## AIP <br> The Journal of <br> Chemical Physics

## Infrared spectroscopy of endohedral HD and D2 in C60

Min Ge, U. Nagel, D. Hüvonen, T. Rõõm, S. Mamone et al.

Citation: J. Chem. Phys. 135, 114511 (2011); doi: 10.1063/1.3637948
View online: http://dx.doi.org/10.1063/1.3637948
View Table of Contents: http://jcp.aip.org/resource/1/JCPSA6/v135/i11
Published by the American Institute of Physics.

## Additional information on J. Chem. Phys.

Journal Homepage: http://jcp.aip.org/
Journal Information: http://jcp.aip.org/about/about_the_journal
Top downloads: http://jcp.aip.org/features/most_downloaded
Information for Authors: http://jcp.aip.org/authors

## ADVERTISEMENT



# Infrared spectroscopy of endohedral HD and $D_{2}$ in $\mathbf{C}_{60}$ 

Min Ge, U. Nagel, ${ }^{1}$ D. Hüvonen, ${ }^{1}$ T. Rõõm, ${ }^{1, \text { a) }}$ S. Mamone, ${ }^{2}$ M. H. Levitt, ${ }^{2}$ M. Carravetta, ${ }^{2}$ Y. Murata, ${ }^{3}$ K. Komatsu, ${ }^{3}$ Xuegong Lei, ${ }^{4}$ and N. J. Turro ${ }^{4}$<br>${ }^{1}$ National Institute of Chemical Physics and Biophysics, Akadeemia tee 23, 12618 Tallinn, Estonia<br>${ }^{2}$ School of Chemistry, Southampton University, Southampton SO17 1BJ, United Kingdom<br>${ }^{3}$ Institute for Chemical Research, Kyoto University, Kyoto 611-0011, Japan<br>${ }^{4}$ Department of Chemistry, Columbia University, New York, New York 10027, USA

(Received 1 July 2011; accepted 22 August 2011; published online 20 September 2011)


#### Abstract

We report on the dynamics of two hydrogen isotopomers, $\mathrm{D}_{2}$ and HD , trapped in the molecular cages of a fullerene $\mathrm{C}_{60}$ molecule. We measured the infrared spectra and analyzed them using a spherical potential for a vibrating rotor. The potential, vibration-rotation Hamiltonian, and dipole moment parameters are compared with previously studied $\mathrm{H}_{2} @ \mathrm{C}_{60}$ parameters [M. Ge, U. Nagel, D. Hüvonen, T. Rõõm, S. Mamone, M. H. Levitt, M. Carravetta, Y. Murata, K. Komatsu, J. Y.-C. Chen, and N. J. Turro, J. Chem. Phys. 134, 054507 (2011)]. The isotropic part of the potential is similar for all three isotopomers. In $\mathrm{HD} @ \mathrm{C}_{60}$, we observe mixing of the rotational states and an interference effect of the dipole moment terms due to the displacement of the HD rotation center from the fullerene cage center. © 2011 American Institute of Physics. [doi:10.1063/1.3637948]


## I. INTRODUCTION

Trapping of the hydrogen molecule inside the $\mathrm{C}_{60}$ molecular cage by means of chemical methods has created a system where the simplest and the lightest molecule is enclosed in the cavity of a molecule with the highest symmetry. ${ }^{1,2}$ The simplicity makes this supra-molecular complex ideal for the studies of non-covalent bondings between $\mathrm{H}_{2}$ and carbon nano-surfaces, the knowledge needed for the design of carbon-based hydrogen storage materials. $\mathrm{H}_{2} @ \mathrm{C}_{60}$ is simple from the theoretical point of view - the high symmetry of $\mathrm{C}_{60}$ brings it to the level of a quantum mechanics textbook example of a vibrating rotor trapped in a spherical potential well. ${ }^{3}$ From the experimental point of view all the trapping sites are equal and there is only one $\mathrm{H}_{2}$ per trapping site. This minimizes the inhomogeneous line broadening effects and $\mathrm{H}_{2}-$ $\mathrm{H}_{2}$ interactions. Indeed, the infrared (IR) absorption lines are well separated and show splittings due to translation-rotation coupling, ${ }^{4}$ consistent with theoretical predictions. ${ }^{5-7}$ The rotational quantum number $J$ is still a good quantum number for $\mathrm{H}_{2}$ despite the translation-rotation coupling. ${ }^{7,8}$ The potential parameters of $\mathrm{H}_{2} @ \mathrm{C}_{60}$ have been determined in both ground and excited vibrational states from the detailed temperature dependence study of IR spectra ${ }^{9}$ and in the ground state by inelastic neutron scattering (INS) spectroscopy. ${ }^{10}$ The introduction of a three site Lennard-Jones potential in the pairwise additive five-dimensional potential energy surface has improved greatly the accuracy of quantum calculations of the coupled translation and rotation eigenstates of $\mathrm{H}_{2} .{ }^{11}$ In general, it has been established that $\mathrm{H}_{2}$ inside the $\mathrm{C}_{60}$ cage is an unhindered vibrating rotor performing coupled translations and rotations in a nearly spherical potential. ${ }^{12}$

The substitution of deuterium for H changes the energy levels of the encapsulated molecule drastically. In $\mathrm{D}_{2} @ \mathrm{C}_{60}$,

[^0]the distances between the energy levels in the vibrational, rotational, and translational manifolds are reduced because of the heavier mass of D. Quantum statistics plays an important role in the dihydrogen wavefunction symmetry. ${ }^{13}$ The nucleus of D is a boson (nuclear spin $I=1$ ) as opposed to H , where it is a fermion, $I=1 / 2$. Thus the rotational states with an even quantum number $J$ have $\mathrm{D}_{2}$ in the state where the total nuclear spin of $\mathrm{D}_{2}$ is either zero or two. This is called ortho$\mathrm{D}_{2}$, while para- $\mathrm{D}_{2}$ has the total nuclear spin one and odd $J$ values. The ortho-para ratio at room temperature is $2: 1$ for $\mathrm{D}_{2}$. As in the case of $\mathrm{H}_{2}$, the conversion between ortho and para species is very slow. This means that at liquid helium temperature ( 4.2 K ) we have only two times more $J$ even (ortho) than $J$ odd (para) $\mathrm{D}_{2}$ although the $J=0$ state lies $E / k_{B}=84 \mathrm{~K}$ below the $J=1$ state. This is in sharp contrast with $\mathrm{H}_{2}$ where the $J$ even (para) species are outnumbered by the $J$ odd (ortho- $\mathrm{H}_{2}$ ) species by three times. Even though the hydrogen molecule is protected by $\mathrm{C}_{60}$ the orthopara conversion rate can be enhanced using a spin catalyst. ${ }^{14}$ The converted $\mathrm{H}_{2} @ \mathrm{C}_{60}$ sample, para-enriched at 77 K with molecular oxygen as a spin catalyst, ${ }^{15}$ was used to identify para and ortho absorption lines in the infrared spectra. ${ }^{9}$

The energy levels of heteronuclear $\mathrm{HD} @ \mathrm{C}_{60}$ are compressed compared to $\mathrm{H}_{2} @ \mathrm{C}_{60}$ as well. However, an additional effect appears, the breaking of two symmetries. First, the inversion symmetry of the diatomic molecule is lost. The consequence is that the center of mass is shifted away from the geometric center and a dipole moment is allowed for rotational transitions, $\Delta J= \pm 1$. Second, there are no ortho and para species since the nuclei of the HD molecule are not identical particles. The consequence is that thermal relaxation between even and odd $J$ levels is fast. All the rotational and translational levels of $\mathrm{HD} @ \mathrm{C}_{60}$ are in thermal equilibrium and at liquid helium temperature only one state with $J=N=0$, is populated. $N$ is the translational quantum number of the spherical oscillator. ${ }^{3}$

The dynamics of endohedral $\mathrm{D}_{2}$ and HD has been studied theoretically ${ }^{8}$ and of HD experimentally. ${ }^{10}$ Three transitions, $N=0 \rightarrow 1, J=0 \rightarrow 1$, and $J=1 \rightarrow 2$, were detected by INS spectroscopy. ${ }^{10}$ An interesting finding is that the energies of two rotary transitions do not scale according to the energies of a free rotor. This was predicted by Olthof et al. in the theoretical study of $\mathrm{CO} @ \mathrm{C}_{60} .{ }^{16}$ Simultaneous rotations of CO and its position vector $\mathbf{R}$ give rise to a rotational structure similar to that of free CO, but the effective rotational constants differ considerably. As theory predicts, the different $J$ and $N$ states are strongly mixed because of the translation-rotation coupling between the heteronuclear rotor and the inner walls of $\mathrm{C}_{60}$ making $J$ not a good quantum number as it is for a homonuclear diatomic in $\mathrm{C}_{60} .^{8,16}$

This paper is a continuation of our IR studies of $\mathrm{H}_{2} @ \mathrm{C}_{60} .{ }^{9}$ We present and analyze the IR spectra of $\mathrm{D}_{2} @ \mathrm{C}_{60}$ and $\mathrm{HD} @ \mathrm{C}_{60}$ and compare the potential, vibrating rotor and dipole moment parameters of the three isotopomers.

