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Welfare Implications and Equilibrium Indeterminacy in a Two-sector Growth Model with Consumption Externalities*

Been-Lon Chen† Yu-Shan Hsu‡ and Kazuo Mino§

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Abstract

One-sector neoclassical growth models reveal that consumption externalities lead to inefficient allocation in a steady state and indeterminate equilibrium toward the steady state only if there is a labor-leisure tradeoff. This paper shows that in a two-sector neoclassical growth model, even without a labor-leisure tradeoff, consumption spillovers easily lead to inefficient allocation in a steady state and indeterminate equilibrium toward the steady state. Consumption spillovers that yield over-accumulation of capital in an otherwise identical one-sector model may lead to under-accumulation of capital in two-sector models depending on relative capital intensities and relative degrees of externalities. Moreover, a two-sector model economy with consumption externalities is less stabilized than an otherwise identical one-sector model economy with consumption externalities.

Keywords: two-sector model; consumption externalities; efficiency; indeterminacy

JEL Classification: E21; E32; O41

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† Correspondence author, Institute of Economics, Academia Sinica, 128 Academia Road, Section 2, Taipei 11529, Taiwan, bchen@econ.sinica.edu.tw.

‡ Department of Economics, National Chung Cheng University, , ecdysh@ccu.edu.tw.

§ Institute of Economic, Research, Kyoto University, mino@kier.kyoto-u.ac.jp.

1. Introduction

In this article, we analyze welfare properties and dynamic properties of a deterministic two-sector neoclassical growth model where average consumption level of the economy affects individuals' felicity. This departure from standard specifications of preferences has been introduced in several growth models in order to account for some empirical phenomena that cannot be explained under more traditional forms of the utility function. These spillovers from the others' consumption could either increase or decrease an individual's utility, and if spillovers from the others' consumption increase the marginal utility of individual consumption, preferences display the typical "keeping up with the Joneses" effect (hereafter, *KUJ* effect) because others' consumption makes more valuable a marginal increase of his/her own consumption (see Gali, 1994).

In one-sector neoclassical growth models with constant time preference rates, only if there is a labor-leisure tradeoff, consumption externalities can lead to inefficient allocation in a steady state and generate indeterminate dynamic equilibrium paths toward the steady state. The reason why the labor-leisure tradeoff plays such a role may be explained by using a negative consumption externality as an example. The negative spillover leads to over-consumption that is associated with inefficiently less leisure time and more labor supply, through changes in the marginal rate of substitution between consumption and leisure. As a higher labor supply increases the marginal product of capital, the level of capital is greater than the efficient level in a steady state (Liu and Turnovsky, 2005). Moreover, if the negative spillover makes the Frisch labor supply to have certain shape, the dynamic equilibrium toward a steady state is indeterminate (Alonso-Carrera *et al.*, 2008).

This paper argues that in a two-sector neoclassical growth model, even without a labor-leisure tradeoff, consumption spillovers can yield not only inefficient allocation in a steady state but also indeterminate dynamic equilibrium paths toward the steady state. We show these results in a two-sector model wherein one sector produces general goods that are used as consumption and investment and the other sector produces only consumption goods. When the consumption goods sector does not use capital, the goods are leisure and our model is reduced to a one-sector model with elastic leisure that was studied by Liu and Turnovsky (2005) and Alonso-Carrera *et al.* (2008). The reasons to have these results lie in the relative prices of these two goods and the factor allocation in a two sectors as follows. In a producer's optimization, the price of general goods relative to consumption goods equals the marginal rate of transformation (hereafter, *MRT*), the ratio of the marginal product in the consumption sector to the marginal product in the general goods sector. In a consumer's optimization, the relative price of general goods equals the marginal rate of substitution (hereafter, *MRS*), the ratio of the marginal utility of general goods to that of consumption goods.

First, to see how inefficiency emerges, we consider as an example a negative externality of general goods consumption. The negative externality leads to a lower individual marginal utility of general goods consumption, so the *MRS* is lower than the *MRS* wherein there is no external effect. A lower *MRS* indicates a lower relative price of general goods than the economy without an external effect. A lower relative price of general goods releases input factors from the general goods sector to the consumption goods sector, so there is less general goods production than the efficient level when there is no external effect. The factor reallocation decreases the marginal product in the consumption goods sector, increases the marginal product in the general goods sector, and thus makes the *MRT* lower than the efficient level. In a steady state, the lower *MRS* is equal to the lower *MRT* and is also equal to the lower relative price, so output, capital and factor allocation all are different from the efficient level.

Next, we explain the indeterminacy in terms of the *KUJ* effect of general goods consumption. Indeterminate equilibrium emerges if self-fulfilling expectations can be supported as equilibrium. Suppose that the representative agent expects an increase in the price of general goods. The agent will reallocate input factors from the consumption goods sector to the general goods sector which increases the *MRT*. Yet, more general goods production increases general goods consumption. If general goods consumption has no *KUJ* effect, more general goods consumption lowers the *MRS* so the *MRS* is smaller than the higher *MRT*. Then, self-fulfilling expectations of a higher price of general goods cannot be supported as equilibrium. However, when general goods consumption has a *KUJ* effect, then more general goods consumption can increase rather than decrease the *MRS*. Hence, this higher *MRS* is equal to the higher *MRT*, and is also equal to the higher relative price of general goods. As a result, self-fulfilling expectations about higher relative prices of general goods can be supported as equilibrium.

We should note that in one-sector growth models, when production externalities establish indeterminacy, it requires that labor supply is elastic and in particular the labor supply and demand curves cross with the "wrong slopes" (e.g., Benhabib and Farmer, 1994; Farmer and Guo, 1994). Moreover, when consumption externalities create indeterminacy in one-sector growth models, it still requires elastic labor supply so the externality can cause the Frisch labor supply to have certain shape, even though the labor supply needs not cross the labor demand with the "wrong slopes" (Alonso-Carrera *et al.*, 2008). In our two-sector model, general goods consumption externalities produce indeterminacy even when the labor supply is inelastic.

Our main findings are as follows. First, even with a negative spillover of general goods consumption, capital may be under- or over-accumulated depending on relative capital intensities and relative degrees of externalities between sectors. This result is different from the one-sector model studied by Liu and Turnovsky (2005) wherein a negative externality of general goods consumption leads to over-accumulation of capital. This finding has important welfare implications about optimal taxation. When general goods consumption has a negative spillover, an optimal capital tax in an otherwise identical one-sector growth model may change to an optimal capital subsidy in a two-sector growth model.

Next, when the general goods sector is more capital intensive, each of general goods consumption externalities and pure consumption externalities can easily establish indeterminacy, but it is easier for the externality from pure consumption to produce indeterminacy. This result is in contrast to existing one-sector models which finds that the leisure externality itself cannot create indeterminacy (e.g., Benhabib and Farmer, 2000; Weder, 2004). Further, if there are symmetric degrees of consumption externalities from both goods, the utility is homothetic and the competitive equilibrium is efficient in a steady state. However, we still find indeterminate equilibrium because symmetric externalities produce different shadow prices of capital between the market and the socially planned economy which cause market failures in transitions. The result is different from an otherwise identical one-sector growth model studied by Alonso-Carrera *et al.* (2008) wherein consumption externalities do not lead to indeterminacy because the utility is homothetic. Finally, no matter whether consumption externalities are from general goods, consumption goods or both, we find that it is much easier for a two-sector growth model to trigger indeterminacy than an otherwise identical one-sector growth model with elastic leisure.

We organize this paper as follows. We set a two-sector model with consumption externalities in Section 2. In Section 3, we study welfare properties. In Section 4, we investigate the dynamic properties. Finally, concluding remarks are found in Section 5; the appendix is in Section 6.

2. The Basic Model

The economy is populated by a representative firm and a representative household. There are two sectors: the general goods sector (y_1) and the consumption goods sector (y_2). The general goods sector produces goods that are used as consumption and investment and the consumption goods sector produces pure consumption goods only. We will also refer to general goods as goods

1 and to pure consumption goods as goods 2.[1] The representative firm hires labor and rents capital in order to produce goods in the two sectors. The representative household has a fixed supply of labor which is normalized to unity and chooses savings and consumption of both goods.

2.1 Technology

The production function is

$$y_i = f^i(k_i, l_i), \ \ i = 1, 2, \tag{1}$$

where k_i and l_i are capital and labor allocated to sector i.

We assume that the function f^i is twice continuously differentiable and is homogenous of degree one with respect to both inputs. Moreover, the function is strictly increasing and strictly concave in inputs and satisfies the Inada condition. Our basic assumption is that sector 1 is more capital intensive than sector 2, but we also consider the opposite case.

2.2 Preference

The representative household supplies all its labor to work and there is thus no leisure activity. The household's utility is affected not only by own consumption but also by average consumption in the society. Let ρ>0 denote the time preference rate, c_i denote own consumption of goods i=1, 2 and $\overline{c}_i$ denote average consumption of goods i. The agent's lifetime utility is represented by

$$U = \int_0^\infty e^{-\rho t} u(c_1, c_2, \overline{c}_1, \overline{c}_2) dt. \tag{2}$$

We assume that the instantaneous utility function is twice continuously differentiable and is strictly increasing and strictly concave in c_1 and c_2. The effect of $\overline{c}_i$ may be positive or negative. It is said that the household is "*admiring*" in good c_i if $\partial u / \partial \overline{c}_i > 0$ and "*jealous*" in good c_i if $\partial u / \partial \overline{c}_i < 0$ (e.g., Dupor and Liu, 2003; Liu and Turnovsky, 2005). Moreover, the consumption activity is described as "*KUJ*" if $\partial^2 u / (\partial c_i \partial \overline{c})_i > 0$ (e.g., Gali, 1994; Ljungqvist and Uhlig, 2000).

2.3 Resource constraints and markets

The resource constraints in the economy at a point in time are given by

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[1] Our two-sector model is a variant of those in Whelan (1993), Rogerson (2008) and Durate and Restuccia (2010). Goods 1 may be thought of as manufacture goods and goods 2 as services goods. Alternatively, goods 2 may be interpreted as home goods as in Greenwood and Hercowitz (1991).

$$k = k_1 + k_2 = sk + (1-s)k, \tag{3a}$$

$$1 = l_1 + l_2 = l + (1-l), \tag{3b}$$

With a fixed labor supply normalized at unity, y_i is output per capita produced in sector i. Notation k is total capital (per capita) in the economy at a point in time and k_1 is capital allocated to sector 1 which accounts for a fraction $s \in (0,1)$ of total capital in the economy. The remaining fraction of total capital 1-s goes to sector 2. The fraction of labor allocated to sector 1 is $l \in (0,1)$ with the remaining fraction going to sector 2.

Finally, the goods market clearance conditions in the economy are

$$\dot{k} = f^1(sk, l) - c_1 - \delta k, \tag{4a}$$

$$c_2 = f^2((1-s)k, 1-l), \tag{4b}$$

where δ is the rate of depreciation of capital.

Note that when sector 2 does not use capital, then goods 2 provides only leisure services. In this case, our model is reduced to a one-sector neoclassical growth model with leisure and leisure externalities. If leisure exhibits no externalities, this is the model studied by Liu and Turnovsk (2005) and Alonso-Carrera *et al.* (2008).

3 Inefficiency of Allocation in a Steady State

3.1 Allocation in a Decentralized Economy and in a Socially Planned Economy

In a competitive market economy, the representative agent takes $\overline{c}_i$ as given by the society. By substituting (4b) into (2), the representative agent's problem is to choose c_1, s, l and k in order to maximize (2) subject to (4a). Denote by λ the shadow price of capital and by p the price of general goods relative pure consumption goods. Then, in addition to the transversality condition, the optimal conditions for c_1, s, l and k are

$$u_1(c_1, c_2, \overline{c}_1, \overline{c}_2) = \lambda, \tag{5a}$$

$$\frac{\lambda}{u_2(c_1, c_2, \overline{c}_1, \overline{c}_2)} = \frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} = p, \tag{5b}$$

$$\frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} = \frac{f_1^2((1-s)k, 1-l)}{f_1^1(sk, l)}, \tag{5c}$$

$$\frac{\dot{\lambda}}{\lambda} = (\rho + \delta) - f_1^1(sk, l). \tag{5d}$$

In these optimal conditions, (5a) equates the marginal utility of general goods to the shadow price of capital, which determines general goods consumption. In (5b), the *MRS* between two goods is equal to the *MRT* between two sectors and is also equal to the relative price of general goods. In (5c), the *MRT* between the two sectors must equal for both capital and labor. Finally, (5d) is the Euler equation that governs how the shadow price of capital changes over time.

In a socially planned economy, the planner internalizes the consumption externality $\overline{c}_i$. Denote by λ^p the shadow price of capital in the planner's problem. Then, in addition to the transversality condition, the optimal conditions for c_1, s, l and k give

$$u_1(c_1, c_2, \overline{c}_1, \overline{c}_2) + u_3(c_1, c_2, \overline{c}_1, \overline{c}_2) = \lambda^p, \tag{6a}$$

$$\frac{\lambda^p}{u_2(c_1, c_2, \overline{c}_1, \overline{c}_2) + u_4(c_1, c_2, \overline{c}_1, \overline{c}_2)} = \frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)}, \tag{6b}$$

$$\frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} = \frac{f_1^2((1-s)k, 1-l)}{f_1^1(sk, l)}, \tag{6c}$$

$$\frac{\dot{\lambda}^p}{\lambda^p} = (\rho + \delta) - f_1^1(sk, l). \tag{6d}$$

3.2 Efficiency of the Allocation in a Decentralized Economy

The equilibrium conditions in a competitive market include (4a)-(4b) and (5a)-(5d) along with $\overline{c}_i = c_i$. The efficient allocation conditions in the socially planned economy are (4a)-(4b) and (6a)-(6d) with $\overline{c}_i = c_i$. In a steady state when $\dot{k} = \dot{\lambda} = \dot{\lambda}^p = 0$, these two sets of conditions are the same except for (5a) and (5b) in the decentralized economy and (6a) and (6b) in the socially planned economy. Combining (5a) and (5b) gives

$$MRS \equiv \frac{u_1(c_1, c_2, \overline{c}_1, \overline{c}_2)}{u_2(c_1, c_2, \overline{c}_1, \overline{c}_2)} = \frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} \equiv MRT, \tag{7a}$$

and (6a) and (6b) yields

$$MRS^p \equiv \frac{u_1(c_1, c_2, \overline{c}_1, \overline{c}_2) + u_3(c_1, c_2, \overline{c}_1, \overline{c}_2)}{u_2(c_1, c_2, \overline{c}_1, \overline{c}_2) + u_4(c_1, c_2, \overline{c}_1, \overline{c}_2)} = \frac{f_2^2((1-s)k, 1-l)}{f_2^1(sk, l)} \equiv MRT. \tag{7b}$$

Since (7a) and (7b) have the same *MRT*, the allocation in the decentralized economy is the same as the allocation in the socially planned economy if the *MRS* in (7a) is the same as the MRS^p in (7b). We obtain the following result.

