

A Different Attitude for Mathematical Formularization between Marxian and Western Economics

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ABSTRACT

Mathematical modeling is more advanced in modern economics than in Marxist economics. But why is this? One answer deals with the characteristics of modern economics, which ignore what mathematics is not good at. Another answer is that Western economics assumes that a human being is a “rational economic man,” prompting economists to use the optimization theory. Many “Marxian economists” do not like this assumption.

However, in my opinion, a “rational economic man” is a materialistic concept of a human being, so I built two types of Marxian models based on the optimization theory: Lenin’s model of imperialism and the “Marxian optimal growth model.”

One more important characteristic of mathematical modeling is that it makes science progress incrementally, which gives modern economics a tendency to avoid challenging researches in a referee system of their journals. However, this type of attitude becomes an obstacle to explain phenomena that have not been seen before or do not fit their ideologies. In this sense, mathematics has to be used by ideologically free scientists; that is, Marxists.

In this paper, we also referred to characteristics of “area studies” in comparing with Marxist methodologies.

Keywords: Mathematical modeling, Marxian economics, Western economics, historical materialism, Peer Review System

JEL Classification Numbers: A10, B24, C60

1 Introduction

Although mathematics is employed in Marxian economics, it is used at a higher level in Western economics. Therefore, this issue gives rise to various controversies, such as whether it denotes the superiority of Western economics or whether the idea expressed using mathematics is much more important. In this paper, I will compare the use of mathematics in Marxian and Western economics.

2 Economics that is neither Western nor Marxian; economics that is both Western and Marxian

Any attempt at comparing these two schools of economics immediately runs into a problem: vagueness in terms of how each school is defined. Marxian economics covers a broad range of topics, and even if it is regarded as being the economics of Marx, we include in it economics that raises objections in some form or another to Marx's propositions. It is also not particularly odd that Nobuo Okishio served as President of the Japan Association of Economics and Econometrics (the previous incarnation of the Japanese Economic Association—essentially, the biggest Japanese economist association in Western economics), even though the theories of economists like Okishio and Michio Morishima were classified under Marxian theory. In other words, while some aspects of Marxian economics are accepted by Western economics, others are not, implying that a variety that is both Western and Marxian can exist.

In fact, the research that my team and I have been pursuing has entailed the construction of such a school of economics. Specifically, we have developed two models, which we present below.

Mathematical model of Lenin's theory of imperialism

This model was expounded in Lenin's book, *Imperialism, the Highest Stage of Capitalism*. Initially, I presented it in the form of an econometric model of international linkages,¹ as the framework of Lenin's theory of imperialism was extremely easy to formularize mathematically and was applicable to the actual economy. The theory was obviously intended to explain the inevitability of imperialist wars, but its logic was as follows: (i) Countries with little capital—that is, underdeveloped countries—offer high profit rates because the marginal productivity of capital is high; therefore, capital flows into them from advanced countries. (ii) Due to the capital inflow, the growth rates of underdeveloped countries outstrip those of advanced countries, enabling them to catch up with the latter. (iii) The uneven development calls for a re-division of the world, so that wars to accomplish the mission become inevitable. Some have argued that this logic was no longer applicable, as no world war has occurred since 1945. As a result of capital movement, however, East Asia has been catching up with the advanced countries, a process that has led to severe and continued trade friction. It is also realistic to view the various wars that have occurred since World War II—the Korean War, Vietnam War, Gulf War, Afghan War, and Iraq War—as wars to re-divide between East and West, or between the U.S.

¹ The Pacific Rim Econometric Model, which forms the basis for this theory, is presented in Onishi (1994a, b, 1995, 1998), etc.

and the forces opposed to it.² In this sense, using actual data to trace the logic of Lenin's theory of imperialism is not particularly difficult. I was thus able to transform the various relationships involved in (i) and (ii) above into an econometric model of international linkages.

