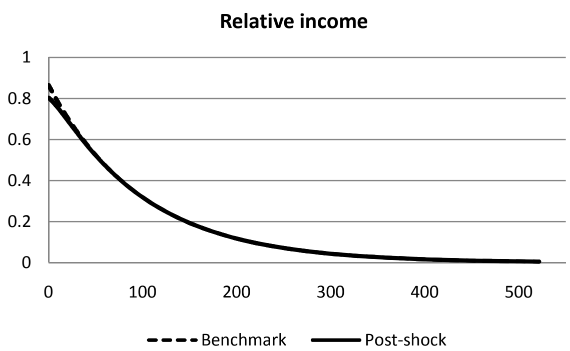

Figure 5: South starts at a 50% steady state

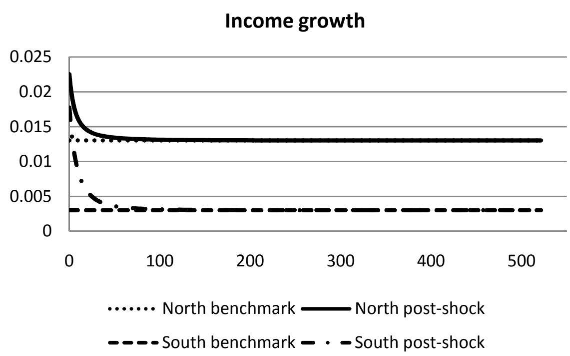

Figure 6: South starts at a 50% steady state

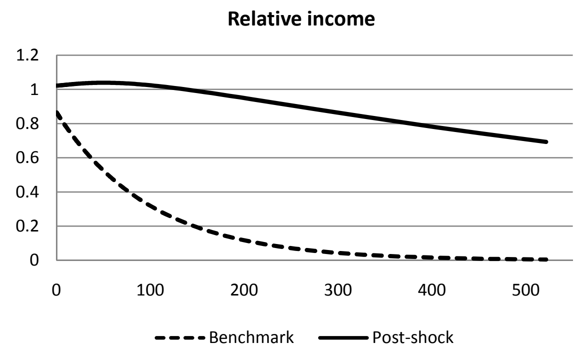

Figure 7: Fall in population growth

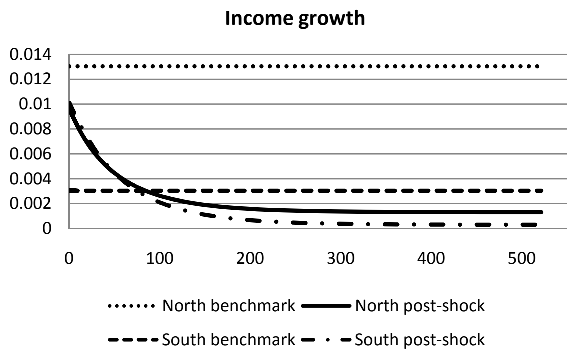

Figure 8: Fall in population growth

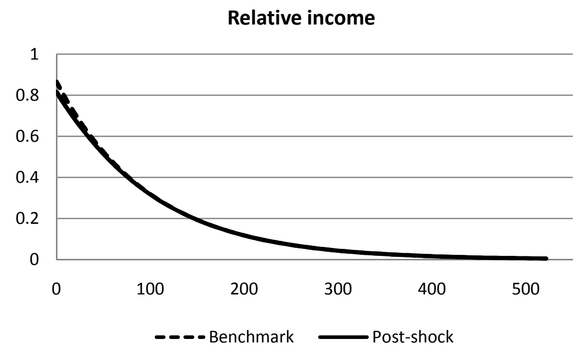

Figure 9: Rise in Northern TFP

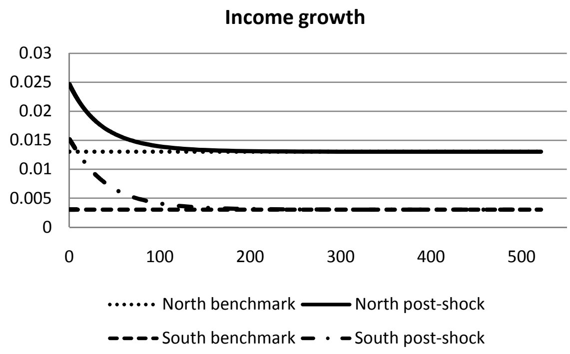

Figure 10: Rise in Northern TFP

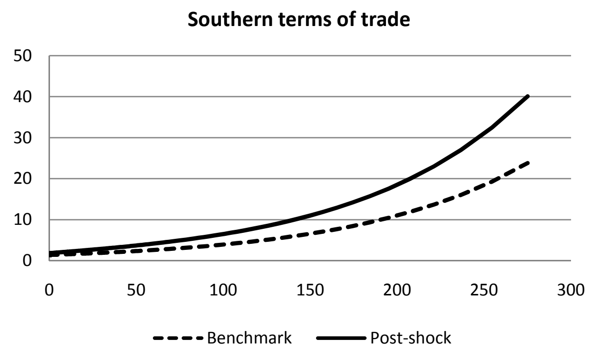

Figure 11: Rise in Northern TFP

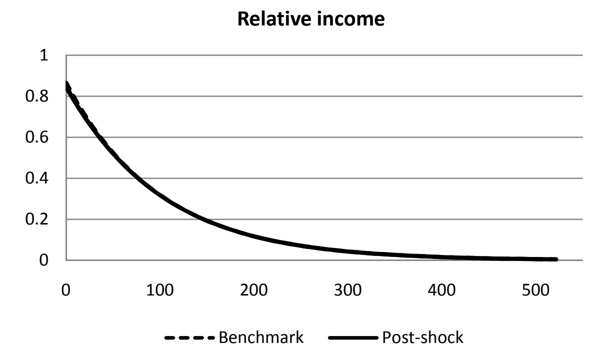

Figure 12: Rise in Southern TFP

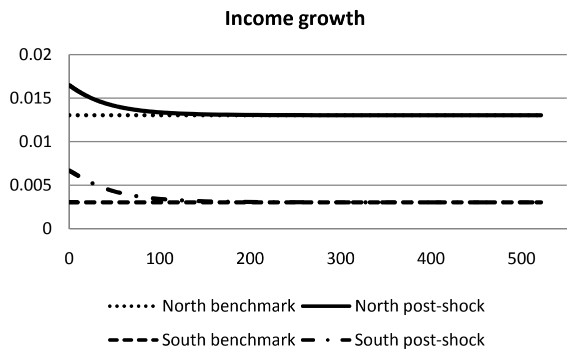

Figure 13: Rise in Southern TFP

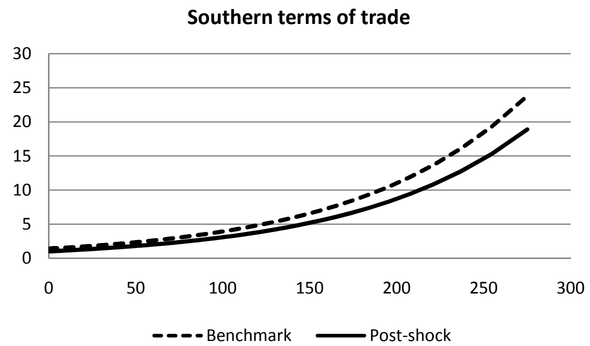

Figure 14: Rise in Southern TFP

Appendix to
“Population Growth and North-South Uneven Development”

(Not for publication)

Hiroaki SASAKI*

A-1: List of 34 countries above the world average in 1980

Switzerland, United States, Luxembourg, Norway, Denmark, Iceland, Canada, Sweden, Netherlands, Austria, Australia, France, Belgium, Finland, Italy, Gabon, Japan, New Zealand, United Kingdom, Israel, Trinidad & Tobago, Barbados, Hong Kong, Singapore, Spain, Greece, Argentina, Ireland, Portugal, Venezuela, Uruguay, South Africa, Mexico, Costa Rica.