Proposition 1. *In a neoclassical growth model with general goods and pure consumption goods, the competitive equilibrium allocation is efficient in a steady state if and only if*

$$\frac{u_1(c_1,c_2,\overline{c}_1,\overline{c}_2)}{u_2(c_1,c_2,\overline{c}_1,\overline{c}_2)}=\frac{u_3(c_1,c_2,\overline{c}_1,\overline{c}_2)}{u_4(c_1,c_2,\overline{c}_1,\overline{c}_2)}. \tag{8}$$

for all feasible c_1 *and* c_2, *where* $\overline{c}_1 = c_1$ *and* $\overline{c}_2 = c_2$.

Condition (8) stipulates that the *MRS* between one's *own* consumption of goods 1 and 2 must be equal to the *MRS* between the *social* consumption of goods 1 and 2. Only when this condition is met, the allocation of consumption, labor and capital in a competitive market is efficient in the long run; otherwise, the allocation is inefficient.

A typical specification of the utility function that satisfies condition (8) is a function that is multiplicatively separable between (c_1, c_2) and $(\overline{c}_1,\overline{c}_2)$ such that

$$u(c_1,c_2,\overline{c}_1,\overline{c}_2)=v(c_1,c_2)h(v(\overline{c}_1,\overline{c}_2)), \tag{9a}$$

where the $h(.)$ is a monotonically increasing or decreasing function.

Another functional form satisfying (8) is that $u(.)$ is weakly separable between $(c_1,\overline{c}_1)$ and $(c_2,\overline{c}_2)$ such that

$$u(c_1,c_2,\overline{c}_1,\overline{c}_2)=V(h(c_1,\overline{c}_1)h(c_2,\overline{c}_2)), \tag{9b}$$

where $h(.)$ is a homothetic function. In this case, (8) is written as $\frac{h_1(c_1,\overline{c}_1)}{h_1(c_2,\overline{c}_2)}=\frac{h_2(c_1,\overline{c}_1)}{h_2(c_2,\overline{c}_2)}$. Since $h(.)$ is homothetic, we obtain $h_1(\frac{c_1}{\overline{c}_1},1)h_2(\frac{c_2}{\overline{c}_2},1)=h_1(\frac{c_2}{\overline{c}_2},1)h_2(\frac{c_1}{\overline{c}_1},1)$, which always holds when $\overline{c}_i = c_i$.

When condition (8) fails to hold, the competitive equilibrium allocation is inefficient in a steady state. Then, in a two-sector growth model, the consumption externality can lead to inefficient allocation even though there is no leisure choice. This result is different from that in the one-sector neoclassical growth model studied by Liu and Turnovsky (2005). In a one-sector growth model the consumption externality produces inefficient allocation in a steady state only when there is a labor-leisure tradeoff. Moreover, in a one-sector growth model, a jealous (i.e., negative) consumption externality leads to over-accumulation of capital and an admiring (i.e., positive) consumption externality results in under-accumulation of capital. In our two-sector growth model, however, it is not the case. As we will show below by using a parametric version, a jealous (or admiring) general goods consumption externality may cause over- or under-accumulation of capital depending on relative capital intensities.

Consider a parametric version of our model with the following constant elasticity of substitution (CES) utility function and Cobb-Douglas production function

$$u(c_1, c_2, \overline{c}_1, \overline{c}_2) = [\gamma(\tfrac{c_1}{\overline{c}_1^{\theta_1}})^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(\tfrac{c_2}{\overline{c}_2^{\theta_2}})^{\frac{\varepsilon-1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}, \quad 0<\gamma\leq 1, \tag{10a}$$

$$f^i(k_i, l_i) = A_i k_i^{\alpha_i} l_i^{1-\alpha_i}, A_i > 0, \ 0 \leq \alpha_i < 1, \ i = 1,2, \tag{10b}$$

where α_i is the capital intensity in sector i. Parameter $\gamma>0$ is the share of goods 1 relative to goods 2 in utility, and θ_i measures the degree of the external consumption effect arising from $\overline{c}_i$. A negative (i.e., jealous) consumption externality emerges if $\theta_i>0$ while a positive (i.e., admiring) consumption externality occurs if $\theta_i<0$. Parameter $\varepsilon>0$ is the elasticity of substitution (hereafter, *ES*) between two goods. Consumption i displays a *KUJ* effect if $\theta_i(\varepsilon-1)<0$.

Several special cases are in order.

1. If $\gamma=1$, goods 2 is not demanded and our model is reduced to the standard one-sector growth model with inelastic labor supply. Then, the competitive equilibrium allocation is efficient in the long run.

2. If $\varepsilon=1$ and $\theta_1=\theta_2=\theta$, (10a) becomes $u = \log[(c_1^{\gamma} c_2^{1-\gamma})(\overline{c}_1^{\gamma}\overline{c}_2^{1-\gamma})^{-\theta}]$, which satisfies (9a). Thus, (8) is satisfied and the competitive equilibrium allocation is efficient in the long run.

3. If $\varepsilon\neq1$ and $\theta_1=\theta_2=\theta$, then it is easy to see that (10a) is homothetic and satisfies (9b). Then, (8) is met and the competitive equilibrium allocation is efficient in the long run.

4. If $\varepsilon\neq1$ and $\theta_1\neq\theta_2$, the degree of externalities is asymmetric. Then, (8) fails to hold and the competitive equilibrium allocation is inefficient in a steady state. Condition (8) fails to satisfy since

$$\frac{u_3(.)}{u_4(.)} = \frac{\theta_1}{\theta_2}\frac{u_1(.)}{u_2(.)} \neq \frac{u_1(.)}{u_2(.)} = \frac{\gamma}{1-\gamma}\left[\frac{(c_1)^{-[1+\theta_1(\varepsilon-1)]}}{(c_2)^{-[1+\theta_2(\varepsilon-1)]}}\right]^{\frac{1}{\varepsilon}} \quad if\ \varepsilon \neq 1, \ \theta_1 \neq \theta_2.$$

The following proposition characterizes the steady-state competitive equilibrium allocation for the parametric version of our model.

Proposition 2. *Suppose* $\varepsilon\neq1$. *Then, in a steady state, capital is over-accumulated if (i)* $\alpha_1>\alpha_2$ *and* $\theta_1>\theta_2$ *or (ii)* $\alpha_1<\alpha_2$ *and* $\theta_1<\theta_2$ *and under-accumulated if (i)* $\alpha_1>\alpha_2$ *and* $\theta_1<\theta_2$ *or (ii)* $\alpha_1<\alpha_2$ *and* $\theta_1>\theta_2$.

To see the properties in proposition 2, we rewrite (3a) as

$$lx_1 + (1-l)x_2 = k, \tag{11a}$$

where $x_1 = \frac{k_1}{l_1} = \frac{sk}{l}$ and $x_2 = \frac{k_2}{l_2} = \frac{(1-s)k}{1-l}$ are the capital intensity in sector 1 and 2, respectively. Then, the production function in (1) becomes

$$y_1 = lg_1(x_1),\ \ y_2 = (1-l)g_2(x_2),$$

where $g_i(x_i) \equiv f^i(x_i,1)$. Then, the relative price satisfies $p = \frac{f_1^2(.)}{f_1^1(.)} = \frac{g_2'(x_2)}{g_1'(x_1)}$ and (5c) becomes

$$\frac{g_2(x_2) - x_2 g_2'(x_2)}{g_2'(x_2)} = \frac{g_1(x_1) - x_1 g_1'(x_1)}{g_1'(x_1)}. \tag{11b}$$

In a steady state, $\rho + \delta = f_1^1(x_1^*,1) = g_1'(x_1^*)$, and, in view of (11b), the steady-state levels of capital intensity, x_1^* and x_2^*, are uniquely determined and are independent of the presence of consumption externalities. The relative price p^* is thus uniquely determined.

From (11a), we obtain $l^* = \frac{k-x_2^*}{x_1^*-x_2^*}$ and $1-l^* = \frac{x_1^*-k}{x_1^*-x_2^*}$ in a steady state. Thus, the steady-state output in each sector is given by $y_1 = \frac{k-x_2^*}{x_1^*-x_2^*} g_1(x_1^*)$ and $y_2 = \frac{x_1^*-k}{x_1^*-x_2^*} g_2(x_2^*)$. Since $c_1 = y_1 - \delta k$ in a steady state, the steady-state expression of (7a) in terms of the utility (10a) is

$$MRS \equiv \frac{u_1(.)}{u_2(.)} = \frac{\gamma}{1\text{-}\gamma}\left[\frac{\left(\frac{x_1^*-k}{x_1^*-x_2^*} g_2(x_2^*)\right)^{1+\theta_2(\varepsilon\text{-}1)}}{\left(\frac{k-x_2^*}{x_1^*-x_2^*} g_1(x_1^*) - \delta k\right)^{1+\theta_1(\varepsilon\text{-}1)}}\right]^{\frac{1}{\varepsilon}} = \frac{g_2'(x_2^*)}{g_1'(x_1^*)} \equiv MRT, \tag{11c}$$

which, under unique values of x_1^* and x_2^*, gives a unique competitive equilibrium value of capital, k^*, in a steady state. When the external effects are internalized, (7b) is expressed as

$$MRS^p \equiv \frac{u_1(.)+u_3(.)}{u_2(.)+u_4(.)} = \frac{\gamma}{1\text{-}\gamma}\left(\frac{1-\theta_1}{1-\theta_2}\right)\left[\frac{\left(\frac{x_1^*-k}{x_1^*-x_2^*} g_2(x_2^*)\right)^{1+\theta_2(\varepsilon\text{-}1)}}{\left(\frac{k-x_2^*}{x_1^*-x_2^*} g_1(x_1^*) - \delta k\right)^{1+\theta_1(\varepsilon\text{-}1)}}\right]^{\frac{1}{\varepsilon}} = \frac{g_2'(x_2^*)}{g_1'(x_1^*)} \equiv MRT, \tag{11d}$$

which uniquely determines a social optimum level of capital, k^p, in a steady state.

If $\theta_1 = \theta_2 = \theta$, (11c) and (11d) yield the same level of capital that coincides with the level of capital stock in the absence of consumption externalities. Alternatively, if $\theta_1 \neq \theta_2$ and $\varepsilon \neq 1$, (11c) gives a level of capital in the competitive equilibrium (k^*) that is different from that in the social optimum (k^p) in (11d).

We now use Figure 1 to illustrate the steady-state levels of capital in the competitive equilibrium (k^*) and in the social optimum (k^p). Note that the *MRT* of (11c) and (11d) is the same and is independent of the value of capital. The *MRS* of (11c) and the MRS^p of (11d) depend on the value of capital. First, if the general goods sector is more capital intensive than the consumption good sector (i.e., $x_1 > x_2$), then the graphs of MRS and MRS^p are negatively sloping, as illustrated in Diagrams A1 and A2 of Figure 1. It is clear that $k^* > k^p$ in Diagram A1 where $\theta_1 > \theta_2$ while $k^* < k^p$ in Diagram A2 where $\theta_1 < \theta_2$. Next, if the general goods sector is less capital intensive than the consumption goods sector (i.e., $x_1 < x_2$), then the graphs of MRS and MRS^p are positively sloping, as illustrated in Diagrams B1 and B2 of Figure 1. Then, $k^* < k^p$ when $\theta_1 > \theta_2$ (Diagram B1) whereas $k^* > k^p$ when $\theta_1 < \theta_2$ (Diagram B2).

[Insert Figure 1 about here]

The factor intensity ranking of the Cobb-Douglas production function in (10b) satisfies

$$\text{sign } (x_1 - x_2) = \text{sign } (\alpha_1 \text{-} \alpha_2).$$

We thus obtain

$$k^* > k^p \text{ either if } \alpha_1 > \alpha_2 \text{ and } \theta_1 > \theta_2 \text{ or if } \alpha_1 < \alpha_2 \text{ and } \theta_1 < \theta_2;$$
$$k^* < k^p \text{ either if } \alpha_1 > \alpha_2 \text{ and } \theta_1 < \theta_2 \text{ or if } \alpha_1 < \alpha_2 \text{ and } \theta_1 > \theta_2.$$

Therefore, we establish proposition 2.

Note that under $\alpha_2=0$, $c_2=(1\text{-}l)$ is leisure. If $\theta_2=0$, then our model is reduced to the one-sector growth model with elastic leisure studied by Liu and Turnovsky (2005). In this case, if the consumption of general goods is jealousy (i.e., $\theta_1>0$), capital is over-accumulated in a one-sector growth model. However, if $\alpha_2>0$, then c_2 is pure consumption goods. Suppose c_2 has no consumption externalities ($\theta_2=0$). Then, proposition 2 implies that a jealousy effect of general goods consumption (thus, $\theta_1>\theta_2=0$) may lead to over-accumulation or under-accumulation of capital. When the general goods are more capital intensive ($\alpha_1>\alpha_2$), capital is over accumulated. However, when the general goods are less capital intensive ($\alpha_1<\alpha_2$), capital is under accumulated.

Our above finding has important welfare implications on optimal taxation. In the case of a one-sector growth model with elastic leisure, a jealousy effect leads to over-accumulation of capital and thus the capital taxation can replicate the competitive market allocation to the efficient allocation. In a two-sector growth model with general goods and pure consumption goods, if the pure consumption goods sector is more capital intensive than the general goods sector, a jealousy effect in general goods consumption leads to under-accumulation of capital. Then, it is the capital subsidy rather than the capital taxation that can replicate the competitive market allocation to the optimal allocation.

4. Indeterminacy in Transitional Dynamics

Consumption externalities cause not only an inefficient allocation in a steady state, but they also generate indeterminate equilibrium paths toward a steady state. This section analyzes the conditions of indeterminacy.

4.1 Conditions of Indeterminacy

Dynamic equilibrium conditions in the competitive market are summarized by (4a)-(4b) and (5a)-(5d) with six variables: c_1, c_2, s, l, λ and k. Different from a one-sector model, our model involves two goods and it is difficult to simplify these equations to a dynamical system with state vector $\{k, c_1\}$. We will simplify them to a system with state vector $\{k, \lambda\}$ as follows.[2]

First, with the help of (4b) and (5a)-(5c), we use (5d) to obtain the Keynes-Ramsey condition.

$$\dot{\lambda} = \lambda[(\rho+\delta) - f_1^1(s(\lambda,k)k, l(\lambda,k))] \equiv J_1(\lambda,k). \tag{12a}$$

Next, with (4b) and (5a)-(5c), we rewrite the general goods market clearance condition (4a).

$$\dot{k} = f^1(s(\lambda,k)k, l(\lambda,k)) - c_1(\lambda,,k) - \delta k \equiv J_2(\lambda,k). \tag{12b}$$

Equations (12a) and (12b) constitute the simplified dynamical system. The steady state is determined by $\dot{k}=0$ and $\dot{\lambda}=0$. The dynamic property is analyzed if we take Taylor's linear expansion of (12a) and (12b) around the steady state (k, λ). The expansion gives

$$\begin{bmatrix} \dot{\lambda} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \lambda(t)-\lambda \\ k(t)-k \end{bmatrix}. \tag{13}$$

The dynamical system (13) includes a state variable whose initial value is given at $k(0)$. There are two roots associated with the Jacobean matrix in (13), denoted by J. The steady state is a saddle if there is only one root with a negative real part and a sink if there are two roots with negative real parts. If the steady state is a sink, the equilibrium path is indeterminate.