Note that the relationships involved in (i) are based on the assumption of diminishing returns to capital, which is the direct opposite of Krugman's assumption. Krugman (1981) modeled the "Lenin case" as he understood it while maintaining this contrary assumption; yet, even if a similar conclusion were reached based on a different assumption, this could not be regarded as a "Leninist model." In Onishi (1997), therefore, I explained the catch-up process described in (i) and (ii), which is based on the assumption of diminishing returns to capital.³ Because Lenin was a Marxian economist, this "Leninist model" is also an example of the "mathematical formularization of Marxian economics."

Formation and development of the "Marxian Optimal Growth Theory"

Another Marxian model that my research team and I have been working on is called the "Marxian Optimal Growth Model." This new field has seen the publication of numerous related papers since it was first presented by Yamashita and Onishi (2002). The aim is to construct a model of capitalism that is more fundamental than previous models. It defines a capitalist society as "a society which has made the accumulation of capital its primary purpose after its industrial revolution" and demonstrates how capitalism arises, develops, and ends. A very brief description of the basic framework is presented below.

Like Marx's model, this model has two sectors: one producing consumption goods and the other, capital goods. Each sector is expressed using the formulas:

$$\text{Consumption-goods sector: } Y(t) = A\{[1 - k(t)]L\}^\beta K(t)^\alpha \quad (1)$$

$$\text{Capital-goods sector: } \dot{K}(t) = Bk(t)L. \quad (2)$$

Y denotes the output of consumer goods; L , a society's total labor force; and K , the capital stock employed in the production of consumption goods. The addition of (t) indicates that these are variables. $\dot{K}(t)$ denotes an increase in the

²See Onishi (2003) for an understanding of this postwar political and economic structure.

³This paper later formed one of the chapters of Onishi (1998). I also presented the model to overseas researchers in Onishi and Liu (2003) and Onishi (2010).

capital stock K in the period t . Depreciation is ignored, so $\dot{K}(t)$ is same as the amount of capital goods produced in the period concerned. A and B show the total factor productivity for each production function. The key points in these formulas are: the total labor force is allocated to the production of consumption goods and capital goods according to the ratio $1 - k(t):k(t)$, and this allocation ratio is set to maximize the diachronic utility of a “representative household.” Expressed in mathematical form, the formula is

$$\max U = \int_0^{\infty} e^{-\rho t} \log Y(t) dt. \quad (3)$$

U is the diachronic utility of a representative household and ρ , the time preference rate.

The description above may elicit various opinions, such as (i) the depreciation rate should be taken into account; (ii) the capital goods sector also employs capital as a factor of production; (iii) the solution needs to be derived not from a model based on a “social planner” like the model above, but from one based on a decentralized market mechanism; and (iv) what would happen if it were expressed at the level of value? However, these issues have been dealt with by Liu (2008), Kanae (2008), and Roxiangul and Kanae (2009), while Onishi and Yamashita (2003), Yamashita (2005), and Onishi (2005) have expanded the model to include two social classes: capitalists, who own a lot of capital goods, and workers, who possess little capital.

The key characteristic that classifies these Marxian models as such is the fact that they mathematically demonstrate the laws governing the birth, development, and death of capitalism. Capital accumulation was useless as a way of expanding output in pre-capitalist (pre-industrial revolution) societies, so there was no need for it. However, the appearance of machinery due to the industrial revolution effected a fundamental change in technological needs, and made capital accumulation essential. Even so, there is an optimal level to capital accumulation.⁴ Once the optimal level is reached, further accumulation becomes unnecessary. At this point, capitalism, defined as “a society which has made the accumulation of capital its primary purpose after its industrial revolution,” comes to an end. All of the social systems in capitalism, such as the political system and the ideology, are designed to encourage the accumulation of capital. Through these models, therefore, the “death of capitalism” has been proved to occur when all these systems become unnecessary. In the past, we had believed historical materialism to be no more than a hypothesis; yet here, for the first time, it has become an object for mathematical proof. For this reason, we believe this mathematical model constitutes the core of Marxian theory.

⁴This level is expressed as $K^* = \frac{\alpha BL}{\beta \rho}$.