## II. THEORY

The model Hamiltonian of the vibrating rotor in the spherical potential, the dipole moment expansion, the calculation of IR absorption line intensities, and the fitting of IR spectra were described in Ref. 9. Here, we outline the consequences of the reduced symmetry of heteronuclear HD as compared to homonuclear $\mathrm{H}_{2}$ and $\mathrm{D}_{2}$. The coordinate $\mathbf{R}$ $=\{R, \Omega\}$ refers to the center of mass displacement of the dihydrogen from the geometric center of $\mathrm{C}_{60}$ and the rotational eigenstates $\left|J M_{J}\right\rangle$ are in the coordinate frame $\mathbf{s}=\left\{s, \Omega_{s}\right\}$, where the diatomic rotates about its center of mass and $\mathbf{s}$ connects the two nuclei of the hydrogen molecule.

The potential transforms according to the $A_{1 g}$ irreducible representation of the symmetry group $I_{h}$. The bipolar spherical harmonics $F_{\lambda m_{\lambda}}^{l j}\left(\Omega, \Omega_{s}\right)$ (Eq. (2) in Ref. 9) of the order $\lambda=0,6,10, \ldots$ transform according to the irreducible representation $A_{1 g}$ of the symmetry group $I_{h} .{ }^{17}$ We consider the case of full spherical symmetry, $\lambda=0$. If $\lambda=0$, then from the triangle rule $\lambda=|l-j|,|l-j|+1, \ldots, l+j$ it follows that $j=l$.

We write the potential energy for a diatomic molecule as ${ }^{v} V={ }^{v} V^{0}+{ }^{v} V^{\prime}$, where the isotropic harmonic term is given by ${ }^{v} V^{0}={ }^{v} V_{00}^{002} R^{2} F_{00}^{00} .{ }^{9}$ We expand the perturbation part of the potential ${ }^{v} V^{\prime}$ in spherical harmonics $F_{\lambda m_{\lambda}}^{l j}$ and in power series of $R$,

$$
\begin{align*}
{ }^{v} V^{\prime}= & { }^{v} V_{00}^{004} R^{4} F_{00}^{00} \\
& +\left({ }^{v} V_{00}^{111} R+{ }^{v} V_{00}^{113} R^{3}\right) F_{00}^{11} \\
& +\left({ }^{v} V_{00}^{222} R^{2}+{ }^{v} V_{00}^{224} R^{4}\right) F_{00}^{22} . \tag{1}
\end{align*}
$$

The powers $n$ of the displacement of the center of mass $R$ are determined by $l: n$ has the same parity as $l$ and $n \geq l$. Here, ${ }^{v} V_{\lambda m_{\lambda}}^{l j n}$ are the expansion coefficients and we limit our expansion to $j_{\max }=2$ and $n_{\max }=4 .{ }^{v} V_{00}^{004}$ is the anharmonic correction to the isotropic coupling, ${ }^{v} V_{00}^{002} .{ }^{v} V_{00}^{224}$ is the anharmonic correction to the anisotropic (translation-rotation) coupling ${ }^{v} V_{00}^{222}$. In the HD@ $\mathrm{C}_{60}$ potential, we have additional anisotropic coupling terms ${ }^{v} V_{00}^{111}$ and ${ }^{v} V_{00}^{113}$, that are absent
in the $\mathrm{H}_{2} @ \mathrm{C}_{60}$ and $\mathrm{D}_{2} @ \mathrm{C}_{60}$ perturbation potential, Eq. (7) in Ref. 9.

We express the induced part of the dipole moment as an interaction between $\mathrm{H}_{2}$ and $\mathrm{C}_{60}$ instead of summation over 60 pairwise induced dipole moments between $\mathrm{H}_{2}$ and carbon atom as was done in Ref. 9. We write the expansion of the dipole moment from the vibrational state $v$ to $v^{\prime}$ in bipolar spherical harmonics and in power series of $R$ as

$$
\begin{equation*}
d_{v^{\prime} v}\left(\mathbf{R}, \Omega_{s}\right)=\frac{4 \pi}{\sqrt{3}} \sum_{l, j, n}{ }^{v^{\prime} v} A_{\lambda m_{\lambda}}^{l j n} R^{n} F_{\lambda m_{\lambda}}^{l j}\left(\Omega, \Omega_{s}\right) . \tag{2}
\end{equation*}
$$

The dipole moment transforms according to the irreducible representation $T_{1 u}$ of the symmetry group $I_{h}$. The spherical harmonics of the order $\lambda=1,5,7, \ldots$ transform according to $T_{1 u}$ of the symmetry group $I_{h} \cdot{ }^{17}$ We use $\lambda=1$ and are interested in $v=0 \rightarrow 1$ transitions. In spherical symmetry it is sufficient to calculate one component of the dipole moment vector (Eq. (17) in Ref. 9), $m_{\lambda}=0$, and if we drop the explicit dependence of $d_{v^{\prime} v}$ on $v, v^{\prime}$ and of ${ }^{v^{\prime} v} A_{\lambda m_{\lambda}}^{l j n}$ on $v, v^{\prime}, \lambda, m_{\lambda}$, the $m_{\lambda}=0$ component of the dipole moment reads

$$
\begin{equation*}
d_{0}\left(\mathbf{R}, \Omega_{s}\right)=\frac{4 \pi}{\sqrt{3}} \sum_{l, j, n} A^{l j n} R^{n} F_{10}^{l j}\left(\Omega, \Omega_{s}\right) \tag{3}
\end{equation*}
$$

As $\lambda=|l-j|,|l-j|+1, \ldots, l+j$ and $\lambda=1$ it must be that $l=|j \pm 1|$. The possible combinations are $(l j)$ $\in\{(01),(10),(12),(21),(23), \ldots\}$. If we restrict the expansion up to $n_{\max }=l_{\text {max }}=1$, we get

$$
\begin{align*}
d_{0}\left(\mathbf{R}, \Omega_{s}\right) & =\frac{4 \pi}{\sqrt{3}} A^{010} F_{10}^{01} \\
& +\frac{4 \pi}{\sqrt{3}}\left(A^{101} F_{10}^{10}+A^{121} F_{10}^{12}\right) R \tag{4}
\end{align*}
$$

Here, $A^{010}$ is zero for homonuclear diatomic molecules. It describes the sum of permanent and induced rotational dipole moments, selection rule $\Delta N=0, \Delta J=0, \pm 1$, but $J=0$ $\rightarrow 0$ is forbidden. The expansion coefficient $A^{101}$ describes the induced dipole moment that is independent of the orientation of the diatomic molecular axis $\mathbf{s}$, selection rule $\Delta N$ $=0, \pm 1, \Delta J=0$, but $N=0 \rightarrow 0$ is forbidden. $A^{121}$ describes the induced dipole moment that depends on the orientation of $\mathbf{s}, \Delta N=0, \pm 1, \Delta J=0, \pm 2$, but $N=0 \rightarrow 0$ and $J=0 \rightarrow 0$ are both forbidden. All terms in Eq. (4) satisfy the selection rule $\Delta \Lambda=0, \pm 1$, but $\Lambda=0 \rightarrow 0$ is forbidden. $\Lambda$ is the total angular momentum $(\boldsymbol{\Lambda}=\mathbf{L}+\mathbf{J})$ quantum number and $L$ is the orbital angular momentum quantum number of the spherical oscillator. ${ }^{3}$

The procedure of fitting IR spectra was described in detail in Ref. 9. The values of quantum numbers we used to define the eigenstates $|v J N L \Lambda\rangle$ were $v=0$ and $1, J \leq 4$, and $N \leq 7$ for $\mathrm{D}_{2} @ \mathrm{C}_{60}, J \leq 4$, and $N$ $\leq 6$ for $\mathrm{HD} @ \mathrm{C}_{60}$. There are 292 para- $\mathrm{D}_{2}, 404$ ortho- $\mathrm{D}_{2}$, and 524 HD states in the selected bases. Energy levels and observed transitions are sketched in Figs. 1 and 2.