Before analyzing dynamic properties, we investigate the existence and uniqueness of steady state in the case without consumption externalities (θ_1=θ_2=0). First, for the slope of the $\dot{k}^*=0$ locus in the (k, λ) plane,[3] we find J^*_{21}>0 and J^*_{22}>0 when k is small and J^*_{22}<0 when k is large.

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[2] The method follows from that found in Benhabib and Farmer (1996) which is a two-sector growth model with one consumption goods and one investment goods. In their model, sector-specific externalities in production are the mechanism leading to indeterminacy.

[3] In what follows, an asterisk superscript is used to denote the case of a two-sector growth model without

The sign of J^*_{21} is positive, since a higher λ (a higher shadow price of capital) attracts more capital and labor to the general goods sector and thus increases general goods production. It also reduces general goods consumption, due to an increased cost of general goods consumption relative to consumption goods. Moreover, for the sign of J^*_{22}, when k is small, because of a high marginal product of capital, larger capital increases more general goods production than general goods consumption. When k is very large, because of a diminishing marginal product of capital, general goods production is increased less than the increase in general goods consumption. Thus, $J^*_{22}>0$ when k is small and $J^*_{22}<0$ when k is large. As a result, the $\dot{k}^* = 0$ locus is downward sloping when k is smaller than a threshold and upward sloping when k is larger than the threshold, just like the $\dot{k}^* = 0$ locus in the standard one-sector growth model. See Figure 2. Moreover, it is easy to show that $\dot{k}^* = -c_1 < 0$ when k=0 and $\dot{k}^* = -\infty < 0$ when k=∞, implying that the locus $\dot{k}^* = 0$ approaches to λ=∞ in both ends of k=0 and k=∞.

[Insert Figure 2 about here]

Next, for the slope of the $\dot{\lambda}^* = 0$ locus, we find $J^*_{11}<0$ and $J^*_{12}>0$. For a given k, a higher λ decreases general goods consumption which increases the marginal utility of general goods consumption and leads to a higher *MRS* between c_1 and c_2 in (5b). In optimum, the marginal product of labor and capital in general goods relative to that in pure consumption goods in (5b) and (5c) needs to increase. Thus, $J^*_{11}<0$. Further, a larger capital has two effects. It decreases the marginal product of capital which directly increases the shadow price of capital. As sector 1 is more capital intensive than sector 2 under construction, the Rybczynski theorem stipulates that larger capital and labor shares are allocated to sector 1. A larger labor share increases the marginal product of capital which indirectly decreases the shadow price of capital. In general, the direct effect dominates the indirect effect and thus $J^*_{12}>0$. As a result, the $\dot{\lambda}^* = 0$ locus is upward sloping in the (λ, k) plane. See Diagrams A1 and A2 in Figure 2. Moreover, it is clear to see $\dot{\lambda}^* = -\infty < 0$ at $(k, \lambda)=(0,0)$ as the Inada condition implies an infinite marginal product of capital. Thus, the $\dot{\lambda}^* = 0$ locus will start from a finite value of k so that at λ=0, the marginal product of capital can equal the sum of the discount rate and the depreciation rate.

The shape of the two loci indicates that the $\dot{\lambda}^* = 0$ locus intersects the $\dot{k}^* = 0$ locus only

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consumption externalities.

once and there is a unique steady state (k, λ). The $\dot{\lambda}^* = 0$ locus may intersect the $\dot{k}^* = 0$ locus at the downward (Diagram A1) or upward (Diagram A2) sloping segment as illustrated in Figure 2.

In the two-sector growth model without consumption externalities, the steady state is a saddle. This indicates that there is only one root with a negative real part and the condition is $Det(J^*)=J^*_{11}J^*_{22}-J^*_{21}J^*_{12}<0$. Under $J^*_{21}>0$ and $J^*_{11}<0$, $Det(J^*)<0$ indicates $(-J^*_{12}/J^*_{11})>(-J^*_{22}/J^*_{21})$ >(or<)0. Thus, the positive slope of the $\dot{\lambda}^* = 0$ locus is larger than the slope of the $\dot{k}^* = 0$. See the steady states E_1 and in Diagrams A1 and A2, Figure 1, wherein the intersection is, respectively, at the negative and positive slope section of the $\dot{k}^* = 0$.

However, if consumption exhibits externalities, the steady state may be a sink. Consider the *KUJ* effect as follows.

Condition *KUJ*: $\partial^2 u / (\partial c_i \partial \overline{c}_i) > 0$.

The *KUJ* effect may lead to $J_{11}>0$ so the locus $\dot{\lambda} = 0$ is negatively sloping. To illustrate this point, suppose that general goods consumption exhibits a *KUJ* effect. Then, when the *KUJ* effect is sufficiently large, a higher λ increases rather than decreases general goods consumption which reduces the *MRS* between c_1 and c_2. In optimum, the marginal product of labor and capital in the general goods sector relative to that in the consumption goods sector in (5b) and (5c) needs to decrease. Thus, $J_{11}>0$, so the locus $\dot{\lambda} = 0$ is negatively sloping.

When the $\dot{\lambda} = 0$ locus is negatively-sloping, the dynamic property may change. In particular, when the negatively-sloping $\dot{\lambda} = 0$ locus is steeper than the locus $\dot{k} = 0$ as illustrated in Diagrams B1 and B2 of Figure 2, the steady state is a sink. This requires two roots with negative real parts and the conditions are $Det(J)=J_{11}J_{22}-J_{21}J_{12}>0$ and $Tr(J)=J_{11}+J_{22}<0$, which are equivalent to $(-J_{12}/J_{11})<(-J_{22}/J_{21})$>(or<)0. Thus, the negatively-sloping $\dot{\lambda} = 0$ locus must be steeper than the $\dot{k} = 0$ locus.

4.2 A Parametric Version

For ease of exposition, in the subsection we use the parametric version of the utility function (10a) and the production function (10b) to illustrate the dynamic property. The utility function stipulates that if $[-\theta_i(\varepsilon-1)]>0$, then goods i consumption exhibits the *KUJ* effect. The 2x2 dynamical equations in (12a) and (12b) are derived as follows.

First, with the production function (10b), factor allocation between sectors in (5c) leads to

$$l = l(s) \equiv [1 + \frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}(\frac{1}{s} - 1)]^{-1}, \tag{14a}$$

where $l'(s) = \frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l^2}{s^2} > 0$. The positive sign is due to the complement of capital and labor. Feasibility of l restricts $s < \frac{\alpha_1(1-\alpha_2)}{\alpha(1-\alpha_1)}$. Note that $\alpha_1 > \alpha_2$ implies $\frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l}{s} > 1$ and thus $l'(s) > \frac{l}{s}$.

Next, the production function (9b) and the pure consumption goods equilibrium (4b) indicate

$$c_2 = c_2(s,k) \equiv c_2(s,k) A_2[(1-s)k]^{\alpha_2}[1-l(s)]^{1-\alpha_2}, \tag{14b}$$

where $\frac{\partial c_2}{\partial s} = -c_2[\frac{\alpha_2}{1-s} + \frac{1-\alpha_2}{1-l}l'(s)] < 0,$

$\frac{\partial c_2}{\partial k} = \frac{\alpha_2 c_2}{k} > 0.$

Intuitively, a smaller share of capital allocated to sector 2 (i.e., a larger *s*) reduces the production and consumption of goods 2. Moreover, larger capital in the economy indicates more capital allocated to sector 2 which increases the production and consumption of goods 2.

Further, with the CES utility, (5a) is $\gamma c_1^{\frac{(\varepsilon-1)(1-\theta_1)}{\varepsilon}} + (1-\gamma)c_2^{\frac{(\varepsilon-1)(1-\theta_2)}{\varepsilon}} = [\lambda c_1^{\frac{[1+\theta_1(\varepsilon-1)]}{\varepsilon}}\gamma^{-1}]^{\varepsilon-1}$ which gives

$$c_1 = c_1(\lambda, c_2), \tag{14c}$$

where $\frac{\partial c_1}{\partial \lambda} = \frac{\varepsilon c_1}{\lambda}\frac{1+p(c_1/c_2)}{B} < 0$ if θ_1=0 ; ambiguous if otherwise;

$\frac{\partial c_1}{\partial c_2} = \text{-}(1-\theta_2)\frac{c_1}{c_2}\frac{1}{B} > 0$ if θ_1=θ_2=0; ambiguous if otherwise;

$B = -1 + [-\theta_1(\varepsilon-1)][1 + p(c_1/c_2)\varepsilon/(\varepsilon-1)] < 0$ if θ_1=0; ambiguous if otherwise.[4]

Thus, if θ_1=θ_2=0, $(\frac{\partial c_1}{\partial \lambda})^* = \text{-}\varepsilon\lambda^{-1}c_1(1 + pc_1/c_2) < 0$. Intuitively, without consumption externalities, a higher shadow price of capital increases the marginal utility of c_1 in (5a) and, with a given c_2, c_1 must decrease in optimum. Moreover, $(\frac{\partial c_1}{\partial c_2})^* = \frac{c_1}{c_2} > 0$ if θ_1=θ_2=0. This result comes because given λ, c_1 and c_2 are complementary in utility. However, with externalities from $\overline{c}_1$, a higher shadow price of capital may increase c_1 if the *KUJ* effect ([-$\theta_1(\varepsilon$-1)]>0) is sufficiently large. In this case, c_1 and c_2 are negatively related.

Furthermore, using (4b), (5a) and (14a)-(14c), the utility function (10a) and the production

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[4] In the parametric version, $p = f_2^2 / f_2^1 = (1-\alpha_2)A_2[(1-s)k/(1-l)]^{\alpha_2}[(1-\alpha_1)A_1(sk/l)^{\alpha_1}]^{-1}$.

function (10b), the equalization of the *MRS* to the *MRT* in (5b) is rewritten as

$$\frac{\gamma\ c_2(s,k)^{\frac{1+\theta_2(\varepsilon-1)}{\varepsilon}}}{(1-\gamma)\ c_1(\lambda,c_2)^{\frac{1+\theta_1(\varepsilon-1)}{\varepsilon}}} = \frac{A_2(1-\alpha_2)}{A_2(1-\alpha_1)} \frac{(1-s)^{\alpha_2}(1-l(s))^{\alpha_2}}{(s)^{\alpha_1}(l(s))^{\alpha_1}} \frac{1}{k^{(\alpha_1-\alpha_2)}},$$

where the *KUJ* effect affects the *MRS* between goods 2 and 1 via $c_1(\lambda, c_2)$. This condition gives

$$s = s(\lambda,k), \tag{14d}$$

where $\frac{\partial s}{\partial \lambda} = [1+\theta_1(\varepsilon-1)]\frac{1}{\varepsilon}\frac{1}{c_1}\frac{\partial c_1}{\partial \lambda}\frac{1}{\Xi} > 0$ if θ_1=θ_2=0; ambiguous if otherwise;

$\frac{\partial s}{\partial k} = [\frac{1+\theta_1(\varepsilon-1)}{\varepsilon}\frac{c_2}{c_1}\frac{\partial c_1}{\partial c_2} - \frac{1+\theta_2(\varepsilon-1)}{\varepsilon}]\frac{\alpha_2}{k}\frac{1}{\Xi} - \frac{\alpha_1-\alpha_2}{k}\frac{1}{\Xi} > 0$ if θ_1=θ_2=0; ambiguous if otherwise;

$\Xi = [\frac{\alpha_2}{1-s} - \frac{\alpha_2}{1-l}l'(s)] + [\frac{\alpha_1}{s} - \frac{\alpha_1}{l}l'(s)] + [\frac{1+\theta_1(\varepsilon-1)}{\varepsilon}\frac{c_2}{c_1}\frac{\partial c_1}{\partial c_2} - \frac{1+\theta_2(\varepsilon-1)}{\varepsilon}][\frac{\alpha_2}{1-s} + \frac{1-\alpha_2}{1-l}l'(s)] < 0$ if θ_1=θ_2=0; ambiguous if otherwise.

Note that $\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^* = 1$ under θ_1=θ_2=0, which gives $\Xi^* = [\frac{\alpha_2}{1-s} - \frac{\alpha_2}{1-l}l'(s)] + [\frac{\alpha_1}{s} - \frac{\alpha_1}{l}l'(s)] < 0.$ Thus, under θ_1=θ_2=0, $(\frac{\partial s}{\partial \lambda})^* = -\frac{1+p(c_1/c_2)}{\lambda^*}\frac{1}{\Xi^*} > 0$ and $(\frac{\partial s}{\partial k})^* = -(\frac{\alpha_1}{\alpha_2} - 1)\frac{\alpha_2}{k}\frac{1}{\Xi^*} > 0.$ Intuitively, without consumption externalities, for a given k, a higher shadow price of capital decreases c_1 which increases the *MRS* between c_1 and c_2. In optimum, a larger share of capital needs to allocate to the general goods sector (i.e., increases s) in order to decrease the marginal product of capital in general goods relative to pure consumption goods so as to increase the *MRT*. For a given λ, if capital increases, since sector 1 is more capital intensive than sector 2, the Rybczynski theorem stipulates that more capital is allocated to sector 1 and thus s increases. However, with consumption externalities, these above relationships are ambiguous. In particular, with the *KUJ* effect, a higher shadow price of capital decreases the share of capital allocated to the general goods sector.

Finally, with the use of (14a)-(14d), the dynamical equations in (12a) and (12b) are

$$\dot{\lambda} = J_1(\lambda,k) \equiv \lambda\{(\rho+\delta) - A_1\alpha_1[\frac{l(s(\lambda,k))}{s(\lambda,k)}]^{1-\alpha_1}\frac{1}{k^{1-\alpha_1}}\}, \tag{15a}$$

$$\dot{k} = J_2(\lambda,k) \equiv A_1 s(\lambda,k)^{\alpha_1} k^{\alpha_1} l(s(\lambda,k))^{1-\alpha_1} - c_1(\lambda, c_2(s(\lambda,k),k)) - \delta k. \tag{15b}$$

A steady state is obtained if $\dot{\lambda} = \dot{k} = 0$.