A-2: List of 63 countries below the world average in 1980

Brazil, Chile, Mauritius, Panama, Romania, Iran, Nicaragua, Algeria, Ecuador, Peru, Malaysia, Paraguay, Colombia, Korea Republic of, Jordan, Tunisia, Guatemala, El Salvador, Dominican Republic, Jamaica, Turkey, Philippines, Zimbabwe, Morocco, Bolivia, Thailand Egypt, Guinea, Cameroon, Honduras, Congo Republic of, Cote d‘Ivoire, Indonesia, Cape Verde, Comoros, Syria, Sri Lanka, Pakistan, Senegal, India, Zambia, Togo, Kenya, Rwanda, Niger, Lesotho, Madagascar, Ghana, Mozambique, Benin, Nigeria, Burundi, Gambia The, Nepal, Mali, Burkina Faso, China, Uganda, Malawi, Chad, Tanzania, Guinea-Bissau, Ethiopia.

*Graduate School of Economics, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan; e-mail: sasaki@econ.kyoto-u.ac.jp

A-3: The reason why consumption of the N good grows at the same rate in both countries

The market clearing condition for the N good can be rewritten as:

$$X_N = C_N^N + C_S^N + g_{K_N} K_N + g_{K_S} K_S,$$

where $I_N = g_{K_N} K_N$ and $I_S = g_{K_S} K_S$. The rates of change of both sides are

$$g_{X_N} = \frac{C_N^N}{X_N} g_{C_N^N} + \frac{C_S^N}{X_N} g_{C_S^N} + \frac{g_{K_N} K_N}{X_N} \left(\hat{g}_{K_N} + \hat{K}_N\right) + \frac{g_{K_S} K_S}{X_N} \left(\hat{g}_{K_S} + \hat{K}_S\right).$$

Since $\hat{g}_{K_N} = 0$ and $\hat{g}_{K_S} = 0$ along the BGP, we can rewrite this equation as

$$g_{X_N} = \frac{C_N^N}{X_N} g_{C_N^N} + \frac{C_S^N}{X_N} g_{C_S^N} + \frac{g_{K_N}^2 K_N}{X_N} + \frac{g_{K_S}^2 K_S}{X_N}.$$

Along the BGP, the left-hand side is constant. Moreover, along the BGP, $g_{C_N^N}$, $g_{C_S^N}$ and the third term of the right-hand side are constant. From this, for the right-hand side as a whole to be constant, it is necessary that C_N^N/X_N, C_S^N/X_N, and K_S/X_N of the right-hand side should be constant. From this, we obtain $g_{C_N^N} = g_{X_N}$, $g_{C_S^N} = g_{X_N}$, and $g_{K_S} = g_{X_N}$.

With these conditions and $g_{X_N} = g_{K_N}$, we can state that along the BGP the following relationship holds:

$$g_{X_N} = g_{K_N} = g_{K_S} = g_{C_N^N} = g_{C_S^N}.$$

A-4: The reason why only 7 out of 16 cases exist

As the text states, g_p^*, $g_{y_N}^*$, $g_{y_S}^*$, and $\partial g_{y_N}^*/\partial n_N$ take either positive or negative values, and consequently, there are $2^4 = 16$ possible sign combinations. In the following analysis, however, we show that only 7 out of 16 are admissible in the model.

First, as stated in the text, for $g_{y_N}^* < 0$, it is at least necessary $\partial g_{y_N}^*/\partial n_N < 0$. Hence, it is impossible that both $\partial g_{y_N}^*/\partial n_N > 0$ and $g_{y_N}^* < 0$. Therefore, we can exclude four cases in Table A. See cells Reason (A) in Table A.

Second, for $g_{y_S}^* > 0$, it is at least necessary that $g_p^* > 0$. Hence, it is impossible that both $g_p^* < 0$ and $g_{y_S}^* > 0$. Therefore, we can exclude three cases in Table A. See cells Reason (B) in Table A.

Third, we can show that it is impossible that both $g_p^* < 0$ and $g_{y_N}^* < 0$. The condition for both $g_p^* < 0$ and $g_{y_N}^* < 0$ is given by

$$-\frac{a[\beta+\gamma(\mu-\theta)]}{\theta-(a+b)(\beta+\gamma\mu)} < \frac{n_N}{n_S} < \frac{a(\mu-\theta)}{\mu(a+b)},$$

where $\theta-(a+b)(\beta+\gamma\mu) < 0$, which is the condition for $\partial g_{y_N}^*/\partial n_N < 0$, is imposed because $\partial g_{y_N}^*/\partial n_N < 0$ is the necessary condition for $g_{y_N}^* < 0$. However, the left-hand side is always larger than the right-hand side, and consequently, there never exists n_N/n_S that satisfies the above equality. Hence, it is impossible that both $g_p^* < 0$ and $g_{y_N}^* < 0$. Therefore, we can exclude one case in Table A. See cell Reason (C) in Table A.

Finally, as long as $n_N < n_S$, the equality $g_{y_N}^* > g_{y_S}^*$ necessarily holds, and consequently, it is impossible that both $g_{y_N}^* < 0$ and $g_{y_S}^* > 0$. Therefore, we can exclude one case in Table A. See cell Reason (D) in Table A.

Taking these results together, we find that $4+3+1+1 = 9$ cases do not exist. In addition, we can present numerical examples for 7 possible cases (see the next section of this appendix). Therefore, only 7 out of 16 cases are possible.