Although these are Marxian models, they also follow the rules of Western economics and can thus be regarded as “economics that is both Western and Marxian.” I often have been dissatisfied with many arguments of “Marxians” that do not have any clear relationship with Marx’s theory. Occasionally, there is even a denial of technology’s critical role in the determination of social systems or of history’s adherence to laws, but such arguments cannot be called Marxian economics. However, since most of them cannot be labeled as Western economics either, they fall into the category of “economics that is neither Western nor Marxian.” We are involved in a different kind of economics, though: “economics that is both Western and Marxian.” We believe that only this type of economics is valuable.

3 The outstanding achievements of Western economics

Previous researches in Western economics that we have referred to

The existence of “economics that is both Western and Marxian” means that its research includes achievements in Western economics or that there are previous researches in that field. Actually, the two models mentioned above also build on the outstanding achievements in Western economics.

For example, Krugman (1981) constitutes a previous research on the Leninist model. Although his understanding of Lenin in said paper is incorrect, he attempted to construct models for the theories of Lenin and Hobson in a famous journal of Western economics. Furthermore, reversing his assumption on capital returns, could lead our Leninist model.

There is even more previous researches in Western economics on the Marxian Optimal Growth Theory, as (i) this model makes the optimal control of the growth rate endogenous and thus puts a target of capital accumulation, (ii) even the optimal growth theory comprises two sectors, consumption and capital goods, and (iii) the model expresses a historical process through which the relative importance of the two sectors changes over time. These are also characteristics of the one-sector optimal growth models developed by Ramsey (1928) and Cass (1965, 1966), the two-sector model first formularized by Uzawa (1964) and others, and the model proposed by Uzawa and Lucas (1988). Uzawa acknowledges that his two-sector model borrows from Marx’s idea, so that it can also be said that this kind of research in Western economics actually builds on research by Marx and on Marxian economics.

From a slightly broader interpretation, Marx, along with Leon Walras, can be said to have been the founders of mathematical economics. His schema of reproduction was groundbreaking in that it provided an equilibrium theory-based formula describing the interdependence of different macroeconomic sectors. Certain mathematical economists in the field of Western economics accept this schema without question, and some of them support the labor theory of

value. Because Marx's schema of reproduction does not employ the concept of marginality, it is considerably different from the current mainstream of Western economics. However, ideas such as Samuelson's theory of substitution have provided a bridge between Marxian linear models and marginalist models. It therefore needs to be recognized that such outstanding researches in Western economics have contributed to the development of the Marx model.

The expansion of our two-class model of Marxian Optimal Growth Theory involved the use of "analytical Marxism" and a formula for exploitation developed by Mitsuchi (1984). Although this research obviously forms a part of Marxian economics, it has also been acknowledged in Western economics. It can thus be considered previous research in the field of "economics that is both Western and Marxian."

Points of contact between Western and Marxian economics

The definitions of Western and Marxian economics are not mutually exclusive in the way that materialism and idealism, male and female, and up and down are. "Economics that is both Western and Marxian" can exist because the two fields use separate criteria in separate dimensions, so that they overlap with each other.

For example, the current *de facto* definition of Western economics is a system formulated as an optimization problem by rational individuals.⁵ Marxian economics, however, does not define people as "irrational individuals." Rather, we can say that the view of people as "rational individuals" is extremely materialistic and should therefore also be regarded as Marxian economics. Nonmainstream economics, which has been seen as distinct from Marxian economics, has been frequently accused of being autotelic in its opposition to mainstream economics, as typified by the neoclassical school. Most of its proponents, however, have simply been trying to get the world to understand that "people are irrational." However, the core meaning of "rational" as it is used here is the assumption of the existence of "homo economicus," which states that human behavior is dictated by whether something is in the person's interest or not. It should also be pointed out that this reflects a materialistic concept of human beings.