To envisage the dynamic property, by differentiating (15a) and (15b) around the steady state, with the use of (14a), we obtain the elements in the Jacobean matrix in (13) as follows.

$$J_{11} = -\lambda(\rho+\delta)(1-\alpha_1)\frac{(\alpha_1-\alpha_2)l}{\alpha_2(1-\alpha_1)s}\frac{\partial s}{\partial \lambda}, \tag{16a}$$

$$J_{12} = \lambda(\rho+\delta)(1-\alpha_1)[\frac{1}{k} - \frac{(\alpha_1-\alpha_2)l}{\alpha_2(1-\alpha_1)s}\frac{\partial s}{\partial k}], \tag{16b}$$

$$J_{21} = \{(c_1+\delta k)[\frac{\alpha_1}{s} + \frac{1-\alpha_1}{l} l'(s)] + c_2[\frac{\alpha_2}{1-s} + \frac{1-\alpha_2}{1-l} l'(s)]\frac{\partial c_1}{\partial c_2}\}\frac{\partial s}{\partial \lambda} - \frac{\partial c_1}{\partial \lambda}, \tag{16c}$$

$$J_{22} = \{(c_1+\delta k)[\frac{\alpha_1}{s} + \frac{1-\alpha_1}{l} l'(s)] + c_2[\frac{\alpha_2}{1-s} + \frac{1-\alpha_2}{1-l} l'(s)]\frac{\partial c_1}{\partial c_2}\}\frac{\partial s}{\partial k} + \frac{c_1}{k}[\alpha_1 - \alpha_2\frac{c_2}{c_1}\frac{\partial c_1}{\partial c_2}] - (1-\alpha_1)\delta. \tag{16d}$$

In the case of $\theta_1=\theta_2=0$ and thus, the two-sector growth model without consumption externalities, (14d) indicates $\frac{\partial s}{\partial \lambda} = (\frac{\partial s}{\partial \lambda})^* > 0$ and $(\frac{\partial s}{\partial k})^* > 0$. Thus, $J^*_{11}<0$. As sector 1 is more capital intensive than sector 2 under construction, an expansion of capital increases capital (sk) and labor (l) allocated to sector 1 and moreover, capital sk is increased proportionally more than the proportional increase of labor l. Thus, $J^*_{12}>0$. As a result, the $\dot{\lambda}^* = 0$ locus is positive sloping: $\frac{d\lambda}{dk}\Big|_{\dot{\lambda}=0} = -J^*_{12} / J^*_{11} > 0.$

Further, the slope of the $\dot{k}^* = 0$ locus is $\frac{d\lambda}{dk}\Big|_{\dot{k}=0} = -J^*_{22} / J^*_{21}$. Under $\theta_1=\theta_2=0$, (13c) indicates $\frac{\partial c_1}{\partial \lambda} = (\frac{\partial c_1}{\partial \lambda})^* < 0$ and thus $J^*_{21}>0$. The sign of J^*_{22} depends on the threshold of k, denoted by $\tilde{k}$, wherein the marginal product of capital minus the effect of capital on consumption is equal to the deprecation rate, and $J^*_{22}>(resp.<)0$ when $k < (resp. >)\tilde{k}$.[5] Thus, $\dot{k}^* = 0$ is a U-shaped locus as illustrated in Figure 2.

In a two-sector growth model without consumption externalities, the steady state is a saddle. The condition is $Det(J^*)<0$ and is equivalent to $-J^*_{12} / J^*_{11} > -J^*_{22} / J^*_{21}$. The condition requires the positive slope of the $\dot{\lambda}^* = 0$ locus to be larger than the slope of the $\dot{k}^* = 0$ locus, as seen in Diagrams A1 and A2, Figure 2. The $\dot{\lambda}^* = 0$ locus needs to start at a positive and finite value of $k = \hat{k} > 0$ in order for the marginal product of capital to equal the sum of the discount rate and the depreciation rate at $\lambda=0$.[6] With consumption externalities, the dynamic property may change.

[5] $\tilde{k}$ is determined by $[D\frac{\partial s}{\partial k} + \alpha_1 A_1 \frac{s^{\alpha_1} l^{1-\alpha_1}}{k^{1-\alpha_1}}] - \frac{\partial c_1}{\partial c_2}\frac{\partial c_2}{\partial k} = \delta$, where $D \equiv \{A_1(\frac{s}{l})^{\alpha_1} k^{\alpha_1}[\alpha_1 \frac{l}{s} + (1-\alpha_1) l'(s)] - \frac{\partial c_1}{\partial c_2}\frac{\partial c_2}{\partial s}\} > 0.$

[6] The value $k = \hat{k}$ is determined by $(\rho+\delta) = A_1\alpha_1[l(s(0,\hat{k}))/s(0,\hat{k})]^{1-\alpha_1}(1/\hat{k})^{1-\alpha_1}$.

4.2.1 Only goods 1 has consumption externalities

First, we consider the externality arising only from general goods consumption; that is, $\theta_1 \neq 0$ and $\theta_2=0$. This is the type of externalities analyzed in existing one-sector models studied by Gali (1994), Dupor and Liu (2003), Liu and Turnovsky (2005) and Alonso-Carrera *et al.* (2008).

With general goods consumption externalities, when there is the *KUJ* effect, self-fulfilling expectations can be supported as equilibrium. To explain the intuition, we use (5a)-(5c) to obtain

$$MRS = \frac{\lambda}{u_2} = \frac{\gamma}{1-\gamma}\left[\frac{c_2}{(c_1)^{1-[-\theta_1(\varepsilon-1)]}}\right]^{\frac{1}{\varepsilon}} = MRT = \frac{f_2^2((1-s)k,(1-l))}{f_2^1(sk,l)} = p. \tag{17}$$

Suppose that the representative agent expects that the price of general goods relative to pure consumption goods will increase (higher p). This raises the *MRT* between general goods and consumption goods. Thus, the agent allocates more capital and labor to the general goods sector (and thus sk and l are increased) which will lower the marginal product in the general goods sector and raise the marginal product in the pure consumption goods sector. Yet, more capital and labor in the general goods sector increases the production of general goods which increases general goods consumption. If there is no consumption externality ($\theta_1=0$), (17) indicates a lower the *MRS* between c_1 and c_2 which will not equal the *MRT*. As a result, anticipations of higher prices of general goods relative to pure consumption goods cannot be supported as equilibrium. Suppose instead that $\theta_1 \neq 0$ and there is the *KUJ* effect, $[-\theta_1(\varepsilon-1)]>0$. If the *KUJ* effect is sufficiently large, then the increase in general goods consumption can raise the *MRS* so as to equal the *MRT*. In this situation, self-fulfilling expectations can be supported as equilibrium.

With goods 1 consumption externalities, the elements of the Jacobean matrix in (16a)-(16d) are: $J^{\theta 1}_{11}$, $J^{\theta 1}_{12}$, $J^{\theta 1}_{21}$ and $J^{\theta 1}_{22}$.[7] The change in the dynamic property comes mainly from the change in the sign of $J^{\theta 1}_{11}$ as a result of the *KUJ* effect being in a proper range. To see this, when $\theta_1 \neq 0$, $(\frac{\partial s}{\partial \lambda})$ in (16a) is

$$\left(\frac{\partial s}{\partial \lambda}\right)^{\theta_1} = \frac{1-(-\theta_1(\varepsilon-1))}{\varepsilon}\frac{1}{c_1}\left(\frac{\partial c_1}{\partial \lambda}\right)^{\theta_1}\frac{1}{\Xi^{\theta_1}}, \tag{18}$$

where $(\frac{\partial c_1}{\partial \lambda})^{\theta_1} = -\frac{\varepsilon c_1}{\lambda}\frac{1+p(c_1/c_2)}{1-[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]}$,

$$\Xi^{\theta_1} = [\tfrac{\alpha_2}{1-s} - \tfrac{\alpha_2}{1-l}l'(s)] + [\tfrac{\alpha_1}{s} - \tfrac{\alpha_1}{l}l'(s)] + \{\tfrac{[1-(-\theta_1(\varepsilon-1))]}{\varepsilon}\tfrac{c_2}{c_1}(\tfrac{\partial c_1}{\partial c_2})^{\theta_1} - \tfrac{1}{\varepsilon}\}(\tfrac{\alpha_2}{1-s} + \tfrac{1-\alpha_2}{1-l}l'(s)) \equiv \Xi^* + \Lambda^{\theta_1},$$

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[7] A superscript θ_i is used to represent the source of externalities from consumption c_i.

$$\Lambda^{\theta_1} = \{\frac{[1-(-\theta_1(\varepsilon-1))]}{\varepsilon}\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^{\theta_1} - \frac{1}{\varepsilon}\}(\frac{\alpha_2}{1-s} + \frac{1-\alpha_2}{1-l}l'(s)),$$

$$(\frac{\partial c_1}{\partial c_2})^{\theta_1} = \frac{c_1}{c_2}\frac{1}{1-[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]}.$$

Now, when the *KUJ* effect (i.e., $[-\theta_1(\varepsilon-1)]>0$) does not make the signs of $(\frac{\partial c_1}{\partial \lambda})^{\theta_1}$ and $(\frac{\partial c_1}{\partial c_2})^{\theta_1}$ to deviate from those of their counterparts of $(\frac{\partial c_1}{\partial \lambda})^* < 0$ and $(\frac{\partial c_1}{\partial c_2})^* > 0$ under $\theta_1=\theta_2=0$, it is possible to get $(\frac{\partial s}{\partial \lambda})^{\theta_1} < 0$ in (17) so $J^{\theta 1}{}_{11}>0$. To get $J^{\theta 1}{}_{11}>0$, note that the *KUJ* effect leads to $\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^{\theta_1} > 1$ and $\frac{[1-(-\theta_1(\varepsilon-1))]}{\varepsilon} < \frac{1}{\varepsilon}$. Moreover, when the value of $[-\theta_1(\varepsilon-1)]>0$ is in a proper range such that $\{\frac{[1-(-\theta_1(\varepsilon-1))]}{\varepsilon}\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^{\theta_1} - \frac{1}{\varepsilon}\} > 0$, then $\Lambda^{\theta_1} > 0$. Further, if $\Lambda^{\theta_1} > 0$ is larger than $-\Xi^* >0$, then $\Xi^{\theta_1} >0$ and thus $(\frac{\partial s}{\partial \lambda})^{\theta_1} < 0$ and $J^{\theta 1}{}_{11}>0$. The same reasoning also gives $(\frac{\partial s}{\partial k})^{\theta_1} < 0$ and thus $J^{\theta 1}{}_{12}>0$ in (15b) and $J^{\theta 1}{}_{21}>0$. As a result, the slope of $\dot{\lambda}=0$ locus is negative: $-J_{12}^{\theta_1} / J_{11}^{\theta_1} < 0$. Hence, when the negative slope of the $\dot{\lambda}=0$ locus is smaller than the slope of the $\dot{k}=0$ locus, the steady state is a sink as illustrated in Diagram B1 and Diagram B2, Figure 2.

To obtain the proper range of the *KUJ* effect under which the steady state is a sink, note that the conditions $Det(J^{\theta 1})>0$ and $Tr(J^{\theta 1})<0$ lead to a relative slope condition. In the Appendix, we have shown that the relative slope condition is met if

$$a_1[-\theta_1(\varepsilon-1)]^2 + b_1[-\theta_1(\varepsilon-1)] + Det(J^*) > 0, \tag{19}$$

where a_1 and b_1 are coefficients that are functions of consumption and the shadow price of capital evaluated at the steady state. This inequality gives the range of the *KUJ* effect wherein the steady state is a sink.

It is worth noting that when $\alpha_2=0$, $c_2=A_2(1-l)$ is leisure.[8] As $\theta_2=0$, our model is then reduced to the one-sector growth model with elastic labor supplies studied by Alonso-Carrera *et al.* (2008). In the model of Alonso-Carrera *et al.* (2008), indeterminacy arises only if consumption externalities make the Frisch labor supply to have certain shape. Different from Alonso-Carrera *et al.* (2008), in our two-sector model, general goods consumption externalities lead to indeterminacy even though

[8] In this case, $s=1$ and there is no relationship (13a). If we normalize $A_2=1$, then (13b) is changed to $c_2=(1-l)$, which is leisure, (13c) is $c_1=c_1(\lambda, 1-l)$ and (13d) is $l^{\alpha_1}[\frac{(1-l)}{c_1(\lambda,1-l)}]^{-[1+(\varepsilon-1)\theta_2]/\varepsilon} = \gamma(1-\alpha_1)A_1(1-\gamma)^{-1}k^{\alpha_1}$, which implies $l=l(\lambda, k)$, where $\frac{\partial l}{\partial \lambda} > 0$ and $\frac{\partial l}{\partial k} > 0$ when $\theta_1=\theta_2=0$ but $\frac{\partial l}{\partial \lambda}$ and $\frac{\partial l}{\partial k}$ are ambiguous if $\theta_1 \neq 0$ or $\theta_2 \neq 0$. Using $l=l(\lambda, k)$, (13c) becomes $c_1=c_1(\lambda, 1-l(\lambda, k))$.

the labor supply is inelastic.

The conditions of indeterminacy in our two-sector model are quantitatively easier to meet than an otherwise identical one-sector model with endogenous leisure. To see this, we calibrate our model economy to the baseline economy without consumption externalities. Thus, $\theta_1=\theta_2=0$ in our baseline model. The capital share in the general goods sector is set at $\alpha_1=0.32$ following Herrendorf and Valentinyi (2008). The pure consumption goods sector may be thought of as the service sector which includes restaurants, transportation services, and financial and retail services. Lee and Wolpin (2006) provided estimates of the share of labor earnings in the service sector in selected years by using the data coming from the Bureau of Economic Analysis in the U.S. Following their estimation, the labor share in the pure consumption sector is set equal to 72%, which is consistent with the average share of labor earnings in the service sector from 1985 to 2000. Hence, the implied value of α_2 is 0.28. Moreover, these two authors also pointed out that as a fraction of total employment, service-sector employment grew from 57 percent to 75 percent between 1950 and 2000. We take this number and set $1-l=0.75$. For the value of the *ES* between two consumption goods, we choose $\varepsilon=1.25$ as our baseline value, which is in the range estimated by Ogaki and Reinhart (1998). Under these values, we use (5b) to calibrate and obtain $\gamma=0.1654$. Then, according to (5c), s is calibrated to 0.2874. If we set the depreciation rate equal $\delta=0.05$ and the discount rate equal $\rho=0.04$, as conventionally suggested, we can use the steady-state condition in (14a) to compute $k^*=5.6179$. Finally, we use (4a) and (7a) to obtain $c_1^*=0.1732$ and $c_2^*=1.1987$. We found that the steady state is a saddle.