FIG. 1. Diagram of the observed IR transitions of $\mathrm{D}_{2} @ \mathrm{C}_{60}$. The rotational quantum number $J$ is even for ortho $-\mathrm{D}_{2}$ states and odd for para- $\mathrm{D}_{2}$ states. Vibrational quantum number is $v$ and translational quantum number is $N$. The additional structure of energy levels $E_{J N L \Lambda}^{v}$ due to orbital angular momentum quantum number $L$ and total angular momentum quantum number $\Lambda$ is not shown and only few lowest values of $v, J$, and $N$ are represented. Black lines are the transitions that start from the ground state, red lines are the transitions that start from the thermally excited states.


FIG. 2. Diagram of the observed IR transitions of $\mathrm{HD}^{@} \mathrm{C}_{60}$. The quantum numbers and the structure of energy levels are same as in Fig. 1. Black lines are the transitions that start from the ground state, red lines are the transitions that start from the thermally excited states. Dashed lines are the unique transitions of HD@ $\mathrm{C}_{60}$ which have selection rules different from the selection rules of $\mathrm{D}_{2} @ \mathrm{C}_{60}$ and $\mathrm{H}_{2} @ \mathrm{C}_{60}$. Arrow lines are the experimentally observed INS transitions Ref. 10.

## III. EXPERIMENT

The endohedral complexes were prepared by "molecular surgery" as described in Refs. 1, 2, $\mathrm{D}_{2} @ \mathrm{C}_{60}$ at Kyoto University and $\mathrm{HD} @ \mathrm{C}_{60}$ at Kyoto University and Columbia University. The filling factor for $\mathrm{D}_{2}$ is $\rho=1.0$. The HD@ $\mathrm{C}_{60}$ sample was a mixture of the hydrogen isotopomers $\mathrm{H}_{2}: \mathrm{HD}: \mathrm{D}_{2}$ with the ratio 1:1:0.2. Since all $\mathrm{C}_{60}$ cages are filled, the filling factor for HD is $\rho=0.45$. Experimental absorption spectra were corrected for the filling factor. The powdered samples were pressed into pellets under vacuum for IR transmission measurements. The diameter of sample pellets was 3 mm and the thickness $d=0.25 \mathrm{~mm}\left(\mathrm{D}_{2} @ \mathrm{C}_{60}\right)$ or $d=0.34$ $\mathrm{mm}\left(\mathrm{HD} @ \mathrm{C}_{60}\right)$. The measurement technique is described in Ref. 9.

## IV. RESULTS

## A. Temperature dependence of the spectra

The temperature dependence of the IR absorption spectra of $\mathrm{HD} @ \mathrm{C}_{60}$ and $\mathrm{D}_{2} @ \mathrm{C}_{60}$ are shown in Figs. 3 and 4. Only the spectral lines involving transitions from the ground translational state, $N=0$, are present at 5 K . As the temperature is increased, additional lines become visible that correspond to the transitions starting from the excited states, see Figs. 1 and 2. Broad lines that are indicated by arrows in Fig. 3(a) are not due to HD because they are present already at 5 K in the spectral region where only a single line is expected according to our model.

The normalized line areas of $\mathrm{D}_{2}$, Fig. 4(d) and HD, Fig. 5, are compared to the temperature dependencies of the normalized population probabilities of the corresponding initial states. The population probabilities are calculated using the eigen-energies obtained from fitting the 90 K spectra with the model Hamiltonian. We see that within the error bars the temperature dependence of the normalized line areas is described very well for the transitions starting from $J=0$ states both in $\mathrm{D}_{2} @ \mathrm{C}_{60}$ and $\mathrm{HD} @ \mathrm{C}_{60}$. In $\mathrm{HD} @ \mathrm{C}_{60}$, the areas of lines that start from $J=1$ state are weaker than the calculated curve above 120 K . In $\mathrm{D}_{2} @ \mathrm{C}_{60}$ the intensity versus temperature analysis can be reliably done for one line only, the $N=0 \rightarrow 1, J=0 \rightarrow 2$ transition at about $3224 \mathrm{~cm}^{-1}$. For other transitions in $\mathrm{D}_{2} @ \mathrm{C}_{60}$ there is an overlap of spectral lines starting from the ground and thermally populated states or an overlap of para and ortho lines, see Fig. 6.

The Hamiltonian parameters were determined by fitting the 90 K spectra because at lower temperatures the higher energy levels in the $v=0$ state are insufficiently populated, while no new lines appear at higher temperatures. The changes in line positions are less than the linewidths over the temperature range explored.

## B. Fitting the experimental spectra

There are much fewer observable IR absorption lines in the spectra of $\mathrm{D}_{2} @ \mathrm{C}_{60}$ and $\mathrm{HD} @ \mathrm{C}_{60}$ than in $\mathrm{H}_{2} @ \mathrm{C}_{60}$ spectra. The missing lines are the fundamental vibrational transition, $\Delta v=1, \Delta N=\Delta J=0$ and the group of lines, $\Delta v$


FIG. 3. Temperature dependence of the infrared absorption spectra of $\mathrm{HD}_{\mathrm{C}} \mathrm{C}_{60}$. In panel (a) the changes of the quantum numbers $N$ and $J$ of the main components of the initial and final states are $\Delta N=1, \Delta J=0$, or $\Delta N=0, \Delta J=1$. Panels (b), (c), and (d) show groups of transitions with the same change of the quantum numbers $J$ and $N$. Spectra are shifted vertically for clarity. The spectra shown in panel (d) are the differential absorption with respect to the 5 K spectrum. The broad lines indicated by arrows are not due to HD.
$=1, \Delta N=-1, \Delta J=0$, placed at lower energies than the fundamental transition. At least some of these lines are needed to determine the vibrational frequency of the diatomic


FIG. 4. Temperature dependence of the infrared absorption spectra of $\mathrm{D}_{2} @ \mathrm{C}_{60}$. Three panels show groups of transitions with the same change of the quantum numbers $J$ and $N$. Spectra are shifted vertically for clarity. The 200 K absorption spectra in panel (b) has been multiplied by five. The spectra shown in panel (c) are the differential absorption with respect to the 5 K spectrum. (d) Temperature dependence of the $3224 \mathrm{~cm}^{-1}$ IR absorption line normalized area $S$ (circles) and calculated normalized thermal population probability $p_{i}$ of the initial state $N=J=0$ (solid line). The error bars of $S$ are from fitting the experimental lines with Gaussians.
and the difference of the translational potential in the ground $(\nu=0)$ and excited $(\nu=1)$ vibrational states.

However, the separation of some energy levels in the ground vibrational state of $\mathrm{HD}^{0} \mathrm{C}_{60}$ is known from the INS experiment. ${ }^{10}$ The numbers we used are $\omega_{N=0 \rightarrow 1}=154.1 \mathrm{~cm}^{-1}, \omega_{J=0 \rightarrow 1}=80.7 \mathrm{~cm}^{-1}$, and $\omega_{J=1 \rightarrow 2}$ $=192.0 \mathrm{~cm}^{-1}$. The INS transition frequencies were included in our fitting program.