Of course, the Marxian approach presumes that people do not act only in their own interest. Thus, Marxian activists do not act for their personal interests and Marxian economists do not conduct research for their personal interest. Their morals are therefore "irrational," at least in the sense described above. However, one's values and the objective observation of reality are completely different things. More accurately, observing real phenomena through completely

⁵This definition of "rationality" includes "bounded rationality."

unclouded eyes is the “righteousness” (justice) of the Marxians, that is, social scientists. If that is indeed the case, “the capitalist and working classes have different interests, and therefore engage in a class struggle.” This view means that we should accept that the laws of social motion can be explained by “interests,” and that workers (and capitalists) are engaged in a struggle for their own interests, rather than justice. Of course, this is not only a behavioral principle of classes at the macro level, but also a behavioral principle of individuals at the micro level (e.g., human behavior within a company).

However, if we accept the “Marxians’ view” that “people do not act only in their own interest,” and deem the notion that people act in their interest to be, say, 70% correct, we must accept that the remaining 30% of behavior is accounted for by other criteria, such as justice and culture. If this were the case, then economics formulated as an optimization problem by rational individuals, or “materialist economics” as defined above, would only be 70% accurate. Such an approach, however, fails to understand the points of Marx’s materialist theory of the economic formation of society, that is, the theory of superstructure. Marx’s theory of materialism states that human consciousness is a reflection of their interests, and the concept of *justice* arose for the purpose of *justifying* interests. In other words, if the various classes advocated only their own interests, they would be unable to satisfy them. Therefore, to justify them, “interest” needs to be rephrased as “*justice*,” and this process is described as “*justification*.” However, this objective is also an “interest,” so “interests” essentially and fundamentally determine society.

The employment of this framework, formulated as an optimization problem, facilitated a dramatic development in the use of mathematics—the approach most suited to optimization. Of course, the formularization of the mechanism rephrasing interest as justice needs to be conducted separately. Even so, we need to return to the original Marxian view of people as materialistic in the sense described above for employing mathematics in Marxian economics.

4 The weaknesses of Western economics

The development and weaknesses of formularizing technology in Western economics

Obviously, the mere employment of the “optimization theory” does not make it Marxian economics. At the very least, historical materialism is not just about explaining people; it also encompasses the fundamental idea that technology determines the nature of a society. In addition, while Western economics has had some success in producing mathematical formulas, it has not succeeded in doing so beyond a certain level. For this reason, our Marxian Optimal Growth Theory also grew out of this field and has been the subject of continued research to develop it.

What this means is that although the branch of Western economics that deals with the long run is “growth theory,” research in this area is always focused on technological progress. In this sense, it can be seen as a system of economics that is more concerned with technology than Marxian economics is. However, since it offers scant acknowledgement of economic systems such as feudalism, capitalism, and communism, it is unable to address issues, such as why the industrial revolution inevitably led to capitalism or why capitalism would come to an end. The Marxian view is that “the hand-powered mill gives you a society with the feudal lord, while the steam-powered mill produces a society with the industrial capitalist.”⁶ In other words, while technologies with different characteristics (i.e., tools, machines,) give rise to different societies, Western economics has been unsuccessful in expressing the qualitative characteristics of such technologies. It has gone no further than the analysis of the quantitative aspects of technology, such as labor productivity and total factor productivity. Even such advances as the Vintage Model have allowed only the age of fixed capital to be included.⁷ Our Marxian Optimal Growth Theory, however, has generated formulas demonstrating that per-person tool accumulation does not lead to an increase in output, so that (after having small amount of tools) the marginal productivity of tools is zero. In contrast, the accumulation of machinery does result in higher output and, therefore, the marginal productivity of machinery is positive. Using the consumer goods production function presented in the first section of this paper, “tool” technology would be expressed as $\alpha = 0$ and “machine” technology, as $\alpha > 0$. Put in another way, the industrial revolution, which is defined as the period when machinery appeared, could be understood as a jump of α from 0 into positive territory. This is an idea that Western economics could probably also have come up with, and it would not have been odd for the concept to be reported in Western economics literature. However, because of Western economists’ lack of interest in the broader issue of fundamental differences in the quality of technology, they did not move to develop models in this area. The problem, however, is not a deficiency in the economists’ ability, but in what they are trying to elucidate—namely, their different interests in different issues.