Based on the baseline parameterization, we quantify combinations of ε and θ_1 under which the general goods consumption externality leads to indeterminacy. See Figure 3. In the left diagram of Figure 3, the shaded area is the region of $(\varepsilon, -\theta_1)$ under which the steady state is a sink. Here, indeterminacy arises only when $\varepsilon>1$ and $-\theta_1>0$. Thus, general goods consumption externalities generate indeterminacy only when the externality is positive. At the empirically plausible value of $\varepsilon=1.25$ as estimated by Ogaki and Reinhart (1998), the smallest absolute value of $-\theta_1$ where indeterminacy can be established is $-\theta_1=4.2\%$. Thus, the required smallest degree of general goods consumption externalities is $(-\theta_1)(\varepsilon-1)/\varepsilon=0.0084$ which is very small and easily met.

[Insert Figure 3 here]

Next, we also quantify the case of the model in Alonso-Carrera *et al.* (2008) under $\alpha_2=0$. The shaded area in the right diagram of Figure 3 is the region of $(\varepsilon, -\theta_1)$ under which the steady state is a sink. In this case, indeterminacy arises when $\varepsilon<1$ and $-\theta_1<0$. Thus, when the labor supply is elastic, the general goods consumption externality generates indeterminacy only when the consumption externality is negative. The estimates in Zabalza *et al.* (1980) indicates that the *ES* between general goods and leisure is around ε=0.5.[9] At ε=0.5, the smallest absolute value of $-\theta_1$ where indeterminacy can be established is $-\theta_1$=-2.2798. Thus, the required smallest degree of goods 1 consumption externalities is $(-\theta_1)(\varepsilon-1)/\varepsilon$=2.2798 which is 270 times larger than the required smallest degree in an otherwise identical two-sector model. The results imply that it is much easier for a two-sector model with consumption externalities to exhibit indeterminacy than for a one-sector model with consumption externalities.

4.2.2 Only goods 2 has consumption externalities. (θ_1=0, $\theta_2\neq$0).

Next, we consider the externality arising only from pure consumption goods. To see why the *KUJ* effect of pure consumption goods can generate indeterminacy, we use (5a)-(5c) to obtain

$$MRS=\frac{\lambda}{u_2}=\frac{\gamma}{1\text{-}\gamma}[\frac{(c_2)^{1\text{-}[\text{-}\theta_2(\varepsilon\text{-}1)]}}{c_1}]^{\frac{1}{\varepsilon}}=MRT=\frac{f_2^2((1-s)k,(1-l))}{f_2^1(sk,l)}=p. \tag{20}$$

Suppose that the representative agent expects that the price of general goods relative to pure consumption goods is increasing (higher *p*). This raises the *MRT* between general goods and consumption goods. Thus, the agent allocates more input to the general goods sector and less input to the consumption sector which reduces the marginal product in the general goods sector, increases the marginal product in the pure consumption goods sector and reduces the production of consumption goods. When the *KUJ* effect of consumption goods ($[-\theta_2(\varepsilon-1)]>0$) is sufficiently large, then pure consumption goods can be consumed less so as to increase the *MRS* and equal the *MRT*. Thus, self-fulfilling expectations can be supported as equilibrium.

In the case, the elements of the Jacobean matrix in (16a)-(16d) are: $J^{\theta 2}{}_{11}$, $J^{\theta 2}{}_{12}$, $J^{\theta 2}{}_{21}$ and $J^{\theta 2}{}_{22}$. The *KUJ* effect may affect the sign of $J^{\theta 2}{}_{11}$ and $J^{\theta 2}{}_{12}$. To see how the *KUJ* effect works when $\theta_2\neq0$, $(\frac{\partial s}{\partial\lambda})$ and $(\frac{\partial s}{\partial k})$ in (16a) and (16b) become, respectively,

$$\left(\frac{\partial s}{\partial\lambda}\right)^{\theta_2}=\frac{1}{\varepsilon}\frac{1}{c_1}\left(\frac{\partial c_1}{\partial\lambda}\right)^{\theta_2}\frac{1}{\Xi^{\theta_2}}, \tag{21a}$$

[9] Using data in the UK, Zabalza *et al.* (1980) found that the ES between income and leisure is 0.25 for men and 1.30 for women. As men are the major labor force, ε=0.5 is about the average of men and women in the labor force estimated by these authors.

$$\left(\frac{\partial s}{\partial k}\right)^{\theta_2} = -\theta_2 \frac{\alpha_2}{k} \frac{1}{\Xi^{\theta_2}} - \frac{\alpha_1 - \alpha_2}{k} \frac{1}{\Xi^{\theta_2}}, \tag{21b}$$

where $(\frac{\partial c_1}{\partial \lambda})^{\theta_2} = -\frac{\varepsilon c_1[1+p(c_1/c_2)]}{\lambda} < 0,$

$$\Xi^{\theta_2} = [\tfrac{\alpha_2}{1-s} - \tfrac{\alpha_2}{1-l} l'(s)] + [\tfrac{\alpha_1}{s} - \tfrac{\alpha_1}{l} l'(s)] - \theta_2[\tfrac{\alpha_2}{1-s} + \tfrac{1-\alpha_2}{1-l} l'(s)] \equiv \Xi^* + \Lambda^{\theta_2},$$

$$\Lambda^{\theta_2} = -\theta_2[\tfrac{\alpha_2}{1-s} + \tfrac{1-\alpha_2}{1-l} l'(s)].$$

Since $(\frac{\partial c_1}{\partial \lambda})^{\theta_2} < 0,$ the *KUJ* effect has to give $\Xi^{\theta_2} > 0$ in order to get $(\frac{\partial s}{\partial \lambda})^{\theta_2} < 0.$ With Ξ^*<0, this is possible only if $\Lambda^{\theta_2} > 0,$ which requires θ_2<0 and, under the *KUJ* effect, ε>1. Moreover, if -θ_2>0 is sufficiently large such that $\Xi^{\theta_2} > 0$, then $(\frac{\partial s}{\partial \lambda})^{\theta_2} < 0$ and thus $J^{\theta 2}{}_{11}$>0. However, the value of -θ_2>0 cannot be too large because a large -θ_2>0 gives a large $(\frac{\partial s}{\partial k})^{\theta_2} > 0$ such that $J^{\theta 2}{}_{12}$<0 and the slope of the $\dot{\lambda}^{\theta_2} = 0$ locus is positive. Thus, it is required that [-θ_2(ε-1)] lies in a range so the slope of the $\dot{\lambda}^{\theta_2} = 0$ locus is negative as illustrated in Diagrams B1 and B2, Figure 2.

Then, a sink arises if $Det(J^{\theta 2})$>0 and $Tr(J^{\theta 2})$<0 which requires that the slope of $\dot{\lambda}^{\theta_2} = 0$ locus be smaller than the slope of $\dot{k}^{\theta_2} = 0.$ In the Appendix, we have shown that the relative slope condition is met under

$$a_2[-\theta_2(\varepsilon - 1)]^2 + b_2[-\theta_2(\varepsilon - 1)] + Det(J^*) > 0, \tag{22}$$

where a_2 and b_2 are coefficients that are functions of consumption and the shadow price of capital evaluated at steady state. Then, we obtain the range of the *KUJ* effect wherein the steady state is a sink.

It is worth noting that when α_2=0, with $\theta_2 \neq 0$ our model is reduced to a one-sector growth model with a leisure externality. Benhabib and Farmer (2000) and Weder (2004) have studied the role of positive leisure externalities in establishing indeterminacy.[10] These authors showed that positive leisure externalities help establish indeterminacy as leisure externalities make it easier for the Frisch labor supply curve to slope down as a function of the real wage. However, using separable utilities, they both found that it is difficult for the leisure externality alone to generate indeterminacy under a plausible parameter space.

[10] Benhabib and Farmer (2000) is a one-sector model with real balances but without capital in the production function. Weder (2004) is a one-sector growth model with externalities in the production function. In these two models, leisure is elastic. Moreover, they both allow for leisure externalities in utility.

To quantify the conditions of indeterminacy in our model, the left diagram of Figure 4 offers quantitative results. The shaded area in the diagram is the region of (ε, -θ_2) under which the steady is a sink. At ε=1.25, the smallest absolute value is $-\theta_2$=0.6% when indeterminacy can be established. The required smallest degree of pure consumption externalities is $(-\theta_2)(\varepsilon-1)/\varepsilon=0.0012$. This required degree is much smaller than the required smallest degree of general goods consumption externalities at 0.0084 in Figure 3. The results indicate that indeterminacy emerges even more easily under pure consumption externalities than under general goods consumption.

[Insert Figure 4 here]

We also quantify the case under α_2=0. The right diagram of Figure 4 offers quantitatively the region of (ε, -θ_2) that leads to indeterminacy. The results indicate that indeterminacy can arise under both positive and negative leisure externalities. At the empirically plausible *ES* between general goods and leisure at ε=0.5, the smallest absolute value is $-\theta_2$=-8.0002 when indeterminacy can be established. This indicates that the required smallest degree of leisure externalities is 8.0002. This required smallest degree is about 4 times larger than the required smallest degree of general goods externalities in a one-sector model (2.2798) in the right diagram of Figure 3. The result thus confirms the conclusions found in Benhabib and Farmer (2000) and Weder (2004) in that the leisure externality alone cannot generate indeterminacy under a plausible parameter space in a one-sector growth model with a non-separable utility.

4.2.3 Goods 1 and 2 exhibit symmetric consumption externalities. ($\theta_1 \neq 0$, $\theta_2 \neq 0$).

We have derived the conditions under which general goods consumption externalities and pure consumption goods externalities each can be a source of indeterminacy. If we combine these conditions, externalities in both types of consumption together can establish indeterminacy.

A more appealing case is $\theta_1=\theta_2=\theta\neq 0$ when consumption externalities from both goods are symmetric. In this case, we have shown in Section 3 that if $\varepsilon\neq 1$, the utility is homothetic. Then, the market equilibrium is efficient in a steady state as the shadow price of capital in the market economy is a fixed proportion of that in the socially planned economy. Yet, symmetric consumption externalities can cause inefficiency in transitions as the shadow price of capital in the market is no longer a fixed proportion of the shadow price of capital in the socially planned economy. As we will see below, when the symmetric *KUJ* effect of consumption externalities is in a proper range, indeterminacy arises.

To see how this works, we use (5a)-(5c) to obtain

$$MRS = \frac{\lambda}{u_2} = \frac{\gamma}{1-\gamma}[\frac{c_2}{c_1}]^{\frac{1-[-\theta(\varepsilon-1)]}{\varepsilon}} = MRT = \frac{f_2^2((1-s)k,(1-l))}{f_2^1(sk,l)} = p. \tag{23}$$

Suppose that the representative agent expects a higher price of general goods relative to pure consumption goods (higher *p*). This raises the *MRT* between general goods and consumption goods. Thus, the agent allocates more input to general goods which reduces the marginal product in the general goods sector and increases the marginal product in the pure consumption goods sector. Then, there are more general goods production and less pure consumption goods production. When there are symmetric consumption externalities from both goods such that $(-\theta(\varepsilon-1))>0$, then general goods may be consumed more and pure consumption goods may be consumed less so as to increase the *MRS* and equal the *MRT*. In this situation, self-fulfilling expectations can be supported as equilibrium.

To derive the conditions, the elements of the Jacobean matrix in (16a)-(16d) are J^θ_{11}, J^θ_{12}, J^θ_{21} and J^θ_{22} which are combinations of those in subsections 4.2.1 and 4.2.2. To see how the *KUJ* effect works when $\theta_1=\theta_2=\theta\neq 0$, $(\frac{\partial s}{\partial \lambda})^\theta$ and $(\frac{\partial s}{\partial k})^\theta$ are, respectively,

$$\left(\frac{\partial s}{\partial \lambda}\right)^\theta = [1-(-\theta(\varepsilon-1))]\frac{1}{\varepsilon}\frac{1}{c_1}\left(\frac{\partial c_1}{\partial \lambda}\right)^\theta \frac{1}{\Xi^\theta}, \tag{24a}$$

$$\left(\frac{\partial s}{\partial k}\right)^\theta = \frac{1-(-\theta(\varepsilon-1))}{\varepsilon}[\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^\theta - 1]\frac{\alpha_2}{k}\frac{1}{\Xi^\theta} - \frac{\alpha_1-\alpha_2}{k}\frac{1}{\Xi^\theta}, \tag{24b}$$

where $(\frac{\partial c_1}{\partial \lambda})^\theta = -\frac{\varepsilon c_1}{\lambda}\frac{1+p(c_1/c_2)}{1-[-\theta(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]}$,

$$(\frac{\partial c_1}{\partial c_2})^\theta = \frac{c_1}{c_2}\frac{1-\theta}{1-[-\theta(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]},$$

$$\Xi^\theta = [\frac{\alpha_2}{1-s} - \frac{\alpha_2}{1-l}l'(s)] + [\frac{\alpha_1}{s} - \frac{\alpha_1}{l}l'(s)] + \frac{1-[-\theta(\varepsilon-1)]}{\varepsilon}[\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^\theta - 1][\frac{\alpha_2}{1-s} + \frac{1-\alpha_2}{1-l}l'(s)] \equiv \Xi^* + \Lambda^\theta,$$

$$\Lambda^\theta = \frac{1-[-\theta(\varepsilon-1)]}{\varepsilon}[\frac{c_2}{c_1}(\frac{\partial c_1}{\partial c_2})^\theta - 1][\frac{\alpha_2}{1-s} + \frac{1-\alpha_2}{1-l}l'(s)].$$

Similar to the case with only general goods consumption externalities (cf. (18)), here the *KUJ* effect $(-\theta(\varepsilon-1))$ appears in a multiplicative term in $(\frac{\partial s}{\partial \lambda})^\theta$ and $(\frac{\partial s}{\partial k})^\theta$. If the *KUJ* effect lies in a range,[11] it is possible to obtain $(\frac{\partial s}{\partial \lambda})^\theta < 0$ and $(\frac{\partial s}{\partial k})^\theta < 0$. Then, $J^\theta_{11}>0$, $J^\theta_{12}>0$ and $J^\theta_{21}<0$. The

[11] In the range of the *KUJ* effect, (i) the signs of $(\frac{\partial c_1}{\partial \lambda})^\theta$ and $(\frac{\partial c_1}{\partial c_2})^\theta$ do not deviate from their

required condition is again $Det(J^{\theta})>0$ and $Tr(J^{\theta})<0$ which gives a negative slope of $\dot{\lambda}=0$ that is steeper than the slope of $\dot{k}=0$. In the Appendix, we have shown that the relative slope condition is met if

$$a_3[-\theta(\varepsilon-1)]^3+b_3[-\theta(\varepsilon-1)]^2+d_3[-\theta(\varepsilon-1)]+Det(J^*)>0, \tag{25}$$

where a_3, b_3 and d_3 are coefficients that are functions of consumption and the shadow price of capital evaluated at the steady state. Then, we obtain the range of the *KUJ* effect wherein the steady state is a sink.