No additional experimental data beside IR data are available for $\mathrm{D}_{2} @ \mathrm{C}_{60}$. Therefore, we estimate the $\mathrm{D}_{2} @ \mathrm{C}_{60}$ fundamental frequency and set it to a fixed value in our fit of the Hamiltonian parameters. The relative change of the frequency $\left[\omega_{0}(\right.$ gas $\left.)-\omega_{0}\left(C_{60}\right)\right] / \omega_{0}$ (gas) depends on the cage and is independent of the hydrogen isotopomer. ${ }^{19}$ Based on our


FIG. 5. Temperature dependence of selected IR absorption line areas $S$ of HD@ $\mathrm{C}_{60}$. Each panel describes transitions starting from one initial state marked by quantum numbers $N$ and $J$. Solid lines are calculated thermal population probabilities $p_{i}$ of the initial states as obtained from the model. $S$ and $p_{i}$ have been normalized to unity at their maxima. The error bars of $S$ are from fitting the experimental lines with Gaussians.


FIG. 6. Experimental absorption spectra of $\mathrm{D}_{2} @ \mathrm{C}_{60}$ at 90 K (dots) together with the fit result with fixed line width, FWHM $=1 \mathrm{~cm}^{-1}$. The ortho $-\mathrm{D}_{2}$ (blue line) and para- $\mathrm{D}_{2}$ (olive line) components of the fit in panel (a) are shifted down in vertical direction for clarity. The total absorbance $\alpha$ (red line) is the sum of the ortho- $\mathrm{D}_{2}$ and para- $\mathrm{D}_{2}$ spectrum. In panels (b) and (c) the para component is not plotted as it does not have any contribution.
fit results, Table VI, we get that for the $\mathrm{H}_{2} \omega_{0}\left(C_{60}\right) / \omega_{0}$ (gas) $=0.9763$ and for the HD $\omega_{0}\left(C_{60}\right) / \omega_{0}($ gas $)=0.9773$. We use the average of these two ratios to calculate the $\mathrm{D}_{2} @ \mathrm{C}_{60}$ fundamental vibrational frequency, $\omega_{0}=2924 \mathrm{~cm}^{-1}$.

The line intensities depend on the ortho-para ratio, and on the absolute value and relative sign of the dipole moment
parameters. We choose $A^{101}>0$. The line intensities depend on the mixing of pure states by the perturbation part of the potential too. However, the perturbation part can be determined by fitting the IR transition frequencies.

We found that it is not possible to fit independently the two dipole moment parameters and the ortho-para ratio in the $D_{2}$ sample for the following reasons: first, the number of observed para lines is small; second, the strongest low temperature para line overlaps with ortho line at $3050 \mathrm{~cm}^{-1}$.

The ratio $n_{o} / n_{p}$ is close to the high temperature equilibrium value in $\mathrm{H}_{2} @ \mathrm{C}_{60}$ even at $5 \mathrm{~K} .{ }^{9}$ Since $\mathrm{H}_{2} @ \mathrm{C}_{60}$ and $\mathrm{D}_{2} @ \mathrm{C}_{60}$ were prepared under similar conditions, the orthopara ratio of the $\mathrm{D}_{2}$ sample in our fitting program was set to the room temperature equilibrium value $n_{o} / n_{p}=2$.

Detailed results of fitting the absorption spectra are shown in Figs. 6 and 7. The experimental and fitted absorption lines are summarized in Tables I and II together with the dominant components of the initial and final eigenstates. Eigenstates are listed in Tables III-V up to the highest states that are involved in the measured infrared transitions of ortho- $\mathrm{D}_{2}$ and HD, and 20 lowest levels of para- $\mathrm{D}_{2}$.

The best fit parameters are listed in Table VI. Because of the limited number of observed lines in the spectra of $\mathrm{D}_{2} @ \mathrm{C}_{60}$ and $\mathrm{HD} @ \mathrm{C}_{60}$, we cannot find the anharmonic corrections to the anisotropic confining potential, thus ${ }^{v} V_{00}^{224}=0$. The potential parameters are positive for all three hydrogen isotopomers. The distance between translational levels increases with the quantum number $N$, if the anharmonic corrections to the isotropic confining potential are greater than zero, ${ }^{v} V_{00}^{004}>0$. In $\mathrm{H}_{2} @ \mathrm{C}_{60}$ and $\mathrm{D}_{2} @ \mathrm{C}_{60}$ the sign of ${ }^{v} V_{00}^{004}$ also determines the order of $L$ levels for a given $N$ : the lowest energy level has the largest $L$ value. The sign of the translationrotation coupling ${ }^{v} V_{00}^{222}$ determines the order of $\Lambda$ levels for given $J, N$, and $L$, see Table VI. The ordering of $L$ and $\Lambda$

TABLE I. Experimental and calculated line positions, $\omega\left(\mathrm{cm}^{-1}\right)$, and absorption line areas, $S_{\omega}\left(\mathrm{cm}^{-2}\right)$, of IR-active modes at 90 K in $\mathrm{D}_{2} @ \mathrm{C}_{60}$. The quantum numbers $J N L \Lambda$ and the relative weights $\left|\xi^{v}\right|^{2}$ of the main components contributing to the initial and final eigenstates are given. Experimental line positions and areas at 5 K are shown for comparison. The ortho and para lines at $3050.2 \mathrm{~cm}^{-1}$ could not be resolved in the experiment and the total area of the line is listed as experimental $S_{\omega}$ in the corresponding parts of the table.

| Experiment, 5 K |  | Experiment, 90 K |  | Fitted |  | Initial state, $v=0$ |  | Final state, $v=1$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\omega$ | $S_{\omega}$ | $\omega$ | $S_{\omega}$ | $\omega$ | $S_{\omega}$ | $J N L \Lambda$ | $\left\|\xi^{0}\right\|^{2}$ | $J N L \Lambda$ | $\left\|\xi^{1}\right\|^{2}$ |
| para lines |  |  |  |  |  |  |  |  |  |
| 3045.1 | 0.98 | 3045.2 | 1.26 | 3045.0 | 0.45 | 1001 | 0.99 | 1111 | 0.97 |
| 3050.2 | 10.77 | 3050.2 | 5.40 | 3049.5 | 1.63 | 1001 | 0.99 | 1112 | 0.96 |
|  |  | 3054.0 | 0.19 | 3054.3 | 0.23 | 1111 | 0.97 | 1222 | 0.93 |
| 3056.8 | 1.55 | 3056.7 | 1.81 | 3056.6 | 1.26 | 1001 | 0.99 | 1110 | 0.96 |
|  |  | 3060.1 | 0.11 | 3060.6 | 0.19 | 1111 | 0.97 | 1221 | 0.65 |
|  |  | 3064.7 | 0.11 | 3065.0 | 0.15 | 1110 | 0.96 | 1201 | 0.61 |
|  |  | 3069.4 | 0.21 | 3070.8 | 0.21 | 1112 | 0.97 | 1201 | 0.61 |
|  |  | ortho lines |  |  |  |  |  |  |  |
|  |  | 3040.9 | 0.17 | 3040.7 | 0.22 | 2002 | 0.99 | 2112 | 0.97 |
|  |  | 3045.2 | 1.26 | 3045.4 | 0.67 | 2002 | 0.99 | 2113 | 0.96 |
|  |  |  |  | 3048.6 | 0.43 | 2002 | 0.99 | 2111 | 0.96 |
| 3050.2 | 10.77 | 3050.2 | 5.40 | 3050.9 | 4.10 | 0000 | 0.99 | 0111 | 0.96 |
|  |  | 3060.1 | 0.11 | 3058.5 | 1.76 | 0111 | 0.97 | 0222 | 0.93 |
|  |  | 3069.4 | 0.21 | 3069.0 | 0.31 | 0111 | 0.97 | 0200 | 0.88 |
| 3224.4 | 1.66 | 3224.3 | 0.86 | 3224.4 | 0.95 | 0000 | 0.99 | 2111 | 0.96 |
|  |  | 3440.3 | 0.09 | 3440.3 | 0.14 | 2002 | 0.99 | 4113 | 0.96 |