Table A: 16 combinations of g_p^*, $g_{y_N}^*$, $g_{y_S}^*$, and $\partial g_{y_N}^*/\partial n_N$ and their feasibility

True or not	g_p^*	$g_{y_N}^*$	$g_{y_S}^*$	$\partial g_{y_N}^*/\partial n_N$	Reason
×	+	−	+	+	(A)
×	+	−	−	+	(A)
×	−	−	+	+	(A)
×	−	−	−	+	(A)
○	+	+	+	−	Case 2
×	−	+	+	−	(B)
○	+	+	−	−	Case 4
○	−	+	−	−	Case 7
×	+	−	+	−	(D)
×	−	−	+	−	(B)
○	+	−	−	−	Case 5
×	−	−	−	−	(C)
○	+	+	−	+	Case 3
×	−	+	+	+	(B)
○	−	+	−	+	Case 6
○	+	+	+	+	Case 1

A-5: Numerical examples for seven possible cases

Case 1

$$\mu = 0.5,\ \theta = 0.3,\ \beta = 0.25,\ a = 0.4,\ b = 0.2,$$
$$\gamma = 0.4,\ n_N = 0.02,\ n_S = 0.03.$$
$$g_p^* = 0.0102857,\ g_{y_N}^* = 0.0130286,\ g_{y_S}^* = 0.00302857,\ \partial g_{y_N^*}/\partial n_N = 0.0857143.$$

Case 2

$$\mu = 0.3,\ \theta = 0.3,\ \beta = 0.5,\ a = 0.3,\ b = 0.3,$$
$$\gamma = 0.4,\ n_N = 0.02,\ n_S = 0.03.$$
$$g_p^* = 0.012,\ g_{y_N}^* = 0.0102,\ g_{y_S}^* = 0.0002,\ \partial g_{y_N^*}/\partial n_N = -0.24.$$

Case 3

$$\mu = 0.5,\ \theta = 0.3,\ \beta = 0.25,\ a = 0.4,\ b = 0.2,$$
$$\gamma = 0.4,\ n_N = 0.01,\ n_S = 0.03.$$
$$g_p^* = 0.00171429,\ g_{y_N}^* = 0.0121714,\ g_{y_S}^* = -0.00782857,\ \partial g_{y_N^*}/\partial n_N = 0.0857143.$$

Case 4

$$\mu = 0.3,\ \theta = 0.2,\ \beta = 0.5,\ a = 0.3,\ b = 0.3,$$
$$\gamma = 0.4,\ n_N = 0.02,\ n_S = 0.03.$$
$$g_p^* = 0.00675,\ g_{y_N}^* = 0.00355,\ g_{y_S}^* = -0.00645,\ \partial g_{y_N^*}/\partial n_N = -0.43.$$

Case 5

$$\mu = 0.3,\ \theta = 0.1,\ \beta = 0.5,\ a = 0.3,\ b = 0.3,$$
$$\gamma = 0.4,\ n_N = 0.02,\ n_S = 0.03.$$
$$g_p^* = 0.0036,\ g_{y_N}^* = -0.00044,\ g_{y_S}^* = -0.01044,\ \partial g_{y_N^*}/\partial n_N = -0.544.$$

Case 6

$\mu = 0.4,\ \theta = 0.1,\ \beta = 0.2,\ a = 0.2,\ b = 0.2,$
$\gamma = 0.1,\ n_N = 0.01,\ n_S = 0.03.$
$g_p^* = -0.000526316,\ g_{y_N}^* = 0.00373684,\ g_{y_S}^* = -0.0162632,\ \partial g_{y_N^*}/\partial n_N = 0.0105263.$

Case 7

$\mu = 0.4,\ \theta = 0.1,\ \beta = 0.3,\ a = 0.2,\ b = 0.3,$
$\gamma = 0.2,\ n_N = 0.01,\ n_S = 0.03.$
$g_p^* = -0.000666667,\ g_{y_N}^* = 0.00402222,\ g_{y_S}^* = -0.0159778,\ \partial g_{y_N^*}/\partial n_N = -0.217778.$

A-6: Determination of the scale-adjusted terms of trade

In the scale-adjusted market clearing condition for the S good,

$$A_S \pi k_S^{1-a-b} - \frac{\gamma}{1-\gamma}(c_N^N + c_S^N) = A_N \beta^{\frac{1}{1-\beta}} \pi^{-\frac{\beta}{1-\beta}} k_N^{\frac{1-\beta+\theta-\mu}{1-\beta}},$$

the left-hand side and the right-hand side can be regarded as functions of π with k_N, k_S, c_N^N, and c_S^N being constant, which are drawn in the followin figure. As the figure shows, the

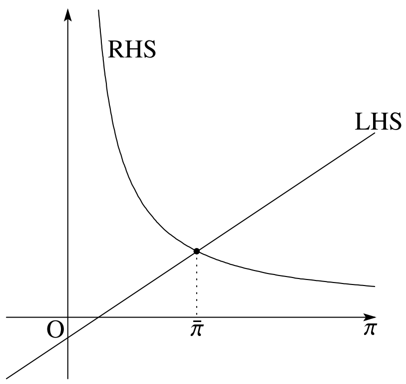

Figure I: Determination of the scale-adjusted terms of trade

intersection of the left-hand side and the right-hand side uniquely determines π. Therefore, the scale-adjusted terms of trade π lead to a function of the other scale-adjusted variables k_N, k_S, c_N^N, and c_S^N.

A-7: Partial derivatives of π

Differentiating both sides of the scale-adjusted market clearing condition for the S good, we obtain the following partial derivatives of π:

$$\begin{aligned}
\frac{\partial \pi}{\partial k_N} &= \frac{A_N(1-\mu-\beta+\theta)\beta^{\frac{1}{1-\beta}}\pi^{-\frac{\beta}{1-\beta}}k_N^{\frac{\theta-\mu}{1-\beta}}}{(1-\beta)\Gamma} > 0,\\
\frac{\partial \pi}{\partial k_S} &= -\frac{A_S(1-a-b)\pi k_S^{-a-b}}{\Gamma} < 0,\\
\frac{\partial \pi}{\partial c_N^N} &= \frac{\partial \pi}{\partial c_S^N} = \frac{\gamma}{(1-\gamma)\Gamma} > 0,\\
&\text{where } \Gamma \equiv A_S k_S^{1-a-b} + \frac{A_N\beta^{\frac{2-\beta}{1-\beta}}\pi^{-\frac{1}{1-\beta}}k_N^{\frac{1-\beta+\theta-\mu}{1-\beta}}}{1-\beta} > 0.
\end{aligned}$$

A-8: Derivation of real income per capita and real consumption per capita

The level of real income per capita

The levels of real income per capita of both countries are respectively given by

$$\begin{aligned}
y_N &= \frac{(1-\beta)X_N}{p_C L_N} = A_N D k_N^{\sigma_1}\pi^{\sigma_2}\exp[(\sigma_3 n_N + \sigma_4 n_S)t],\\
y_S &= \frac{pY_S}{p_C L_S} = A_S E k_S^{\sigma_5}\pi^{\sigma_6}\exp[(\sigma_7 n_N + \sigma_8 n_S)t],
\end{aligned}$$

where D and E are defined as follows:

$$D = (1-\beta)\beta^{\frac{\beta}{1-\beta}}\gamma^{\gamma}(1-\gamma)^{1-\gamma}, \quad E = \gamma^{\gamma}(1-\gamma)^{1-\gamma}.$$