The peer-review system and the lack of interest in fundamental issues

Another weakness, this time in the framework itself, needs to be pointed out. It is easy to improve mathematical formularization incrementally by making

⁶From p. 166 of the 4th volume of the Collected Works of Marx and Engels.

⁷Other models analyzing the “quality of technology” include the Quality Ladder Model, which deals with quality, and the Product Variety Model, which covers product lines. Both models, however, are concerned with the quality of products, not production methods.

subtle changes in the assumptions on which it is based. This will likely either spark debate among Western economists concerning intricate details or, conversely, dampen interest in questioning the framework itself. The problem, therefore, is that analysis becomes “piecemeal” and is compounded by performance evaluation systems that only look at the number of papers published in famous academic journals.

People take for granted that the peer-review system is designed to select the good research; ironically, however, the very system discourages researchers from taking risks. Papers that are not rejected by anyone are generally unadventurous. On the other side, those that are authored by Marxian economists (i.e., nonempirical papers), who rarely employ the peer-review system, have always been adventurous. The tendency of some Marxian economists to say much about social problems shows their extremely lax attitude towards violating academic standards.

Meanwhile, it is also very important to note that Western economists have neglected fundamental issues, such as in the collapse of state-capitalism, the recent economic crisis, and the Iraq War without questioning their framework at all. It seems that Western economists, who are only concerned about getting their papers through the peer-review process, are generally disinclined to come to grips with fundamental issues.

This problem has been strikingly evident in the attitudes of Western economists when discussing the recent crisis. For instance, even though the present financial crisis has been studied in a narrow sense, with a great deal of work conducted on shedding light on the conditions that led to self-realizing price increases during the bubble, and a lot of analyses carried out on the causes of the failure of financial engineering techniques to predict prices, no interest in the bigger picture—why “dependence on finance” emerged in the U.S., Britain, and other countries—has been shown.⁸ For example, the fact that the recent crisis has seen the hub of the world economy shift from the U.S. to East Asia is deemed a mere coincidence, and the crisis has not been positioned as part of a larger framework and major historical phase.

Moreover, since Western economics makes no distinction between productive and unproductive labor, it has induced countries such as Britain, the U.S., and Iceland⁹ to be dependent on finance. This can also be considered a problem related to the lack of interest in “fundamental issues,” i.e., the structure of society and the economy. The formularization of optimization problems using

⁸It can also be said that while they have discussed the merits and demerits of individual policy measures, they have not attempted to explain the nature of the superstructure itself. For example, they have not asked themselves why such policy measures appear.

⁹Marxian economists have also tended to pay little attention to the productive labor theory. However, because the surplus value theory, which forms a part of it, is a “fundamental issue” rather than a government policy-level issue, these are key problems that must be addressed by Marxian economics.

mathematics and the inability to analyze issues that are excluded from this framework are two sides of the same coin.

5 The tasks of Marxian economists for mathematical formularization

“Theory-oriented” and “comprehension-oriented”

It is important to understand the nature of Western economics not only in the context of the presence or absence of interest in fundamental issues, but also in its “theory-oriented” characteristics. Using the aforementioned optimization theory, Western economics has succeeded in providing quite a simple description of society and the economy as a whole, so that its view has become very theoretical. This characteristic represents a stark contrast with “area studies,” which are focused on taking various factors into account.

This characteristic is also evident, for example, in the field of research on China, which I am engaged in. The *Japan Association for Modern China Studies*, Japan’s principal academic association for research on the modern China since 1950, is organized as a “comprehensive” academic association covering the fields of law, politics, history, sociology, and literature. This is an acknowledgment that China and matters related to it cannot be understood through separate studies conducted on economics, law, politics, history, sociology, literature, and thought. In other words, it recognizes the need for a “general analysis of social structure.” For example, the agendas for the association’s national and local annual conferences always carry multidisciplinary topics. A single issue is selected for each conference, and researchers from the fields of economics, law, politics, sociology, history, and literature work on it together. I, for one, have learned a lot from the presentation from fields other than economics at these conferences. Actually, the point is that the economists that attended have mainly been Marxian economists.