TABLE II. Experimental and calculated line positions, $\omega\left(\mathrm{cm}^{-1}\right)$, and absorption line areas, $S_{\omega}\left(\mathrm{cm}^{-2}\right)$, of IR-active modes at 90 K in $\mathrm{HD} @ \mathrm{C}_{60}$. The quantum numbers $J N L \Lambda$ and the relative weights $\left|\xi^{v}\right|^{2}$ of the main components contributing to the initial and final eigenstates are given. Experimental line positions and areas at 5 K are shown for comparison.

| Experiment, 5 K |  | Experiment, 90 K |  | Fitted |  | Initial state, $v=0$ |  | Final state, $v=1$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\omega$ | $S_{\omega}$ | $\omega$ | $S_{\omega}$ | $\omega$ | $S_{\omega}$ | $J N L \Lambda$ | $\left\|\xi^{0}\right\|^{2}$ | $J N L \Lambda$ | $\left\|\xi^{1}\right\|^{2}$ |
| 3710.8 | 23.45 | 3670.9 | 1.67 | 3670.7 | 1.57 | 1001 | 0.80 | 1112 | 0.64 |
|  |  | 3680.0 | 0.76 | 3680.1 | 0.81 | 1112 | 0.67 | 1223 | 0.55 |
|  |  | 3686.5 | 0.12 | 3681.5 | 0.06 | 1110 | 0.68 | 1201 | 0.50 |
|  |  |  |  | 3685.1 | 0.11 | 1111 | 0.95 | 1222 | 0.62 |
|  |  |  |  | 3691.2 | 0.16 | 2002 | 0.67 | 2113 | 0.48 |
|  |  | 3698.5 | 1.34 | 3698.6 | 1.31 | 1001 | 0.80 | 1110 | 0.64 |
|  |  | 3703.9 | 1.38 | 3704.0 | 1.70 | 1001 | 0.80 | 1111 | 0.94 |
|  |  | 3710.9 | 8.14 | 3711.1 | 7.94 | 0000 | 0.95 | 0111 | 0.73 |
|  |  | 3713.6 | 0.11 | 3713.7 | 0.19 | 1112 | 0.67 | 1222 | 0.62 |
|  |  | 3716.9 | 0.46 | 3716.4 | 0.19 | 1110 | 0.68 | 1221 | 0.67 |
|  |  |  |  | 3717.4 | 0.12 | 2002 | 0.67 | 2111 | 0.54 |
|  |  | 3727.7 | 3.24 | 3727.4 | 2.27 | 0111 | 0.74 | 0222 | 0.48 |
|  |  | 3737.0 | 0.56 | 3736.6 | 0.46 | 0111 | 0.74 | 0200 | 0.56 |
|  |  | 3740.9 | 1.43 | 3740.4 | 1.97 | 1001 | 0.80 | 2002 | 0.66 |
|  |  | $3743.8$ | $0.20$ | 3742.6 | 0.27 | 1111 | 0.95 | 2112 | 0.63 |
|  |  |  |  | 3758.5 | 0.07 | 1111 | 0.95 | 2111 | 0.54 |
|  |  | 3761.8 | 0.32 | 3760.9 | 0.21 | 1112 | 0.67 | 2113 | 0.48 |
|  |  |  |  | 3913.8 | 0.02 | 0111 | 0.74 | 1310 | 0.42 |
| 3906.7 | 1.05 | 3906.8 | 0.45 | 3906.7 | 0.41 | 0000 | 0.95 | 1201 | 0.50 |
|  |  |  |  | 3986.9 | 0.02 | 1001 | 0.80 | 1310 | 0.42 |
| 3988.9 | 1.600 | 3988.9 | 0.62 | 3989.6 | 0.53 | 0000 | 0.95 | 2111 | 0.54 |
|  |  |  |  | 3992.8 | 0.03 | 1001 | 0.80 | 1311 | 0.51 |
|  |  |  |  | 3997.7 | 0.007 | 1111 | 0.95 | 1422 | 0.38 |
|  |  |  |  | 4144.4 | 0.008 | 0111 | 0.74 | 0400 | 0.36 |
|  |  | 4149.9 | 0.40 | 4149.9 | 0.27 | 1001 | 0.8 | 3112 | 0.64 |

TABLE III. First 20 calculated energy levels of para- $\mathrm{D}_{2} @ \mathrm{C}_{60}$ for the ground $(v=0)$ and excited $(v=1)$ states. The excitation energies $\Delta E_{\text {para }}^{0}$ and $\Delta E_{\text {para }}^{1}$ (in $\mathrm{cm}^{-1}$ ) are relative to the ground-state energy $E_{1001}^{0}$ $=1696.21 \mathrm{~cm}^{-1}$ and $E_{1001}^{1}=4618.18 \mathrm{~cm}^{-1} \cdot\left|\xi^{v}\right|^{2}$ is the contribution of the dominant quantum basis $|J N L \Lambda\rangle$ to the particular eigenstate.

|  | $v=0$ |  |  |  | $v=1$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $J N L \Lambda$ | $\Delta E_{\text {para }}^{0}$ | $\left\|\xi^{0}\right\|^{2}$ |  | $J N L \Lambda$ | $\Delta E_{\text {para }}^{1}$ | $\left\|\xi^{1}\right\|^{2}$ |
| 1001 | 0.0 | 0.99 |  | 1001 | 0.0 | 0.99 |
| 1111 | 122.9 | 0.97 |  | 1111 | 123.1 | 0.97 |
| 1112 | 126.6 | 0.97 |  | 1112 | 127.5 | 0.96 |
| 1110 | 132.3 | 0.96 |  | 1110 | 134.7 | 0.96 |
| 1222 | 254.0 | 0.94 |  | 1222 | 255.2 | 0.93 |
| 1223 | 259.0 | 0.93 |  | 1223 | 261.1 | 0.90 |
| 1221 | 259.5 | 0.70 |  | 1221 | 261.6 | 0.65 |
| 1201 | 271.2 | 0.67 |  | 1201 | 275.4 | 0.61 |
| 3003 | 292.6 | 0.98 |  | 3003 | 282.1 | 0.96 |
| 1333 | 390.5 | 0.90 |  | 1333 | 393.0 | 0.87 |
| 1334 | 396.4 | 0.86 |  | 1334 | 398.6 | 0.70 |
| 1332 | 397.6 | 0.76 |  | 1332 | 401.6 | 0.71 |
| 1311 | 406.2 | 0.83 |  | 3113 | 404.8 | 0.94 |
| 1312 | 413.6 | 0.61 |  | 1311 | 410.2 | 0.80 |
| 3113 | 415.3 | 0.97 |  | 3112 | 410.8 | 0.86 |
| 3114 | 420.4 | 0.93 |  | 3114 | 412.7 | 0.75 |
| 1310 | 421.1 | 0.81 | 1312 | 421.9 | 0.59 |  |
| 3112 | 422.0 | 0.85 | 1310 | 428.6 | 0.78 |  |
| 1444 | 536.6 | 0.85 | 3224 | 536.9 | 0.78 |  |
| 1445 | 542.2 | 0.65 | 3223 | 537.5 | 0.83 |  |
|  |  |  |  |  |  |  |

levels of $\mathrm{HD}^{@} \mathrm{C}_{60}$ is very different from $\mathrm{H}_{2} @ \mathrm{C}_{60}$ and $\mathrm{D}_{2} @ \mathrm{C}_{60}$ due to strong mixing of states. However, for all three isotopomers only the states with the same $\Lambda$ are mixed, if the potential is spherical.

Some of the lines in the 90 K spectrum or at lower $T$ show small splittings: the $3056.8 \mathrm{~cm}^{-1}$ and $3224.4 \mathrm{~cm}^{-1}$ lines in $\mathrm{D}_{2} @ \mathrm{C}_{60}$ (Fig. 4(b)) and the $3698.5 \mathrm{~cm}^{-1}, 3710.8 \mathrm{~cm}^{-1}, 3906.7 \mathrm{~cm}^{-1}$, and $3988.9 \mathrm{~cm}^{-1}$ lines in $\mathrm{HD} @ \mathrm{C}_{60}$ (Fig. 3). These splittings are not described by our model, which assumes local spherical symmetry, and the split lines are treated as single experimental lines in the fit. As discussed before ${ }^{9}$ in the case of $\mathrm{H}_{2} @ \mathrm{C}_{60}$, the local icosahedral symmetry of the $\mathrm{C}_{60}$ cage is too high to give rise to such splittings. The shape of spectral lines showing this type of splitting is very similar in $\mathrm{H}_{2} @ \mathrm{C}_{60}, \mathrm{D}_{2} @ \mathrm{C}_{60}$, and even in $\mathrm{HD} @ \mathrm{C}_{60}$. This splitting is independent of the isotopomer and depends on the initial and/or on the final state defined by quantum numbers $v, J, N, L$, and $\Lambda$.