Moreover, σ_i ($i = 1, \ldots, 8$) are defined as follows:

$$\sigma_1 = \frac{1-\beta+\theta-\mu}{1-\beta},\ \sigma_2 = -\frac{\beta}{1-\beta}-\gamma,\ \sigma_3 = \frac{\theta-(a+b)(\beta+\gamma\mu)}{\beta(a+b)+(\mu-\theta)},\ \sigma_4 = \frac{a[\beta+\gamma(\mu-\theta)]}{\beta(a+b)+(\mu-\theta)},$$
$$\sigma_5 = 1-a-b,\ \sigma_6 = 1-\gamma,\ \sigma_7 = \frac{\mu[1-\gamma(a+b)]}{\beta(a+b)+(\mu-\theta)},\ \sigma_8 = -\frac{(\mu-\theta)(1-a\gamma)+\beta b}{\beta(a+b)+(\mu-\theta)}.$$

The level of real consumption per capita

The levels of real consumption per capita in both countries are respectively given by

$$c_N = \frac{C_N}{L_N} = \frac{(C_N^N)^{1-\gamma}\left(\dfrac{\gamma}{1-\gamma}\dfrac{C_N^N}{p}\right)^{\gamma}}{L_N} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} c_N^N \pi^{-\gamma} \exp[(\sigma_3 n_N + \sigma_4 n_S)t],$$
$$c_S = \frac{C_S}{L_S} = \frac{(C_S^N)^{1-\gamma}\left(\dfrac{\gamma}{1-\gamma}\dfrac{C_S^N}{p}\right)^{\gamma}}{L_S} = \left(\frac{\gamma}{1-\gamma}\right)^{\gamma} c_S^N \pi^{-\gamma} \exp[(\sigma_7 n_N + \sigma_8 n_S)t].$$

Terms of trade

The terms of trade are given by

$$p = \pi \exp[(\delta n_N + \varepsilon n_S)t].$$

The growth rate of real income per capita

The growth rates of real income per capita in both countries are respectively given by

$$g_{y_N} = \sigma_1 g_{k_N} + \sigma_2 g_\pi + \sigma_3 n_N + \sigma_4 n_S,$$
$$g_{y_S} = \sigma_5 g_{k_S} + \sigma_6 g_\pi + \sigma_7 n_N + \sigma_8 n_S.$$

The growth rate of the scale-adjusted terms of trade, g_π, is obtained by differentiating the scale-adjusted market clearing condition for the S good:

$$g_\pi = \frac{1}{\Gamma\pi}\left[\frac{A_N(1-\beta+\theta-\mu)\beta^{\frac{1}{1-\beta}}}{1-\beta}\pi^{-\frac{\beta}{1-\beta}} k_N^{\frac{\theta-\mu}{1-\beta}} \cdot \dot{k}_N - A_S(1-a-b)\pi k_S^{-a-b} \cdot \dot{k}_S + \frac{\gamma}{1-\gamma}(\dot{c}_N^N + \dot{c}_S^N)\right].$$

Along the BGP, we have $g_{k_N}^* = g_{k_S}^* = g_\pi^* = 0$. Therefore, we can see that along the BGP, we obtain $g_{y_N}^* = \sigma_3 n_N + \sigma_4 n_S$, $g_{y_S}^* = \sigma_7 n_N + \sigma_8 n_S$.

The growth rate of real consumption per capita

The growth rates of real consumption per capita in both countries are respectively given by

$$g_{c_N} = g_{c_N^N} - \gamma g_\pi + \sigma_3 n_N + \sigma_4 n_S,$$
$$g_{c_S} = g_{c_S^N} - \gamma g_\pi + \sigma_7 n_N + \sigma_8 n_S.$$

Along the BGP, we have $g^*_{c_N^N} = g^*_{c_S^N} = g^*_\pi = 0$. Therefore, in each country, the growth rate of real consumption per capita is equal to that of real income per capita.