Our analysis indicates that in a two-sector growth model, symmetric consumption externalities lead to indeterminacy even when the utility is homothetic. The result is different from that in the one-sector growth model by Alonso-Carrera *et al.* (2008) wherein consumption externalities do not lead to indeterminacy when the utility is homothetic. The difference arises because there is a relative price of the two goods in our model.

Figure 5 offers quantitative results about the region of (ε, -θ) under which the steady state is a sink (see left diagram). At the empirically plausible value of ε=1.25, the smallest absolute value is $-\theta$=0.5% when indeterminacy can be established. The required smallest degree of symmetric consumption externalities is $-\theta(1-\varepsilon)/\varepsilon$=0.001 which is smaller than the required smallest degree of 0.0084 when there is only the general goods consumption externality. Hence, the pure consumption externality helps the general goods consumption externality to establish indeterminacy.

[Insert Figure 5 here]

When α_2=0, there is the leisure externality.[12] With $\theta_1=\theta_2=\theta$, quantitative results are in the right diagram of Figure 5. Notice that this case may be thought of as the case of the general goods consumption externality with an additional symmetric leisure externality. Thus, similar to the left diagram of Figure 3 with the general goods consumption externality, indeterminacy here arises only when externalities are positive ($-\theta_1=\theta>0$). However, with additional symmetric, positive leisure externalities $-\theta_2=\theta>0$, the required degree of general goods consumption externalities here is much larger than that in Figure 3. For example, at ε=1.25, the smallest absolute value here in Figure 5 is $-\theta$=50.76% when indeterminacy can be established, as opposed to $-\theta_1$=4.2% in Figure 3. Thus, a positive leisure externality does not help the general goods consumption externality to establish

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counterparts of $(\frac{\partial c_1}{\partial \lambda})^*<0$ and $(\frac{\partial c_1}{\partial c_2})^*>0$ under $\theta_1=\theta_2=0$ and (ii) $\Lambda^{\theta}>-\Xi^*>0$.

[12] In the case under α_2=0, as there is the leisure externality, our model is different from the Alonso-Carrera *et al.* (2008) model which has no leisure externality.

indeterminacy. This result is different from the findings uncovered by Benhabib and Farmer (2000) and Weder (2004) wherein the leisure externality helps money in production and externalities in production to establish indeterminacy.

4.2.4 Goods 2 is more capital intensive than goods 1

We have so far assumed the plausible case that the general goods sector is more capital intensive than the pure consumption goods sector, $\alpha_1>\alpha_2$. Nevertheless, from the theoretical point of view, it might be interesting to consider the opposite case that $\alpha_1<\alpha_2$. As shown in Section 3, with the jealousy effect of general goods consumption, the case of $\alpha_1<\alpha_2$ produces different welfare properties in that otherwise optimal capital taxes may become subsidies. It is interesting to investigate quantitatively local dynamic properties under a different intensity of capital.

To this end, we set α_1=0.28 and α_2=0.32 so the general goods sector is less capital intensive. We recalibrate the model following the same method used in subsection 4.2.1. The baseline parameter values are not different but the values of the steady state change.[13] We quantify the value of *ES* between the two goods and the degree of externalities to see whether the steady state is a sink. The results are as follows.

First, if the *ES* between the two goods is smaller than unity ($\varepsilon<1$), the steady state is always a saddle. Second, if the *ES* between the two goods is larger than one ($\varepsilon>1$), whether the steady state is a sink or not depends on the source of consumption externalities and is as follows.

Case 1. $\theta_1\neq0$ and $\theta_2=0$

In this case, only general goods consumption has externalities. We find that the steady state is either a saddle or a source, not a sink. The result is thus different from those in the case of $\alpha_1>\alpha_2$ as illustrated in the left diagram in Figure 3.

Case 2. $\theta_1=0$ and $\theta_2\neq0$

In this case, only the consumption of pure consumption goods has externalities. We find that the steady state is a sink only if $1<\varepsilon<1.45$. See the left diagram of Figure 6. As we see from the diagram, a sink arises only if the value of $-\theta_2$ lies above a threshold. Note that different from the corresponding left diagram of Figure 4 wherein the threshold of the degree of consumption

[13] The baseline parameters are α_1=0.28, α_2=0.32, ρ=0.04, δ=0.05, $\theta_1=\theta_2$=0. Then γ and s are recalibrated to 0.1267 and 0.216, respectively. Under these parameter values we obtain k^*=5.5995, c_1^*=0.1087 and c_2^*=1.3202.

externalities is constant, the threshold here is increasing in the value of the *ES* between the two goods ε. Moreover, for a given value of ε, the required smallest degree of consumption externalities here is much larger. For example, at ε=1.25, the required smallest degree of consumption externalities here is 4.544, as opposed to 0.0012 in Figure 4. Even at ε=1.01, the required smallest degree of consumption externalities that gives a sink here is as large as 0.176736.

[Insert Figure 6 here].

Case 3. $\theta_1=\theta_2=\theta\neq 0$.

In this case, the two types of consumption have symmetric externalities. We find that the steady state is a sink only if $1<\varepsilon<1.1$ (the right diagram of Figure 6). In this range of the *ES*, a sink arises if the value of $-\theta$ is larger than a large threshold. Note that different from the corresponding left diagram of Figure 5 wherein the required smallest degree of consumption externalities is constant, the threshold here is increasing in the value of the *ES* between the two goods. Moreover, for a given value of ε, the required degree of consumption externalities is large. For example, at ε=1.1, the smallest value of $-\theta$ that gives a sink here is 148.6 and thus the required smallest degree of consumption externalities is $-\theta(\varepsilon-1)/\varepsilon$=13.52727, which is very large as compared to 0.0004545 in the left diagram of Figure 5. Even at ε=1.01, the required smallest degree of consumption externalities that creates a sink here is as large as 0.38683.

To summarize the results, when sector 1 is less capital intensive than sector 2, we find that the equilibrium indeterminacy is possible only when there are consumption externalities from goods 2. However, the required degree of consumption externalities that gives rise to an indeterminacy is much larger than that when sector 1 is more capital intensive than sector 2. The required degree of consumption externalities is quantitatively less plausible.

5. Concluding Remarks

In one-sector neoclassical growth models, consumption externalities can produce inefficient allocation in a steady state and create indeterminate equilibrium paths toward a steady state only if there is a labor-leisure tradeoff. In our paper, we have shown that in a two-sector neoclassical growth model with general goods and consumption goods, even if there is no labor-leisure tradeoff, consumption spillovers can yield inefficient allocation in a steady state and generate indeterminate equilibrium paths toward a steady state.

In our two-sector model, the factor reallocation between sectors and the relative price of the two goods are the mechanisms that generate these results. Consumption externalities change the

MRS and affect the relative price and thus the factor reallocation between sectors so the allocation is inefficient in a steady state. Moreover, equilibrium paths toward a steady state are indeterminate because these externalities can influence the *MRS* and the *MRT* in such a way that self-fulfilling expectations about relative prices of the two goods can be supported as equilibrium.

We find that even with negative general goods consumption externalities, capital may be over- or under-accumulated depending on relative capital intensities and relative degrees of externalities between sectors. The result has important welfare implications as to whether it is optimal to tax or to subsidize capital in the case with consumption externalities. When the general goods sector is more capital intensive, general goods consumption externalities generate indeterminacy more easily in a two-sector model than in a one-sector model. Moreover, pure consumption goods are leisure if capital is not an input in producing pure consumption goods, pure consumption externalities easily cause indeterminacy although it is difficult for leisure externalities to generate indeterminacy and leisure. Further, even when there are symmetric degrees of consumption externalities so the utility is homothetic, the allocation is efficient in a steady state but equilibrium paths toward the steady state are indeterminate. As a result, no matter whether consumption externalities are from general goods, consumption goods or both, it is much easier to exhibit indeterminacy in a two-sector growth model than in a one-sector growth model.

Our results thus imply that a two-sector model economy with consumption externalities is less stabilized than a one-sector model economy with consumption externalities. In the case of a one-sector neoclassical growth model with production externalities, existing studies found that income taxes may stabilize the economy when its equilibrium is otherwise indeterminate and thus less stabilized (e.g., Guo and Lansing, 1998). In our two-sector model with consumption externalities, depending on relative capital intensities and relative degrees of externalities, it may be capital taxes or capital subsidies that can be used to stabilize the economy. It is interesting to study whether taxing or subsidizing capital or other activities can stabilize a two-sector economy with consumption externalities. This is an avenue for further research.

6. Appendix: Stability Conditions

Differentiating the general case in (15a) and (15b), the elements in the Jacobean matrix are:

$$J_{11} = -\frac{(1+pc_1/c_2)(\varrho+\delta)(1-\alpha_1)}{s}\left[-1+\frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l}{s}\right]\frac{1+\theta_1(\varepsilon\text{-}1)}{B}\frac{1}{\Xi},$$

$$J_{12} = \frac{\lambda(\varrho+\delta)(1-\alpha_1)}{s}\left\{\frac{s}{k}-\left[-1+\frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l}{s}\right]\left[\frac{1+\theta_1(\varepsilon\text{-}1)}{\varepsilon}\frac{1\text{-}\theta_2}{B}+\frac{1+\theta_2(\varepsilon\text{-}1)}{\varepsilon}+\frac{\alpha_1-\alpha_2}{\alpha_2}\right]\frac{\alpha_2}{k}\frac{1}{\Xi}\right\},$$

$$J_{21} = \frac{(1+pc_1/c_2)}{\lambda B}\{[1+\theta_1(\varepsilon-1)]\frac{\Gamma}{\Xi} - \varepsilon c_1\},$$

$$J_{22} = [\frac{1+\theta_1(\varepsilon\text{-}1)}{\varepsilon}\frac{1\text{-}\theta_2}{B} + \frac{1+\theta_2(\varepsilon\text{-}1)}{\varepsilon} + \frac{\alpha_1-\alpha_2}{\alpha_2}]\frac{\alpha_2}{k}\frac{\Gamma}{\Xi} + \frac{\alpha_1 c_1}{k} + \frac{(1-\theta_2)}{B}\frac{\alpha_2 c_1}{k} - (1-\alpha_1)\delta,$$

where $B = -1 + [-\theta_1(\varepsilon-1)][1 + p(c_1/c_2)(\varepsilon/\varepsilon-1)]$,

$$\Xi = -[\frac{1+\theta_1(\varepsilon-1)}{\varepsilon}\frac{1-\theta_2}{B} + \frac{1+\theta_2(\varepsilon-1)}{\varepsilon}]\Psi + \Xi^*,$$

$$\Psi = [\frac{\alpha_2}{1-s} + \frac{1-\alpha_2}{1-l}l'(s)],$$

$$\Xi^* = [\frac{\alpha_2}{1-s} - \frac{\alpha_2}{1-l}l'(s)] + [\frac{\alpha_1}{s} - \frac{\alpha_1}{l}l'(s)] < 0,$$

$$\Gamma = (c_1 + \delta k)(\frac{\alpha_1}{s} + \frac{1-\alpha_1}{l}l') - \frac{c_1(1-\theta_2)}{B}\Psi .$$

In the baseline model of the two-sector growth model without consumption externalities ($\theta_1=\theta_2=0$), the elements in the Jacobean matrix are as follows.[14]

$$J_{11}^* = \frac{(1+pc_1/c_2)(\varrho+\delta)(1-\alpha_1)}{s}[-1 + \frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l}{s}]\frac{1}{\Xi^*} < 0,$$

$$J_{12}^* = \frac{\lambda^*(\varrho+\delta)(1-\alpha_1)}{s}\{\frac{s}{k} - [-1 + \frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l}{s}]\frac{\alpha_1-\alpha_2}{k}\frac{1}{\Xi^*}\} > 0,$$

$$J_{21}^* = \frac{(1+pc_1/c_2)}{\lambda^*}\{[(c_1+\delta k)(\frac{\alpha_1}{s} + \frac{1-\alpha_1}{l}l') + c_1\Psi]\frac{1}{\Xi^*} - \varepsilon c_1\} > 0,$$

$$J_{22}^* = \frac{\alpha_1-\alpha_2}{k}[(c_1+\delta k)(\frac{\alpha_1}{s} + \frac{1-\alpha_1}{l}l') + c_1\Psi]\frac{1}{\Xi^*} + \frac{(\alpha_1-\alpha_2)c_1}{k} - (1-\alpha_1)\delta \gtrless 0$$

$$if\ \frac{\alpha_1-\alpha_2}{k}[(c_1+\delta k)(\frac{\alpha_1}{s} + \frac{1-\alpha_1}{l}l') + c_1\Psi]\frac{1}{\Xi^*} + \frac{(\alpha_1-\alpha_2)c_1}{k} \gtrless (1-\alpha_1)\delta.$$

1: $\theta_1=\theta_2=0$

In a standard two-sector growth model wherein $\theta_1=\theta_2=0$, the steady state is a saddle, which indicates that the dynamical system has one root with a negative real part and one root with a positive real part. The required condition is $Det(J^*)=J_{11}^* J_{22}^* - J_{21}^* J_{12}^* < 0$.

2: the Case of $\theta_1\neq0$ and $\theta_2=0$

The elements in the Jacobean matrix are:

$$J^{\theta_1}{}_{11} = -\frac{1+\theta_1(\varepsilon\text{-}1)}{B}\frac{\Xi^*}{\Xi^*+\Xi^{\theta_1}}J_{11}^*,$$

$$J^{\theta_1}{}_{12} = \frac{\lambda}{\lambda^*}J_{12}^* + \frac{\lambda(\varrho+\delta)(1-\alpha_1)}{s}[1 - \frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l}{s}]\frac{\alpha_1-\alpha_2}{k}\frac{1-D^{\theta_1}}{\Xi^*},$$

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[14] Notice that $\alpha_1>\alpha_2$ implies $\frac{\alpha_1(1\text{-}\alpha_2)}{\alpha_2(1\text{-}\alpha_1)}\frac{l}{s} > 1$.

$$J^{\theta_1}{}_{21} = -\frac{[1+\theta_1(\varepsilon-1)]}{\lambda B}\frac{(1+pc_1/c_2)}{\Xi^*+\Xi^{\theta_1}}\{\frac{\lambda^*\Xi^*}{(1+pc_1/c_2)}J^*_{21} + \frac{[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]c_1\Psi}{B} - \varepsilon c_1\Xi^*\} - \frac{\varepsilon c_1(1+pc_1/c_2)}{\lambda B},$$

$$J^{\theta_1}{}_{22} = D^{\theta_1}J^*_{22} + \frac{[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]c_1}{B}[\frac{(\alpha_1-\alpha_2)\Psi D^{\theta_1}}{k\Xi^*} + \frac{\alpha_2}{k}] + [\frac{(\alpha_1-\alpha_2)c_1}{k} - (1-\alpha_1)\delta](1-D^{\theta_1}),$$

where $\Xi^{\theta_1} = -[\frac{1+\theta_1(\varepsilon-1)}{\varepsilon}\frac{1}{B}+\frac{1}{\varepsilon}]\Psi$ and $D^{\theta_1} = (1-\frac{\alpha_2}{\alpha_1-\alpha_2}\frac{\Xi^{\theta_1}}{\Psi})\frac{\Xi^*}{\Xi^*+\Xi^{\theta_1}}$.