## V. DISCUSSION

## A. Hamiltonian parameters

The potential parameters ${ }^{v} V_{00}^{l j n}$ and vibration-rotation Hamiltonian parameters, $\omega_{0}, B_{e}, \alpha_{e}$, and $D_{e}$, are listed in Table VI for all three isotopomers. The harmonic ${ }^{v} V_{00}^{002}$ and anharmonic ${ }^{v} V_{00}^{004}$ part of the isotropic potential are


FIG. 7. Experimental absorption spectra of $\mathrm{HD} @ \mathrm{C}_{60}$ at 90 K (dots) together with the fit result with fixed line width, FWHM $=1 \mathrm{~cm}^{-1}$. The changes of the quantum numbers $J$ and $N$ for the transitions in panel (a) vary from line to line and are $\Delta J=0, \Delta N=1$, or $\Delta J=1, \Delta N=0$. Some lines in the experimental spectrum do not belong to HD and are indicated by arrows.
similar for three isotopomers. There is a slight trend that these two parameters are larger for the heavier isotopomers. If the separation of two energy levels, $E_{01111}^{1}-E_{0000}^{1}$ is accounted for in ortho- $\mathrm{D}_{2} @ \mathrm{C}_{60}, 126.7 \mathrm{~cm}^{-1}$ (Table IV), and in para$\mathrm{H}_{2} @ \mathrm{C}_{60}, 184.4 \mathrm{~cm}^{-1}$, ${ }^{9}$ we get the ratio 1.4566 , which is larger than the mass scaling, $\sqrt{m_{D_{2}} / m_{H_{2}}}=\sqrt{2}$. This tells us that the splitting of energy levels is larger in $\mathrm{H}_{2} @ \mathrm{C}_{60}$ even though the potential parameters are smaller. The controversy is resolved if one takes into account the anharmonic energy correction to the difference $E_{01111}^{1}-E_{0000}^{1}$. Beside the dependence on ${ }^{v} V_{00}^{002}$ and ${ }^{v} V_{00}^{004}$, which are larger for $\mathrm{D}_{2}$, the linear energy correction in the perturbation theory to this difference decreases with the increase of the molecular mass as $m^{-0.5}$.

The displacement of the center of mass from the bond center changes the dynamics of HD in the $\mathrm{C}_{60}$ cage drastically. In our model, this change is represented by the ${ }^{v} V_{00}^{111}$ and ${ }^{v} V_{00}^{113}$ potential parameters. These potential terms shift

TABLE IV. Calculated energy levels of ortho- $\mathrm{D}_{2} @ \mathrm{C}_{60}$ for the ground ( $v$ $=0)$ and excited $(v=1)$ states up to $J N L \Lambda=4113$. The excitation energies $\Delta E_{\text {ortho }}^{0}$ and $\Delta E_{\text {ortho }}^{1}$ (in $\mathrm{cm}^{-1}$ ) are relative to the ground-state energy $E_{0000}^{0}=1637.56 \mathrm{~cm}^{-1}$ and $E_{0000}^{1}=4561.71 \mathrm{~cm}^{-1} .\left|\xi^{v}\right|^{2}$ is the contribution of the dominant quantum basis $|J N L \Lambda\rangle$ to the particular eigenstate.

| $v=0$ |  |  | $v=1$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $J N L \Lambda$ | $\Delta E_{\text {ortho }}^{0}$ | $\left\|\xi^{0}\right\|^{2}$ | $J N L \Lambda$ | $\Delta E_{\text {ortho }}^{1}$ | $\left\|\xi^{1}\right\|^{2}$ |
| 0000 | 0.0 | 0.99 | 0000 | 0.0 | 0.99 |
| 0111 | 125.9 | 0.97 | 0111 | 126.7 | 0.96 |
| 2002 | 175.7 | 0.99 | 2002 | 169.2 | 0.98 |
| 0222 | 258.1 | 0.94 | 0222 | 260.2 | 0.93 |
| 0200 | 268.0 | 0.90 | 0200 | 270.8 | 0.88 |
| 2112 | 298.7 | 0.97 | 2112 | 292.3 | 0.97 |
| 2113 | 302.5 | 0.97 | 2113 | 297.0 | 0.96 |
| 2111 | 305.0 | 0.96 | 2111 | 300.2 | 0.96 |
| 0333 | 395.7 | 0.90 | 0333 | 399.6 | 0.88 |
| 0311 | 410.8 | 0.82 | 0311 | 415.7 | 0.79 |
| 2223 | 430.5 | 0.94 | 2223 | 425.4 | 0.93 |
| 2222 | 431.3 | 0.84 | 2222 | 426.3 | 0.81 |
| 2224 | 435.3 | 0.93 | 2224 | 431.3 | 0.92 |
| 2221 | 437.1 | 0.94 | 2221 | 433.5 | 0.92 |
| 2220 | 440.4 | 0.93 | 2220 | 437.8 | 0.92 |
| 2202 | 445.6 | 0.80 | 2202 | 442.6 | 0.77 |
| 0444 | 543.8 | 0.89 | 0444 | 552.0 | 0.88 |
| 2334 | 567.8 | 0.90 | 4004 | 562.0 | 0.98 |
| 2333 | 568.0 | 0.86 | 2334 | 564.3 | 0.89 |
| 0422 | 568.2 | 0.78 | 2333 | 564.6 | 0.83 |
| 2332 | 572.0 | 0.80 | 2332 | 569.3 | 0.75 |
| 2335 | 573.5 | 0.89 | 2335 | 571.2 | 0.88 |
| 2331 | 577.8 | 0.88 | 2331 | 576.8 | 0.86 |
| 0400 | 579.1 | 0.74 | 0422 | 579.5 | 0.76 |
| 2312 | 583.2 | 0.74 | 2312 | 581.3 | 0.68 |
| 4004 | 583.6 | 0.99 | 2313 | 588.7 | 0.74 |
| 2313 | 589.3 | 0.78 | 0400 | 591.9 | 0.71 |
| 2311 | 592.4 | 0.80 | 2311 | 592.5 | 0.77 |
| 0555 | 696.7 | 0.86 | 4114 | 684.9 | 0.96 |
| 4114 | 706.5 | 0.97 | 4115 | 690.5 | 0.96 |
| 4115 | 710.9 | 0.97 | 4113 | 691.9 | 0.96 |
| 4113 | 712.0 | 0.97 | 0555 | 709.1 | 0.85 |

the HD center of mass away from the cage center. HD cannot rotate in the cage unless the center of mass is moving along the circular trajectory, in a classical view. The rotational quantum number $J$ is not a good quantum number any more. ${ }^{8}$ The spherical symmetry is preserved and $\Lambda(\boldsymbol{\Lambda}=\mathbf{L}+\mathbf{J})$ is still a good quantum number.