A-9: Transitional dynamics in Case 1

South starts with capital half of its steady-state value

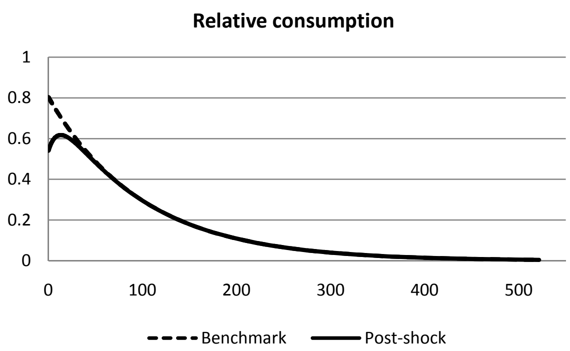

Figure II: South starts at a 50% steady state

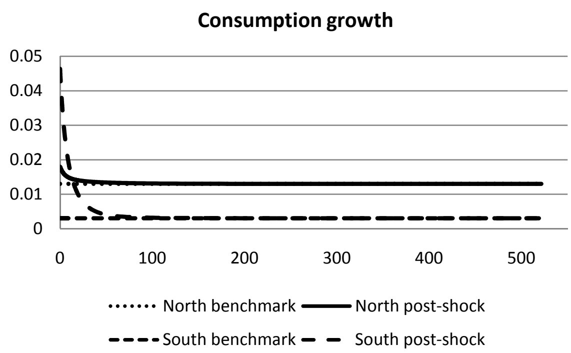

Figure III: South starts at a 50% steady state

Both n_N and n_S decrease by tenth

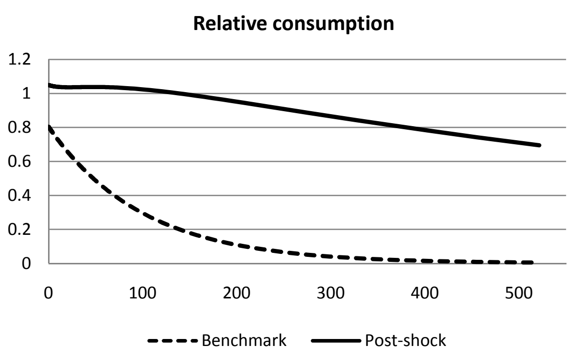

Figure IV: Fall in the population growth

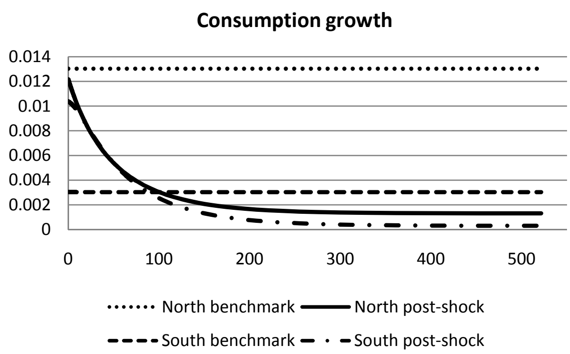

Figure V: Fall in the population growth

Northern TFP level rises

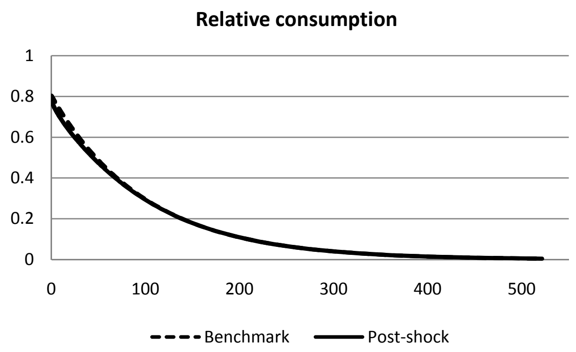

Figure VI: Rise in the Northern TFP

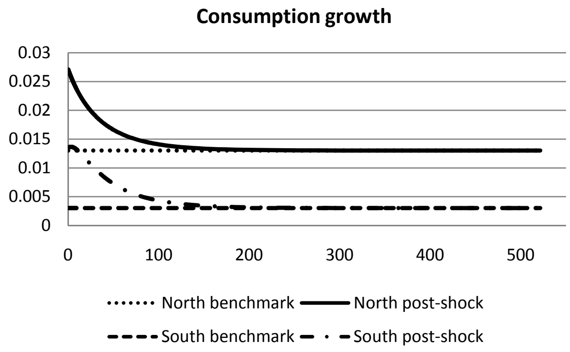

Figure VII: Rise in the Northern TFP

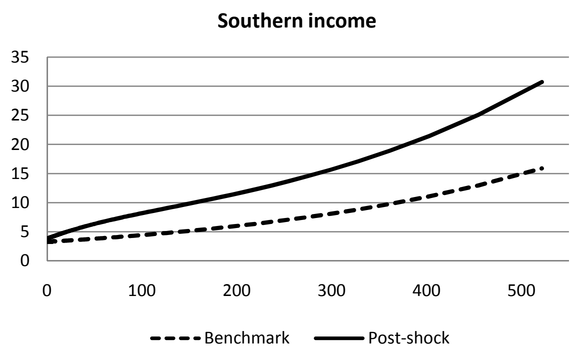

Figure VIII: Rise in the Northern TFP

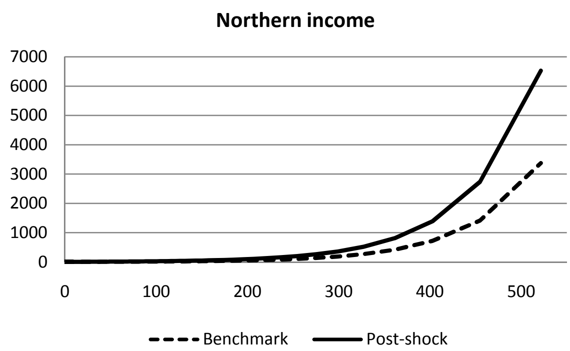

Figure IX: Rise in the Northern TFP

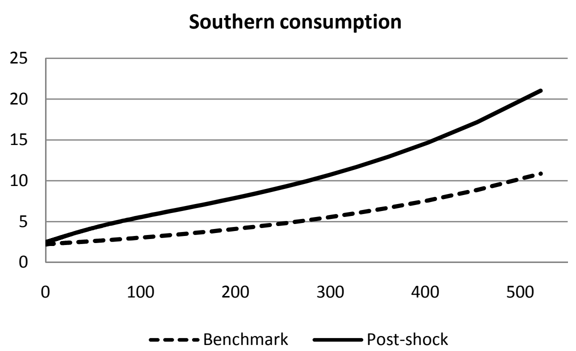

Figure X: Rise in the Northern TFP

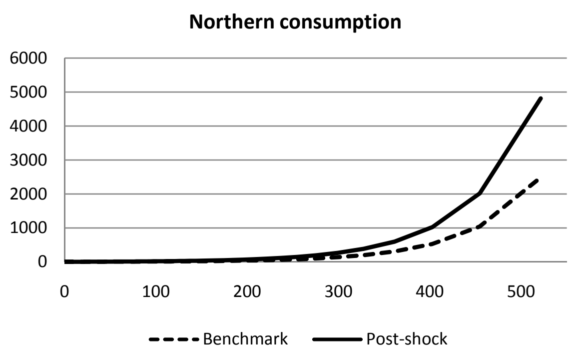

Figure XI: Rise in the Northern TFP

Southern TFP level rises

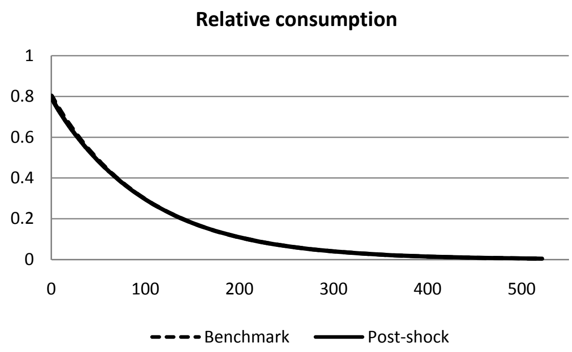

Figure XII: Rise in the Southern TFP

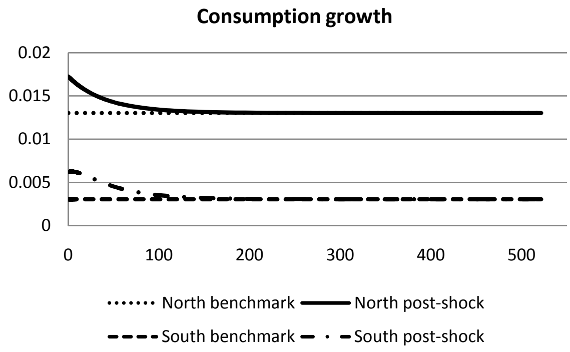