The steady state is a sink if the Jacobean matrix *J* has two roots with negative real parts, whose conditions are $Tr(J^{\theta 1})<0$ and $Det(J^{\theta 1})>0$. These conditions require (i) $-J^*_{11}>J^*_{22}$ and (ii) the slope condition:

$$\left.\frac{dc_1}{dk}\right|_{\dot{c}_1=0} = -\frac{J^{\theta_1}{}_{12}}{J^{\theta_1}{}_{11}} > \left.\frac{dc_1}{dk}\right|_{\dot{k}=0} = -\frac{J^{\theta_1}{}_{22}}{J^{\theta_1}{}_{21}}. \quad \text{(A1)}$$

Denote

$$a_1 \equiv \frac{[1+p(c_1/c_2)(\varepsilon/\varepsilon-1)]\Xi^2}{(1+pc_1/c_2)^2}[Det(J^*)+\Omega_1] - \frac{p(c_1/c_2)(1/(\varepsilon-1))\Xi^*\Psi}{(1+pc_1/c_2)^2}[Det(J^*)+\Omega_1+(1+\frac{\alpha_2}{\alpha_1-\alpha_2}\frac{\Xi^*}{\Psi})(\Omega_2-\Omega_3-\frac{J^*_{11}c_1(\alpha_1-\alpha_2)}{k}\frac{\Psi}{\Xi^*})],$$

$$b_1 \equiv -\frac{[1+p(c_1/c_2)\varepsilon/(\varepsilon-1)]\Xi^2}{(1+pc_1/c_2)^2}\Omega_1 - \frac{p(c_1/c_2)(1/(\varepsilon-1))\Xi^*\Psi}{(1+pc_1/c_2)^2}[\Omega_4-(1+\frac{\alpha_2}{\alpha_1-\alpha_2}\frac{\Xi^*}{\Psi})\Omega_2] - \frac{[2+p(c_1/c_2)\varepsilon/(\varepsilon-1)]\Xi^2}{(1+pc_1/c_2)^2}\Omega_4,$$

where

$$\Omega_1 = \frac{J^*_{12}c_1(1+pc_1/c_2)}{\lambda^*}(\varepsilon-\frac{\Psi}{\Xi^*}) + \frac{J^*_{11}c_1(\alpha_1-\alpha_2)}{k}\frac{\Psi}{\Xi^*}(1+\frac{\alpha_2}{\alpha_1-\alpha_2}\frac{\Xi^*}{\Psi}),$$

$$\Omega_2 = \frac{J^*_{11}[(\alpha_1-\alpha_2)c_1-(1-\alpha_1)\delta k-J^*_{22}k]}{k} - \frac{(\varrho+\delta)(1-\alpha_1)(\alpha_1-\alpha_2)}{sk\Xi^*}[1-\frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l}{s}][J^*_{21}\lambda^* - \varepsilon c_1(1+pc_1/c_2)],$$

$$\Omega_3 = -(1+pc_1/c_2)(\varrho+\delta)(1-\alpha_1)[1-\frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l}{s}]\frac{\alpha_1-\alpha_2}{sk}\frac{c_1\Psi}{\Xi^{*2}},$$

$$\Omega_4 = Det(J^*) + \frac{1}{\lambda^*}J^*_{12}\varepsilon c_1(1+pc_1/c_2).$$

Then, the relative slope condition that yields a sink is rewritten as

$$a_1[-\theta_1(\varepsilon-1)]^2 + b_1[-\theta_1(\varepsilon-1)] + Det(J^*) > 0. \quad \text{(A2)}$$

If we set (A2) equal zero, we obtain two critical values for $[-\theta_1(\varepsilon-1)]$, denoted by ζ_1 and ζ_2,

$$\zeta_1, \zeta_2 = \frac{1}{2a_1}\{-b_1 \pm [(b_1)^2 - 4a_1 Det(J^*)]^{1/2}\}.$$

Let $\zeta_1<\zeta_2$ when $a_1>0$. Thus, when $a_1<0$, $\zeta_1>\zeta_2$. Then, we have the following results.

(i) If $a_1>0$, (A1) requires $[-\theta_1(\varepsilon-1)]<\zeta_1$ or $[-\theta_1(\varepsilon-1)]>\zeta_2$.

(ii) If $a_1<0$, (A1) require $\zeta_2<[-\theta_1(\varepsilon-1)]<\zeta_1$.

If we combine Conditions *KUJ* and (A2), the required conditions of a steady state that is a sink are summarized as follows.

(i) $0<[-\theta_1(\varepsilon-1)]<\zeta_1$ or $[-\theta_1(\varepsilon-1)]>\max\{\zeta_2, 0\}$ if $a_1>0$;

(ii) $\max\{\zeta_2, 0\}<[-\theta_1(\varepsilon-1)]<\zeta_1$ if $a_1<0$.

3: the Case of θ_1=0 and θ_2≠0

In this case, the elements in the Jacobean matrix are:

$$J^{\theta_2}{}_{11} = \frac{\Xi^*}{\Xi^* - \theta_2 \Psi} J^*_{11},$$

$$J^{\theta_2}{}_{12} = \frac{\lambda}{\lambda^*} J^*_{12} + \frac{\lambda(\varrho+\delta)(1-\alpha_1)}{s}[1 - \frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l}{s}]\frac{\alpha_1-\alpha_2}{k}\frac{1-D^{\theta_2}}{\Xi^*},$$

$$J^{\theta_2}{}_{21} = \frac{1}{\lambda(\Xi^* - \theta_2\Psi)}[\Xi^* \lambda^* J^*_{21} + \theta_2 c_1 (1 + pc_1 / c_2)(1 - \varepsilon\Psi)],$$

$$J^{\theta_2}{}_{22} = D^{\theta_2} J^*_{22} + \theta_2 c_1 [\frac{(\alpha_1-\alpha_2)\Psi D^{\theta_2}}{k\Xi^*} + \frac{\alpha_2}{k}] + [\frac{(\alpha_1-\alpha_2)c_1}{k} - (1-\alpha_1)\delta](1 - D^{\theta_2}),$$

where $D^{\theta_2} = (1 - \frac{\alpha_2}{\alpha_1-\alpha_2}\frac{\Xi^{\theta_2}}{\Psi})\frac{\Xi^*}{\Xi^* + \Xi^{\theta_2}}$.

To obtain two roots with negative real parts, the conditions are $Tr(J^{\theta 2})=J^{\theta 2}{}_{11}+J^{\theta 2}{}_{22}<0$ and $Det(J^{\theta 2})=J^{\theta 2}{}_{11}J^{\theta 2}{}_{22}-J^{\theta 2}{}_{12}J^{\theta 2}{}_{21}<0$ which lead to the relative slope condition that gives a sink.

Denote

$$a_2 = \frac{\Psi^2}{(\varepsilon-1)(1+pc_1/c_2)^2}[(1 + \frac{\alpha_2}{\alpha_1-\alpha_2}\frac{\Xi^*}{\Psi})\frac{\Xi^*}{\Psi}\frac{\Omega_3}{(1+pc_1/c_2)} - \frac{J^*_{12}c_1}{\lambda^*}],$$

$$b_2 \equiv \frac{\Xi^*\Psi}{(\varepsilon-1)(1+pc_1/c_2)^2}[Det(J^*) - \frac{J^*_{11}c_1(\alpha_1-\alpha_2)}{k} + \frac{J^*_{12}c_1(1+pc_1/c_2)(1-\varepsilon\Xi^*/\Psi)}{\lambda^*} + (1 + \frac{\alpha_2}{\alpha_1-\alpha_2}\frac{\Xi^*}{\Psi})(\Omega_2 + \frac{\varepsilon\Xi^*}{\Psi}\Omega_3) - \frac{\alpha_2 c_1 J^*_{11}\Xi^*}{k\Psi}],$$

Then, the relative slope condition that gives a sink is rewritten as

$$a_2[-\theta_2(\varepsilon-1)]^2 + b_2[-\theta_2(\varepsilon-1)] + Det(J^*) > 0. \qquad \text{(A3)}$$

If we set (A3) equal zero, we obtain the two critical values for [-$\theta_2(\varepsilon$-1)], denoted by η_1 and η_2. et$\eta_1<\eta_2$ when $a_2>0$. Thus, when $a_2<0$, $\eta_1>\eta_2$. Then, we have the following results.

(i) If $a_2>0$, (A3) requires [-$\theta_2(\varepsilon$-1)]< η_1 or [-$\theta_2(\varepsilon$-1)]> η_2.

(ii) If $a_2<0$, (A3) requireη_2<[-$\theta_2(\varepsilon$-1)]< η_1.

If we combine Conditions *KUJ* and (A3), the required conditions of a steady state that is a sink are summarized as

(i) 0<[-θ_2(ε-1)]<η_1 or [-θ_2(ε-1)]>max{η_2, 0} if a_2>0;

(ii) max{η_2 ,0}<[-θ_2(ε-1)]< η_1 if a_2<0.

4: the Case of $\theta_1=\theta_2=\theta$≠0

In the case, the elements in the Jacobean matrix are:

$$J^{\theta}{}_{11} = -\frac{1+\theta(\varepsilon\text{-}1)}{B}\frac{\Xi^*}{\Xi^*+\Xi^{\theta}}J^*_{11},$$

$$J^{\theta}{}_{12} = \frac{\lambda}{\lambda^*} J^*_{12} + \frac{\lambda(\varrho+\delta)(1-\alpha_1)}{s}[1 - \frac{\alpha_1(1-\alpha_2)}{\alpha_2(1-\alpha_1)}\frac{l}{s}]\frac{\alpha_1-\alpha_2}{k}\frac{1-D^{\theta}}{\Xi^*},$$

$$J^{\theta}{}_{21} = -\frac{[1+\theta(\varepsilon-1)]}{\lambda B}\frac{(1+pc_1/c_2)}{\Xi^*+\Xi^{\theta}}\{\frac{\lambda^*\Xi^*}{(1+pc_1/c_2)}J^*_{21} + \frac{[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)]\varepsilon/(\varepsilon-1)c_1\Psi}{B} - \varepsilon c_1\Xi^*\} - \frac{\varepsilon c_1(1+pc_1/c_2)}{\lambda B},$$

$$J^{\theta}{}_{22} = D^{\theta}J^*_{22} + \frac{[-\theta_1(\varepsilon-1)][1+p(c_1/c_2)]\varepsilon/(\varepsilon-1)c_1}{B}[\frac{(\alpha_1-\alpha_2)\Psi D^{\theta}}{k\Xi^*} + \frac{\alpha_2}{k}] + [\frac{(\alpha_1-\alpha_2)c_1}{k} - (1-\alpha_1)\delta](1-D^{\theta}),$$

where $\Xi^{\theta} = -\frac{1+\theta(\varepsilon-1)}{\varepsilon}[1+\frac{1-\theta}{B}]\Psi$ and $D^{\theta_1} = (1-\frac{\alpha_2}{\alpha_1-\alpha_2}\frac{\Xi^{\theta}}{\Psi})\frac{\Xi^*}{\Xi^*+\Xi^{\theta}}$.

Denote

$$a_3 \equiv \frac{\Xi^*\Psi}{(\varepsilon-1)(1+pc_1/c_2)}[\Omega_4 + (1+\frac{\alpha_2\Xi^*}{\alpha_1-\alpha_2})\Omega_2],$$

$$b_3 \equiv \frac{\Xi^{*2}}{(1+pc_1/c_2)^2}[(1+p\frac{c_1}{c_2}\frac{\varepsilon}{\varepsilon-1})\Omega_4 - (1+\frac{\alpha_2\Xi^*}{\alpha_1-\alpha_2})\frac{\varepsilon\Omega_3(1+pc_1/c_2)}{\Psi}] + \frac{\Xi^*\Psi\varepsilon c_1}{(\varepsilon-1)(1+pc_1/c_2)}[(1+\frac{\alpha_2\Xi^*}{\alpha_1-\alpha_2})\frac{J^*_{11}(\alpha_1-\alpha_2)}{k} + \frac{J^*_{12}(1+pc_1/c_2)}{\lambda^*}],$$

$$d_3 \equiv \frac{\Xi^{*2}(1+p\frac{c_1}{c_2}\frac{\varepsilon}{\varepsilon-1})}{(1+pc_1/c_2)}\frac{J^*_{12}\varepsilon c_1}{\lambda^*} - \frac{\Xi^{*2}(2+p\frac{c_1}{c_2}\frac{\varepsilon}{\varepsilon-1})}{(1+pc_1/c_2)^2}\Omega_4 + \frac{\Xi^*\Psi}{(\varepsilon-1)(1+pc_1/c_2)}[Det(J^*) + (1+\frac{\alpha_2\Xi^*}{\alpha_1-\alpha_2})(\Omega_2 + \frac{\varepsilon\Xi^*\Omega_3}{\Psi} - \frac{J^*_{11}\varepsilon c_1(\alpha_1-\alpha_2)}{k})],$$

The relative slope condition that yields a sink is rewritten as

$$a_3[-\theta(\varepsilon-1)]^3 + b_3[-\theta(\varepsilon-1)]^2 + d_3[-\theta(\varepsilon-1)] + Det(J^*) > 0. \quad \text{(A4)}$$

Define $a=b_3/a_3$, $b=d_3/a_3$, and $d=Det(J^*)/a_3$. If we set (A4) equal 0, we obtain three critical values of $[-\theta(\varepsilon-1)]$: $m+n-a/3$, $m\omega+n\omega^2-a/3$and $m\omega^2+n\omega-a/3$, in which

$$m = \frac{1}{3}\sqrt[3]{-\frac{27d-9ab+2a^3}{2} + \sqrt{(\frac{27d-9ab+2a^3}{2})^2 + (3b-a^2)^3}},$$

$$n = \frac{1}{3}\sqrt[3]{-\frac{27d-9ab+2a^3}{2} - \sqrt{(\frac{27d-9ab+2a^3}{2})^2 + (3b-a^2)^3}},$$

$$\omega = \frac{-1+\sqrt{3}i}{2}.$$

Denote the three critical values as φ_1, φ_2 and φ_3. Let φ_3 be the largest and φ_1 be the smallest value of the three critical values. Then, the required conditions of a steady state that is a sink are summarized as

(i) $\max\{0, \varphi_1\}<[-\theta(\varepsilon-1)]<\max\{0,\varphi_2\}$ *or* $[-\theta(\varepsilon-1)]>\max\{0, \varphi_3\}$, *if* $a_3>0$;

(ii) $0<[-\theta(\varepsilon-1)]<\max\{0,\varphi_1\}$ *or* $\max\{0, \varphi_2\}<[-\theta(\varepsilon-1)]<\max\{0, \varphi_3\}$, *if* $a_3<0$.