The rotational and translational states are mixed in quantum mechanical terms. As an example, we analyze three states where the main component is $N=0$ and $J=0,1$, or 2 . It was pointed out in the INS study that the energies of these levels do not scale as is expected for the rotational states of a diatomic rotor. ${ }^{10}$ Our fit shows that the $J=1$ state of HD has $80 \% J=1, N=0, L=0$ component, Table V, and the next main component with $18 \%$ weight has quantum numbers $J=0, N=1, L=1$. The linear potential ${ }^{v} V_{00}^{111}$ couples the two components and they both have $\Lambda=1$. The next rotational level is $67 \%$ of $J=2, N=0, L=0$, and $28 \%$ of $J$ $=0, N=2, L=2$. Here, the ${ }^{v} V_{00}^{111}$ has zero matrix element between the two components. However, the coupling goes

TABLE V. Calculated energy levels of $\mathrm{HD} @ \mathrm{C}_{60}$ for the ground $(v=0)$ and excited $(v=1)$ states up to $J N L \Lambda=3112$. The excitation energies $\Delta E_{H D}^{0}$ and $\Delta E_{H D}^{1}$ (in $\mathrm{cm}^{-1}$ ) are relative to the ground-state energy $E_{0000}^{0}$ $=1964.05 \mathrm{~cm}^{-1}$ and $E_{0000}^{1}=5517.66 \mathrm{~cm}^{-1} .\left|\xi^{v}\right|^{2}$ is the contribution of the dominant quantum basis $|J N L \Lambda\rangle$ to the particular eigenstate.

| $v=0$ |  |  | $v=1$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $J N L \Lambda$ | $\Delta E_{H D}^{0}$ | $\left\|\xi^{0}\right\|^{2}$ | $J N L \Lambda$ | $\Delta E_{H D}^{1}$ | $\left\|\xi^{1}\right\|^{2}$ |
| 0000 | 0.0 | 0.95 | 0000 | 0.0 | 0.94 |
| 1001 | 80.3 | 0.80 | 1001 | 76.6 | 0.80 |
| 0111 | 153.3 | 0.74 | 0111 | 157.5 | 0.73 |
| 1112 | 202.4 | 0.67 | 1112 | 197.4 | 0.64 |
| 1110 | 225.2 | 0.68 | 1110 | 225.3 | 0.64 |
| 1111 | 231.0 | 0.95 | 1111 | 230.6 | 0.94 |
| 2002 | 272.1 | 0.67 | 2002 | 267.0 | 0.66 |
| 0222 | 318.1 | 0.47 | 0222 | 327.2 | 0.48 |
| 0200 | 324.0 | 0.61 | 1223 | 328.9 | 0.55 |
| 1223 | 334.5 | 0.59 | 0200 | 336.3 | 0.56 |
| 1201 | 357.5 | 0.51 | 1201 | 353.1 | 0.50 |
| 1222 | 365.4 | 0.70 | 1222 | 362.6 | 0.62 |
| 1221 | 384.0 | 0.70 | 1221 | 388.0 | 0.67 |
| 2113 | 413.3 | 0.54 | 2113 | 409.8 | 0.48 |
| 2112 | 420.6 | 0.72 | 2112 | 420.0 | 0.63 |
| 2111 | 432.6 | 0.60 | 2111 | 436.0 | 0.54 |
| 1334 | 474.7 | 0.53 | 1334 | 468.5 | 0.49 |
| 0333 | 481.0 | 0.44 | 0333 | 489.8 | 0.37 |
| 0311 | 500.1 | 0.44 | 1312 | 495.6 | 0.44 |
| 1312 | 501.2 | 0.46 | 1333 | 503.8 | 0.48 |
| 1333 | 507.5 | 0.56 | 1310 | 513.5 | 0.42 |
| 1310 | 515.7 | 0.46 | 1311 | 519.4 | 0.51 |
| 1311 | 522.6 | 0.54 | 0311 | 519.7 | 0.39 |
| 3003 | 533.3 | 0.83 | 3003 | 525.6 | 0.58 |
| 1332 | 535.6 | 0.54 | 1332 | 537.9 | 0.38 |
| 2224 | 562.8 | 0.43 | 2224 | 560.0 | 0.35 |
| 2222 | 565.5 | 0.57 | 2222 | 569.2 | 0.42 |
| 2223 | 575.3 | 0.50 | 1333 | 578.1 | 0.38 |
| 2202 | 588.8 | 0.33 | 2202 | 590.9 | 0.22 |
| 2220 | 594.6 | 0.52 | 2220 | 605.2 | 0.46 |
| 2221 | 600.3 | 0.49 | 2221 | 613.0 | 0.44 |
| 1445 | 628.1 | 0.45 | 2335 | 620.6 | 0.44 |
| 0444 | 652.0 | 0.42 | 2334 | 653.7 | 0.50 |
| 2334 | 658.0 | 0.45 | 2313 | 653.8 | 0.40 |
| 1423 | 661.1 | 0.38 | 3114 | 655.0 | 0.49 |
| 3113 | 667.8 | 0.88 | 3113 | 656.7 | 0.81 |
| 3112 | 677.3 | 0.62 | 2311 | 675.1 | 0.49 |
| 1422 | 678.6 | 0.41 | 1422 | 675.1 | 0.38 |
| 2311 | 680.8 | 0.48 | 3112 | 676.6 | 0.64 |

over an intermediate state what is $67 \% J=1, N=1, L=1$, $15 \% J=0, N=2, L=2$, and $15 \% J=2, N=0, L=0$. These states are $\Lambda=2$ states.

The potential terms ${ }^{v} V_{00}^{111}$ and ${ }^{v} V_{00}^{113}$ shift the HD center of mass from the $\mathrm{C}_{60}$ cage center. Using the potential values from Table VI we find that the minimum of the HD potential in the $v=0$ state is shifted $0.1383 \AA$ along the bond and in the $v=1$ state by $0.1387 \AA$. Using the free HD bond length $s_{0}=0.74142 \AA$ (Ref. 18) we get for the deuteron distance from the cage center $0.3560 \AA$ and for the proton distance $0.3854 \AA$ in the ground vibrational state. The corresponding distances from the center of mass of free HD molecule are $0.24714 \AA$ and $0.49428 \AA$. As one can see, inside $\mathrm{C}_{60}$ the
two nuclei are placed fairly symmetrically relative to the cage center.

Another feature of HD is an enlarged anisotropic potential ${ }^{v} V_{00}^{222}$, Table VI. ${ }^{v} V_{00}^{222}$ is defined in the coordinate system where the rotation of the HD molecule is about its center of mass.

The relative change of the rotational constant is similar in $\mathrm{H}_{2}$ and $\mathrm{D}_{2}$ with respect to free molecule value. $B_{e}$ depends on the nucleus-nucleus distance what is determined by electronelectron interactions. Since the electron-electron interactions, in the first approximation, are same for the two isotopomers, the relative change of $B_{e}$ is independent of mass.

Among the rotation and vibration constants of $\mathrm{HD} @ \mathrm{C}_{60}$, the centrifugal correction $D_{e}$ to the rotational constant $B_{r}$, $B_{r}=B_{e}-\alpha_{e}(v+1 / 2)-D_{e} J(J+1),{ }^{9,18}$ is anomalously different from its gas phase value compared to other two isotopomers, Table VI. Positive $D_{e}$ means that the faster the molecule rotates, the longer is the bond. We speculate that since the rotation center of HD inside the cage is further away from the deuteron, the centrifugal force on the deuteron increases and the bond is stretched more than in the free HD molecule.

## B. Dipole moment

The dipole moment parameters $A^{l j n}$ are listed in Table VI. $A^{101}$ and $A^{121}$ coefficients are the largest for the lightest isotopomer. These coefficients do not have explicit dependence on the vibrational coordinate and were defined in Eq. (2) as ${ }^{v^{\prime} v} A^{l j n}=\left\langle\psi_{v^{\prime}}^{v i b}(s)\right| A^{l j n}(s)\left|\psi_{v}^{v i b}(s)\right\rangle$. We expand the coefficients in powers of $\delta s=s-s_{0}$ at the bond length $s_{0}, A^{l j n}(s)=A^{l j n}\left(s_{0}\right)+\left.\frac{d}{d s} A^{l j n}\right|_{s_{0}} \delta s$ $+\ldots$. The second term of this expansion gives the $v$ $=0 \rightarrow 1$ transition, $\left\langle\psi_{1}^{v i b}(s)\right| s\left|\psi_{0}^{v i b}(s)\right\rangle \sim \mu^{-0.25}$ where $\mu$ is the reduced mass of the hydrogen molecule. ${ }^{3}$ The ratio of the $A^{101}$ coefficients of $\mathrm{H}_{2}$ and $\mathrm{D}_{2}$ is $\sim 1.3$, Table VI. This is not very different from the estimate using the reduced masses, $2^{0.25} \approx 1.19$. The ratio of the anisotropic coefficients, $A^{121}$, is almost 1.5 , however with the large error bar dominated by the error of the $D_{2}$ coefficient.