Figure XIII: Rise in the Southern TFP

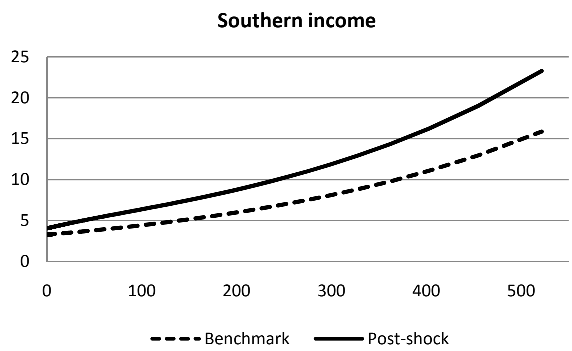

Figure XIV: Rise in the Southern TFP

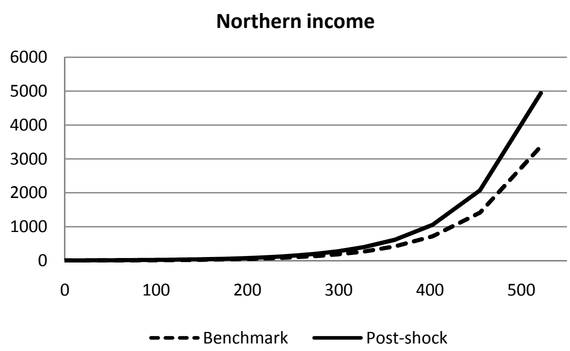

Figure XV: Rise in the Southern TFP

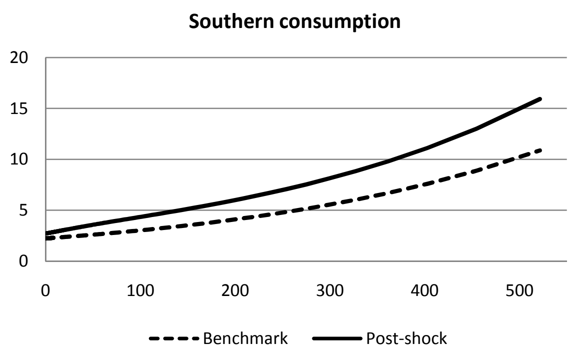

Figure XVI: Rise in the Southern TFP

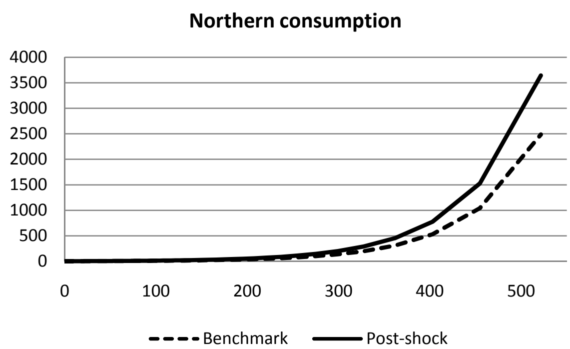

Figure XVII: Rise in the Southern TFP

A-10: Tansitional dynamics for Case 3

South starts with capital half of its steady-state value

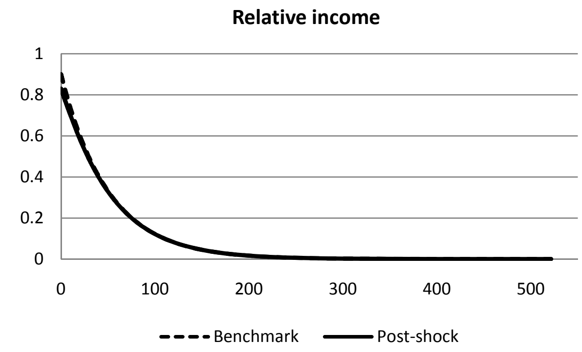

Figure XVIII: South starts at a 50% steady state

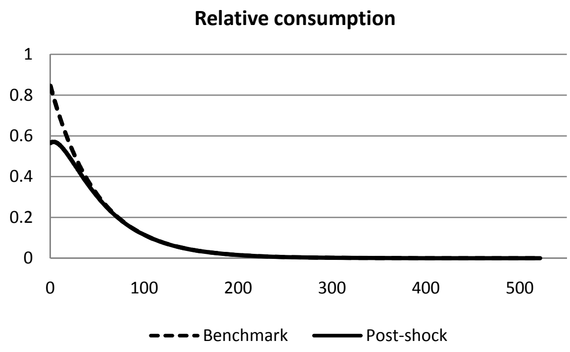

Figure XIX: South starts at a 50% steady state

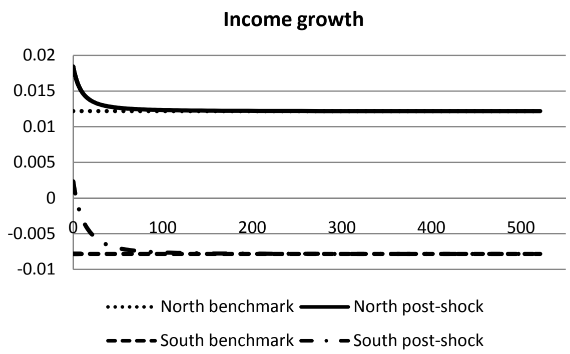

Figure XX: South starts at a 50% steady state

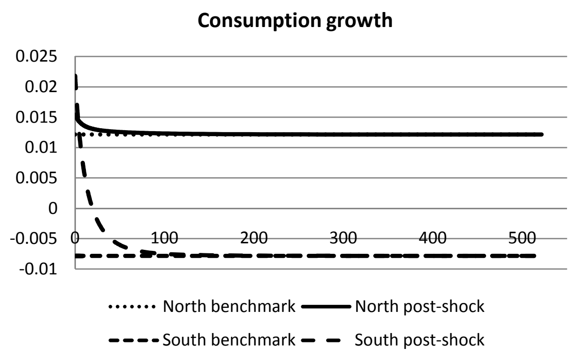

Figure XXI: South starts at a 50% steady state

Both n_N and n_S decrease by tenth

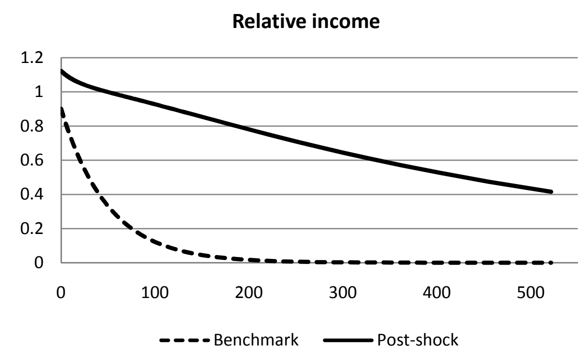

Figure XXII: Fall in the population growth

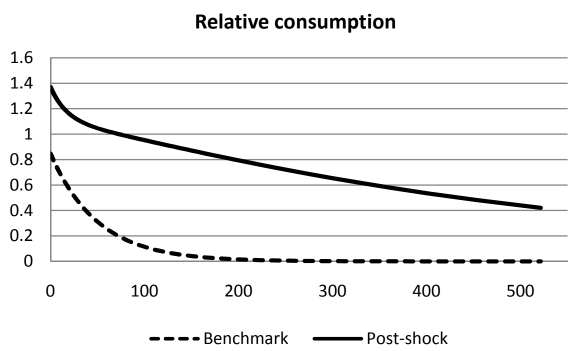

Figure XXIII: Fall in the population growth

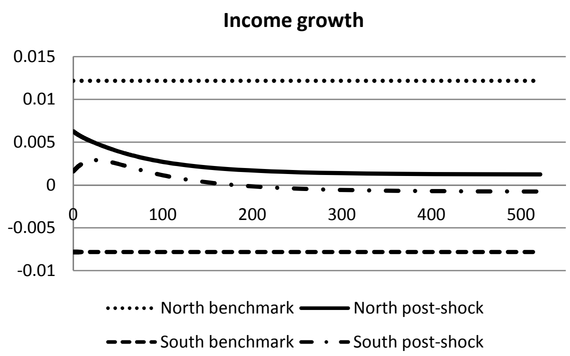

Figure XXIV: Fall in the population growth