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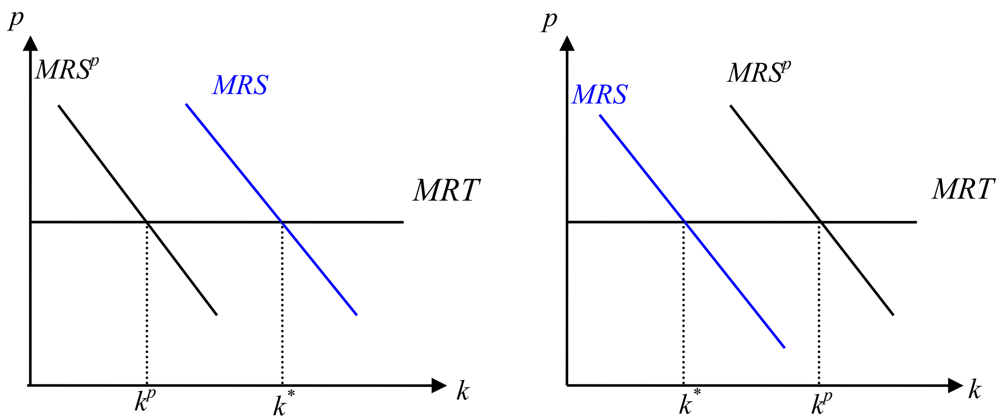
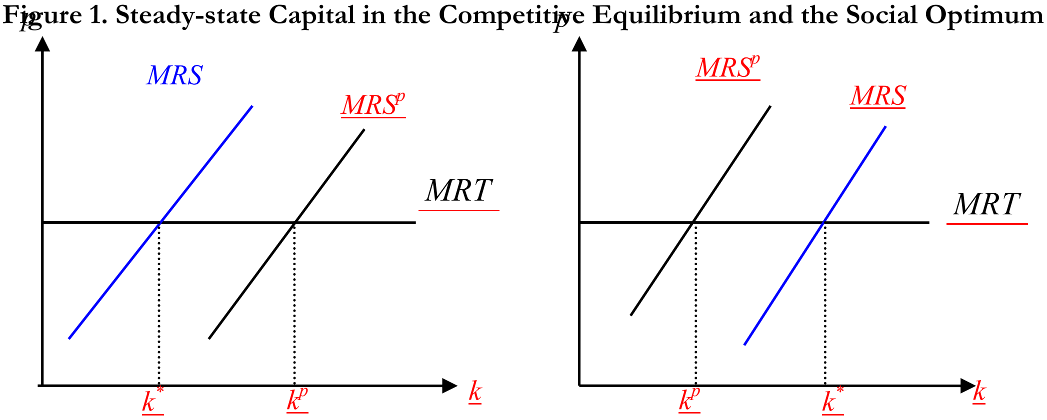


A1. $x_1 > x_2$ **and** $\theta_1 > \theta_2$

A2. $x_1 > x_2$ **and** $\theta_1 < \theta_2$

Figure 1. Steady-state Capital in the Competitive Equilibrium and the Social Optimum



B1. $x_1 < x_2$ **and** $\theta_1 > \theta_2$

B2. $x_1 < x_2$ **and** $\theta_1 < \theta_2$

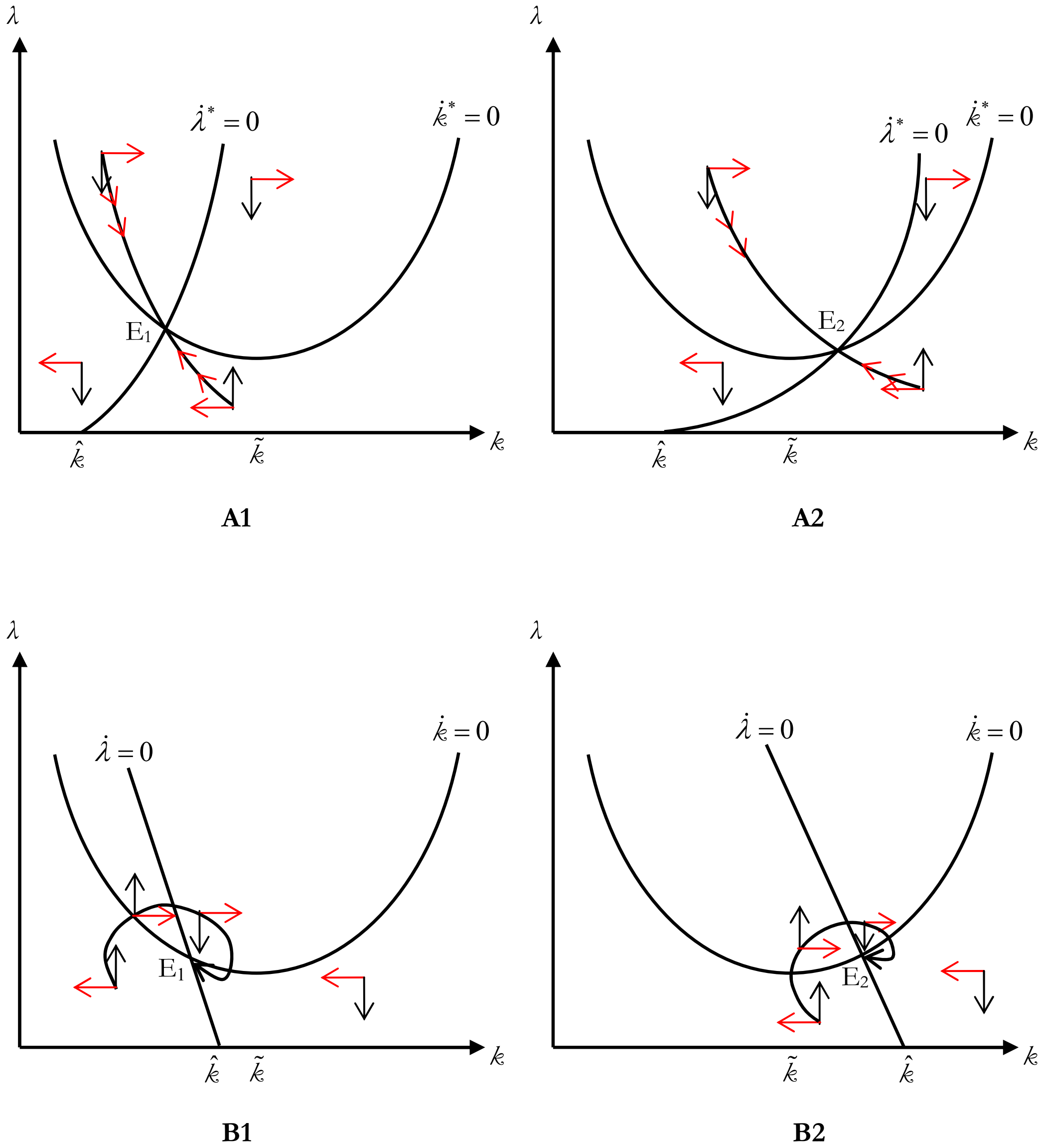


Figure 2: Two-sector Growth Model with a Sufficiently Large *KUJ* Effect: a Sink

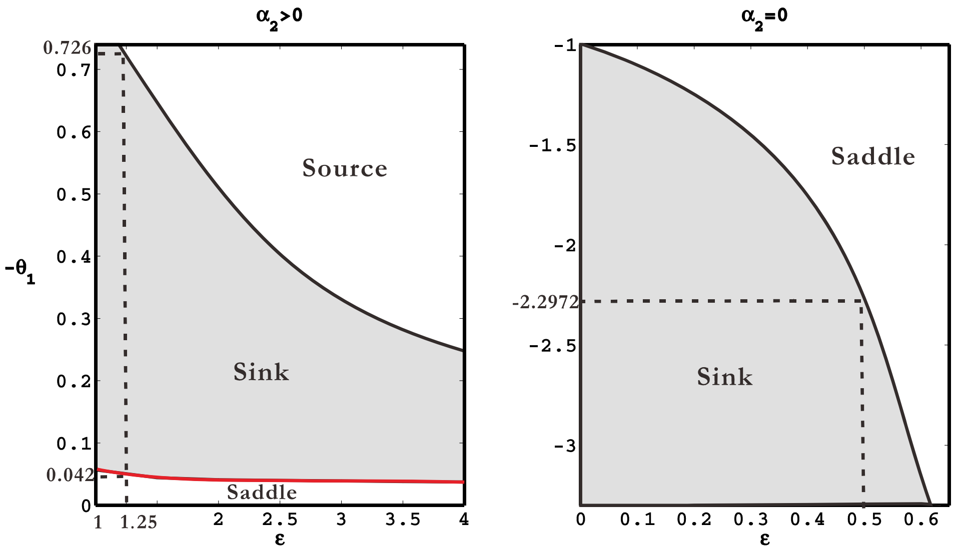


Figure 3: $\theta_1 \neq 0$

(Baseline parameters: α_1=0.32, α_2=0.28, δ=0.05, ρ=0.04, γ=0.1654, θ_1=θ_2=0.)

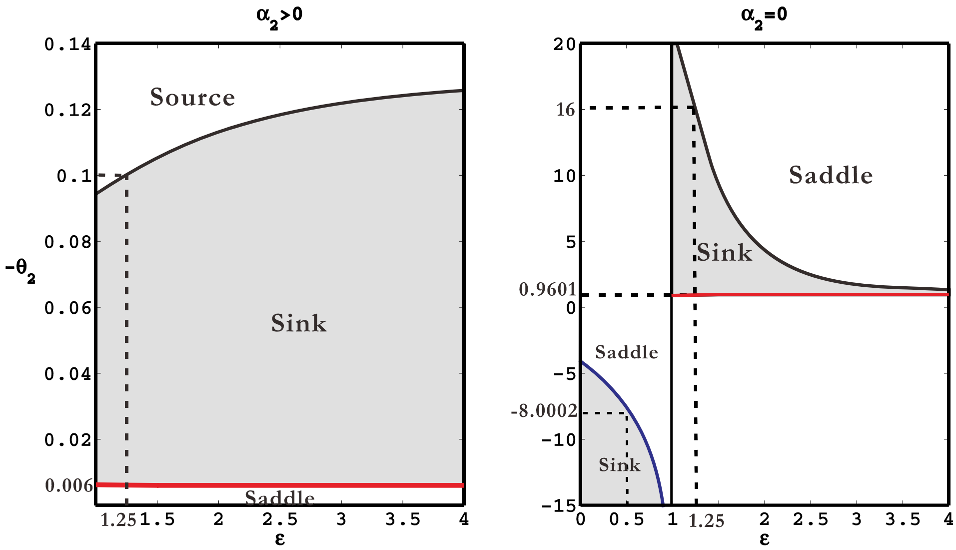


Figure 4: $\theta_2 \neq 0$

(Baseline parameters: same as Figure 3.)

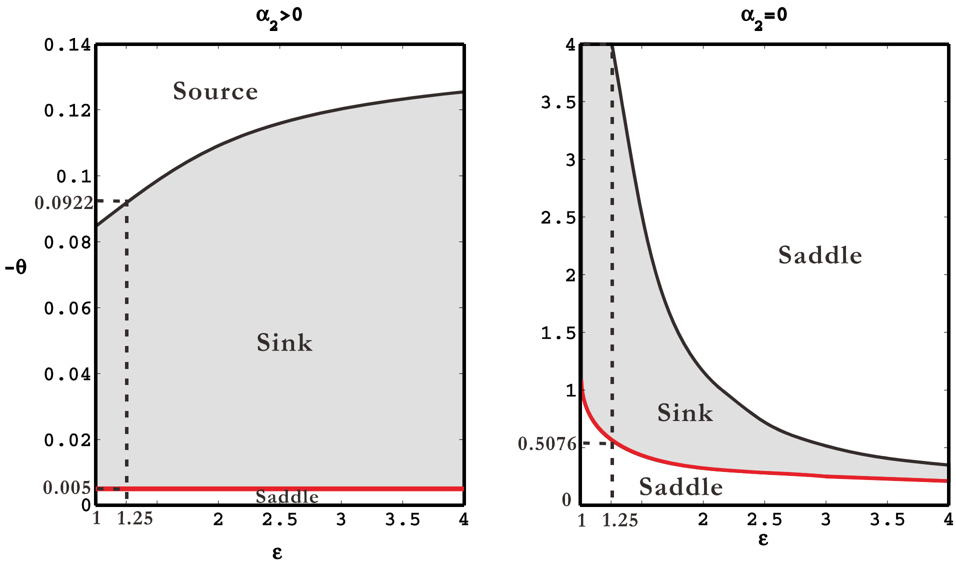


Figure 5: $\theta_1 = \theta_2 = \theta \neq 0$
(Baseline parameters: same as Figure 3.)

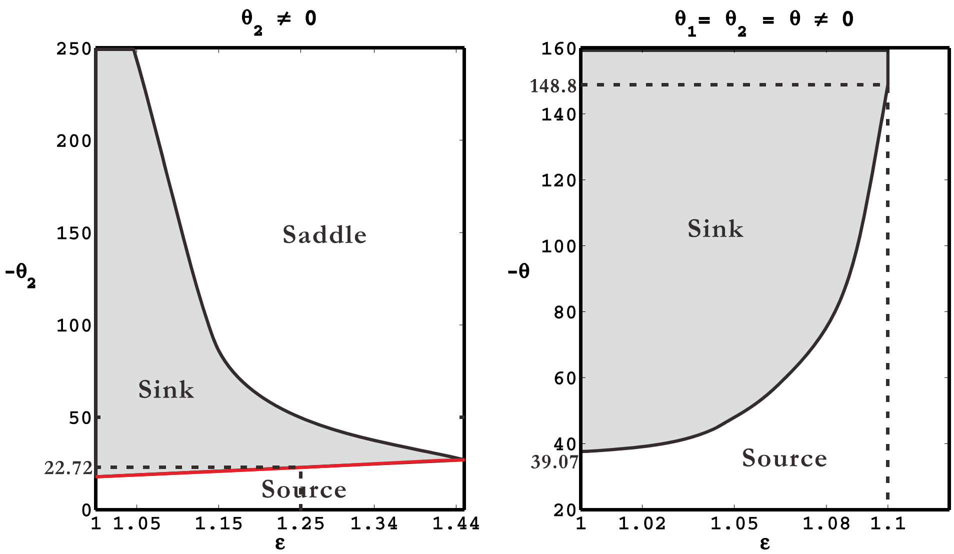


Figure 6: Factor Intensity Reversal
(Baseline parameters: α_1=0.28, α_2=0.32, δ=0.05, ρ=0.04, γ=0.1654, θ_1=θ_2=0.)