The situation changed several years back, however. A few years ago, Western economists with an interest in China launched their own academic association, the *Japanese Research Association for Chinese Economy*, as a vehicle for using the Chinese economy to test various theories in Western economics. This has led to some extremely odd research being conducted, which tends to be concerned only with the “application” of these theories to China (although this trend has weakened in recent years). Reflecting the dominance of Western economics over Marxian economics, this association already has more economists in its ranks than the *Japan Association for Modern China Studies* (though the latter still has more members as a whole). This reality makes one wonder what Western and Marxian economics actually are. I think Marxian economics, i.e. various propositions concerning historical materialism, also can provide a lot of individual “partial hypotheses” which can be tested empirically.

Here is an example: One of the materialist propositions is that a shift in the hub of the world economy will lead to a similar shift in the hub of political power. In other words, a cause-and-effect relationship exists that can be expressed using mathematical formulas. This can be tested by analyzing various indicators of political and economic power for the Pax Britannica and Pax Americana eras, and indicators that express the transition to the Asian era. The diminishing returns under Lenin's theory of imperialism (discussed at the beginning of this paper) can be tested by the estimating production functions for different countries. In addition, the technological shift from tools to machinery that occurred due to the industrial revolution can also be tested by estimating production functions from before and after the industrial revolution. Other hypotheses can also be tested. Marxian economists need to ponder hard on why they have not sought to prove their own theories in this manner.

Nevertheless, the reason Marxian economists have not followed that path is understandable: their hypotheses have always been "comprehensive," which makes the meaning of partial theories unclear. As a result, researchers prefer to discuss the validity of a hypothesis as a whole. For instance, when debating things, such as the complex problems in Japan during the country's high growth period or the structural changes wrought by the financial crisis in the U.S., it is impossible to test such characteristics as individual hypotheses separately. This is because the hypotheses are descriptive and that is all that happens.

Western economics, which has come to make heavy use of mathematics, has adopted a style in which, for example, a conclusion will be reached when there are diminishing returns in the economy, but not when there are increasing returns (an approach that is only possible with the use of mathematical formulas). Thus, as long as it can be ascertained whether or not the economy exhibits diminishing returns, the crux of the hypothesis can be checked. With Marxian economics, however, this style has failed to catch on due to a difference in the orientation, which depends on whether or not it has been successfully formulated using mathematics.

The "comprehensiveness" of Marxian economics involves looking at the economies of different countries as a single organism formed from the intertwining of the various "base" and "superstructure" factors. This research approach has actually come about as a result of the *base/superstructure* theory, that is, the framework of historical materialism. However, the failure to express things using mathematical formulas has been enough to separate it from scientific methodology, which is grounded in the development and verification of hypotheses. This is a shame.

The challenge of applying mathematics to Marxian economics

Therefore, the conclusion here is that the challenge of expressing Marxian economics in mathematical terms forms part of a broader range of problems that encompass the modernization of "empirical" approaches. However, for as

long as we are concerned with the application of mathematics to Marxian economics, not Western economics, the work must include unique elements. This implies that it must express all or some of the historical laws governing all social systems, including the *base* and the *superstructure*. This is not an easy task, but not an impossible one either.

I, however, believe that the key to supporting this challenge is for Marxian economists to reconstruct their view of history. For example, is the modern Chinese economy capitalist or socialist? Was the China under Mao an example of state socialism or state capitalism? Because it is impossible to discuss historical laws without an accurate understanding of issues such as these, research must again be done on concepts such as capitalism, feudalism, and socialism. Therefore, it would be acceptable for the reorganization of such concepts not to involve mathematics directly, or that this type of work be sufficiently active. A lot of the research that I have conducted has been at that kind of stage, or carried out with that kind of objective.

At any rate, it is essential for Marxian economics to employ mathematics, and the researchers involved will need to change their attitudes. While recognizing the strengths and weaknesses of Western economics, they must exert continued efforts in this area.

(This work was supported by JSPS Asian-Core Program.)

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