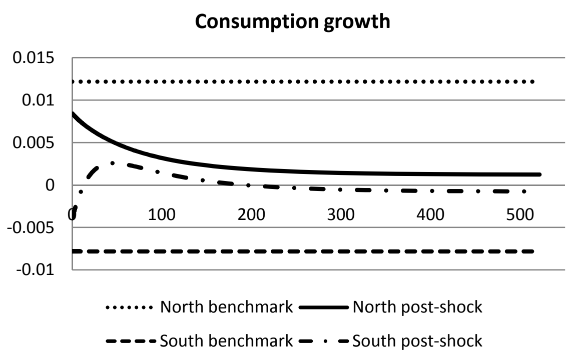

Figure XXV: Fall in the population growth

Northern TFP level rises

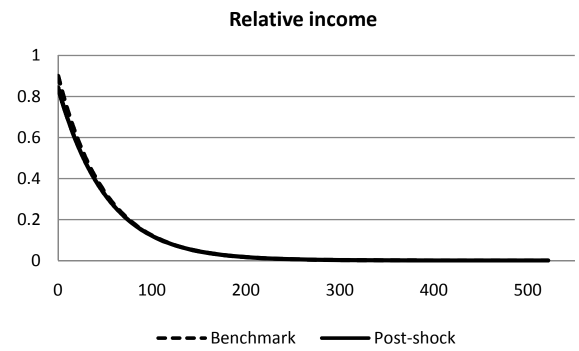

Figure XXVI: Rise in the Northern TFP

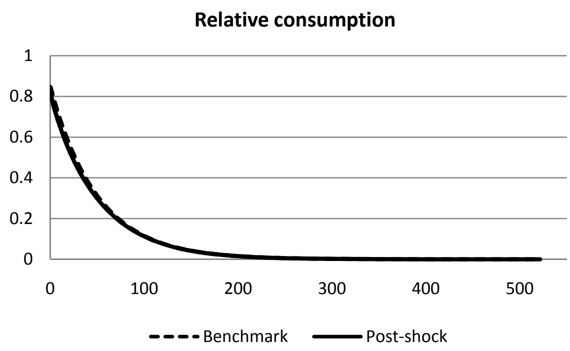

Figure XXVII: Rise in the Northern TFP

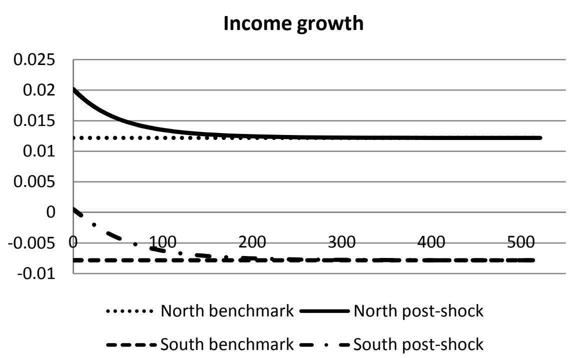

Figure XXVIII: Rise in the Northern TFP

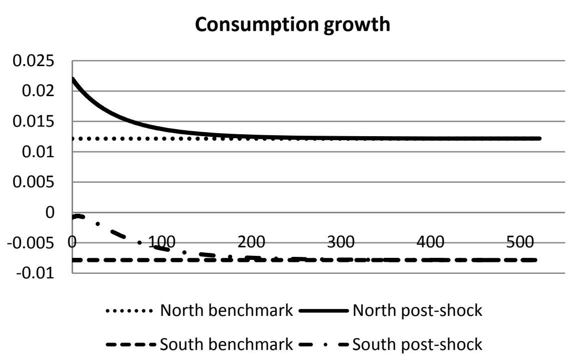

Figure XXIX: Rise in the Northern TFP

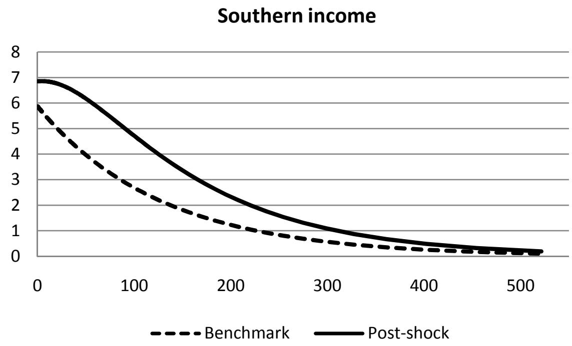

Figure XXX: Rise in the Northern TFP

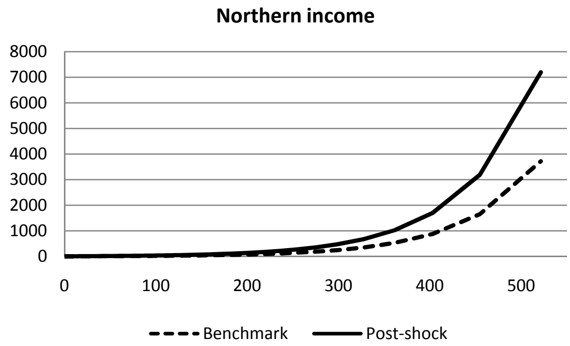

Figure XXXI: Rise in the Northern TFP

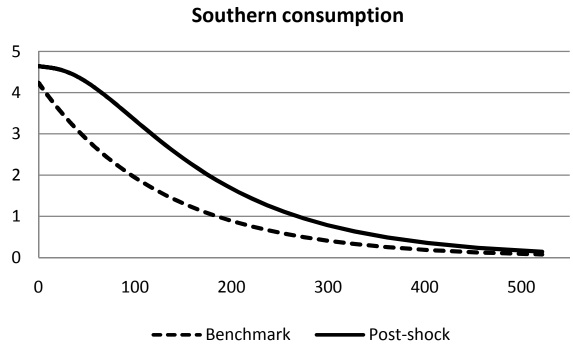

Figure XXXII: Rise in the Northern TFP

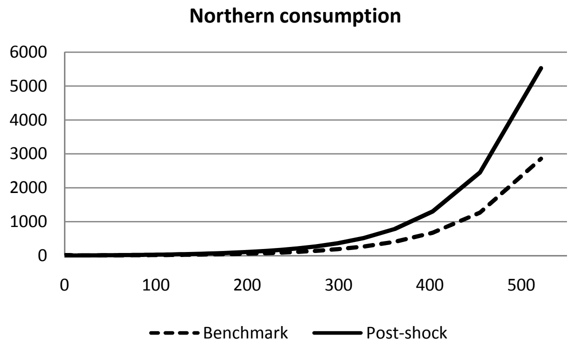

Figure XXXIII: Rise in the Northern TFP

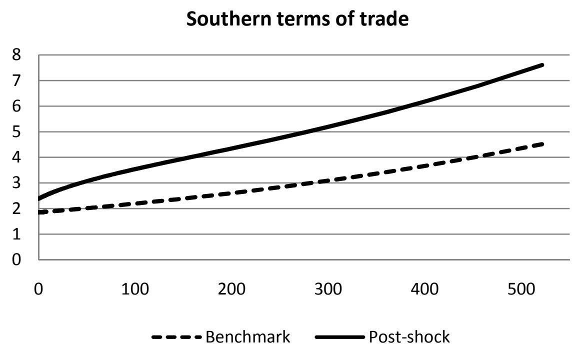

Figure XXXIV: Rise in the Northern TFP

Southern TFP level rises

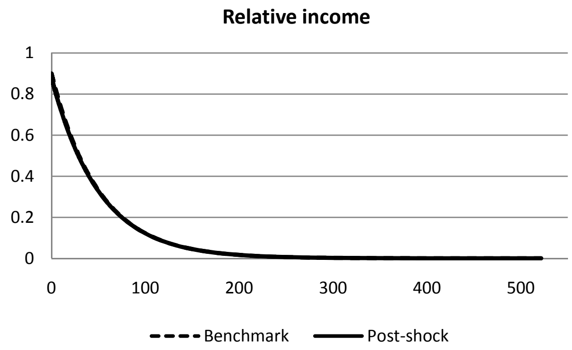

Figure XXXV: Rise in the Southern TFP

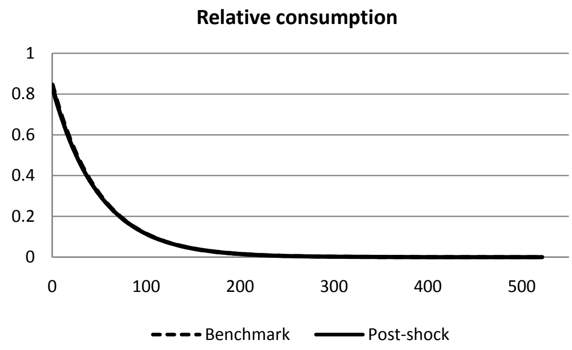

Figure XXXVI: Rise in the Southern TFP

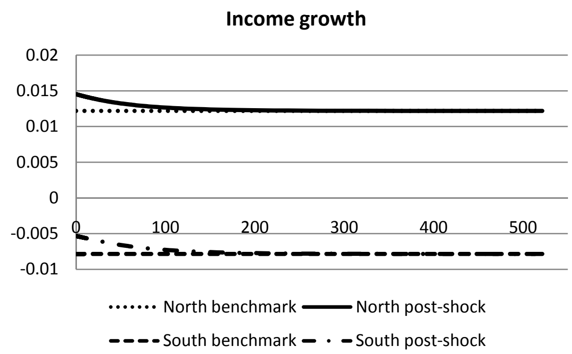

Figure XXXVII: Rise in the Southern TFP

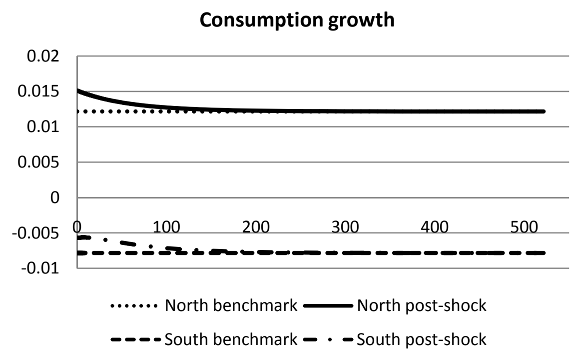

Figure XXXVIII: Rise in the Southern TFP

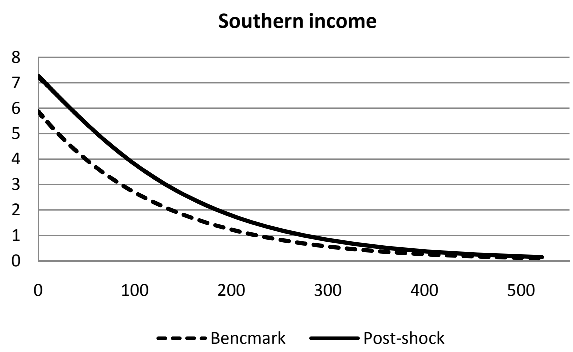

Figure XXXIX: Rise in the Southern TFP

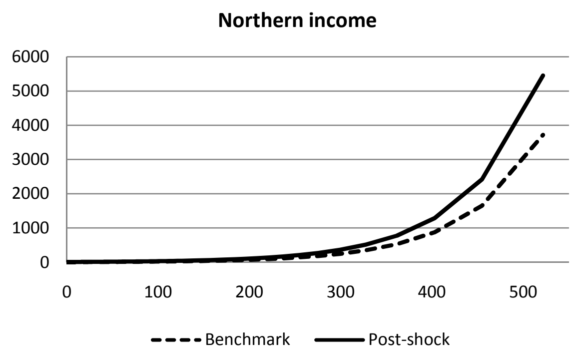

Figure XL: Rise in the Southern TFP

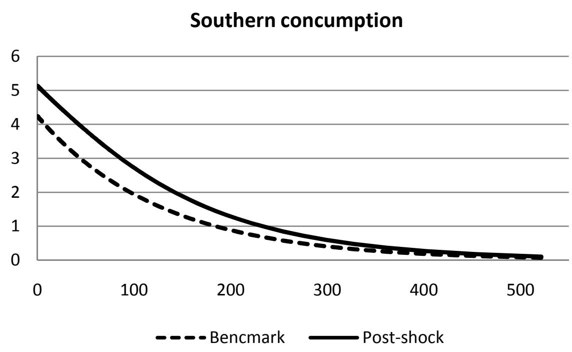

Figure XLI: Rise in the Southern TFP

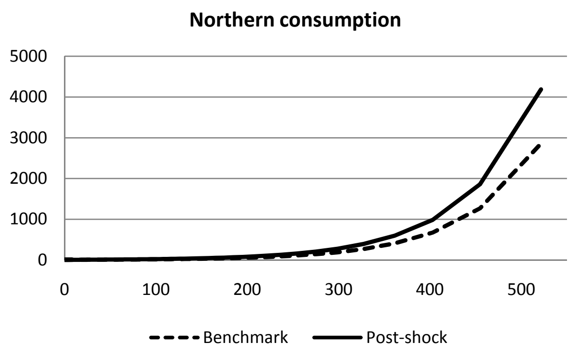

Figure XLII: Rise in the Southern TFP

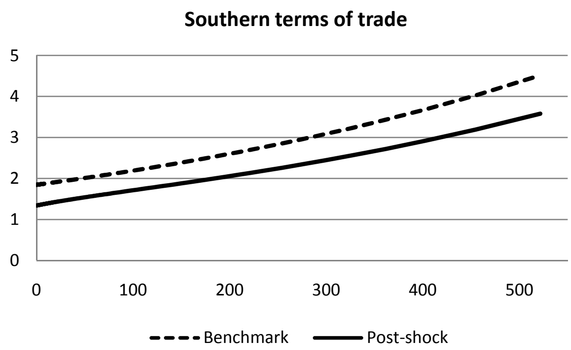

Figure XLIII: Rise in the Southern TFP

A-11: BGP values of scale-adjusted variables

Table B: BGP values of scale-adjusted variables for Cases 1 and 3

		Benchmark	$n_N, n_S \downarrow$	$A_N \uparrow$	$A_S \uparrow$
k_N^*	Case 1	60.0915	152.726	143.269	80.2773
	Case 3	165.708	428.238	395.078	221.372
k_S^*	Case 1	62.3989	166.711	148.77	83.3597
	Case 3	179.078	328.564	426.954	239.234
c_N^{N*}	Case 1	3.76061	8.37809	8.96453	5.02306
	Case 3	7.52789	15.8203	17.9478	10.0566
c_S^{N*}	Case 1	3.02189	9.80976	7.20471	4.03699
	Case 3	6.37007	18.9112	15.1874	8.50988
π^*	Case 1	1.40754	2.19213	2.37064	1.11645
	Case 3	1.8467	3.17795	3.11027	1.46478