We found that in the $\mathrm{D}_{2}$ sample several absorption lines were rendered invisible. There are other factors, beside the $A$ coefficients dependence on the vibrational coordinate, that reduce the $\mathrm{D}_{2} @ \mathrm{C}_{60}$ IR line intensities compared to $\mathrm{H}_{2} @ \mathrm{C}_{60}$. From Eq. (17) of Ref. 9, we get that the line area is $S(\omega)$ $\left.\sim \omega\left(A^{l j 1}\right)^{2}\left|\left\langle\psi_{N=1}^{1}\right| R\right| \psi_{N=0}^{0}\right\rangle\left.\right|^{2}$. The matrix element of the radial part depends on the molecule mass as $\left.\left|\left\langle\psi_{N=1}^{1}\right| R\right| \psi_{N=0}^{0}\right\rangle\left.\right|^{2}$ $\sim m^{-0.5}$, Ref. 3. The transition frequency $\omega$ is approximately equal to the vibrational frequency $\omega_{0} \sim \mu^{-0.5}$. If one uses 1.3 as the experimental ratio of the $A^{101}$ coefficients of $\mathrm{H}_{2}$ and $\mathrm{D}_{2}$, the estimate of the relative line intensities of the two isotopomers is $S_{H_{2}} / S_{D_{2}}=3.3$.

While the coefficients $A^{101}$ and $A^{121}$ are zero for the free hydrogen molecule, the third coefficient $A^{010}$, allowed by symmetry in a heteronuclear diatomic, describes the dipole moment of a rotating molecule. However, inside $\mathrm{C}_{60}$ it could be different from the free HD value. The permanent dipole

TABLE VI. Values of the fitted parameters $\kappa_{i}$ for HD and $\mathrm{D}_{2} @ \mathrm{C}_{60}$ compared to the results for $\mathrm{H}_{2} @ \mathrm{C}_{60}$ (Ref. 9). The vibrational and rotational constants of a free molecule of the three hydrogen isotopomers are shown for comparison; gas phase $\omega_{0}$ is calculated including all terms $(v+1 / 2)^{k}$ up to $k=3$ (Ref. 18). In $\mathrm{HD} @ \mathrm{C}_{60}$ and $\mathrm{D}_{2} @ \mathrm{C}_{60}$ the parameter ${ }^{v} V_{00}^{224}$ is set to zero. Currently and previously used dipole moment expansion parameters are related as ${ }^{\prime} v A^{l j 1}=20 B_{v^{\prime} v}[l j]$, where $B_{v^{\prime} v}[l j]$ is the pairwise $\mathrm{H}_{2}$-carbon induced dipole moment expansion parameter given by Eqs. (13) and (14) in Ref. 9.

| $\kappa_{i}$ | $\mathrm{H}_{2} @ \mathrm{C}_{60}$ |  | HD@ $\mathrm{C}_{60}$ |  | $\mathrm{D}_{2} @ \mathrm{C}_{60}$ |  | Unit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $v=0$ | $v=1$ | $v=0$ | $v=1$ | $v=0$ | $v=1$ |  |
| ${ }^{v} V_{00}^{002}$ | $14.277 \pm 0.034$ | $15.952 \pm 0.032$ | $16.355 \pm 0.065$ | $17.483 \pm 0.048$ | $16.781 \pm 0.080$ | $16.456 \pm 0.056$ | $\mathrm{Jm}^{-2}$ |
| ${ }^{v} V_{00}^{004}$ | $(2.2105 \pm 0.0099) 10^{21}$ | (2.1920 $\pm 0.0085) 10^{21}$ | $(1.884 \pm 0.024) 10^{21}$ | $(2.133 \pm 0.015) 10^{21}$ | $(2.159 \pm 0.041) 10^{21}$ | $(2.390 \pm 0.022) 10^{21}$ | $\mathrm{Jm}^{-4}$ |
| ${ }^{v} V_{00}^{222}$ | $0.5629 \pm 0.0059$ | $1.199 \pm 0.012$ | $1.313 \pm 0.040$ | $2.037 \pm 0.058$ | $1.11 \pm 0.22$ | $1.40 \pm 0.14$ | $\mathrm{Jm}^{-2}$ |
| ${ }^{v} V_{00}^{224}$ | $(2.211 \pm 0.023) 10^{20}$ | $(1.030 \pm 0.011) 10^{20}$ | 0 | 0 | 0 | 0 | $\mathrm{Jm}^{-4}$ |
| ${ }^{v} V_{00}^{111}$ | ... | ... | $(3.1102 \pm 0.0095) 10^{-10}$ | $(3.251 \pm 0.010) 10^{-10}$ | $\ldots$ | $\ldots$ | $\mathrm{Jm}^{-1}$ |
| ${ }^{v} V_{00}^{113}$ |  |  | $(4.25 \pm 0.18) 10^{10}$ | $(7.12 \pm 0.11) 10^{10}$ | $\ldots$ | ... | $\mathrm{Jm}^{-3}$ |
| $n_{o} / n_{p}$ | $2.890 \pm 0.045$ |  | - |  | 2 |  |  |
| $A^{010}$ | - |  | $(-1.54 \pm 0.15) 10^{-32}$ |  |  |  | Cm |
| $A^{101}$ | $(9.06 \pm 0.26) 10^{-22}$ |  | $(7.01 \pm 0.69) 10^{-22}$ |  | $(6.85 \pm 0.73) 10^{-22}$ |  | C |
| $A^{121}$ | $(-4.26 \pm 0.42) 10^{-22}$ |  | $(-2.67 \pm 0.27) 10^{-22}$ |  | $(-2.9 \pm 1.5) 10^{-22}$ |  | C |
|  | $\mathrm{H}_{2} @ \mathrm{C}_{60}$ | $\mathrm{H}_{2}$ | HD@ $\mathrm{C}_{60}$ | HD | $\mathrm{D}_{2} @ \mathrm{C}_{60}$ | $\mathrm{D}_{2}$ |  |
| $\omega_{0}$ | $4062.37 \pm 0.28$ | 4161.18 | $3549.66 \pm 0.39$ | 3632.20 | 2924 | 2993.69 | $\mathrm{cm}^{-1}$ |
| $B_{e}$ | $59.865 \pm 0.074$ | 60.853 | $45.38 \pm 0.14$ | 45.655 | $29.888 \pm 0.089$ | 30.444 | $\mathrm{cm}^{-1}$ |
| $\alpha_{e}$ | $2.974 \pm 0.025$ | 3.062 | $1.695 \pm 0.044$ | 1.986 | $1.086 \pm 0.045$ | 1.079 | $\mathrm{cm}^{-1}$ |
| $D_{e}$ | $(4.832 \pm 0.051) 10^{-2}$ | $4.71 \times 10^{-2}$ | $(1.535 \pm 0.097) 10^{-1}$ | $2.605 \times 10^{-2}$ | $(8.45 \pm 3.71) 10^{-3}$ | $1.141 \times 10^{-2}$ | $\mathrm{cm}^{-1}$ |

moment of the free HD molecule corresponding to the $v$ $=0 \rightarrow 1, J=0 \rightarrow 1$ transition is $5.0 \times 10^{-5} \mathrm{D},{ }^{20}$ while the endohedral HD dipole moment is $\left|A^{010}\right|=1.54$ $\times 10^{-32} \mathrm{Cm}=4.77 \times 10^{-3} \mathrm{D}$, Table VI. The dipole moment $A^{010}$ of HD inside $\mathrm{C}_{60}$ is enhanced by two orders of magnitude because of the mutual induced polarization of HD and $\mathrm{C}_{60}$.

Despite the enhancement of the rotational dipole moment $A^{010}$ we did not detect the rotational transition $J=0 \rightarrow 1$, $\Delta N=0$ in the IR spectra of $\mathrm{HD} @ \mathrm{C}_{60}$. Indeed, our model fit gives a rather small area $S=0.07 \mathrm{~cm}^{-2}$ for this line at $3630.2 \mathrm{~cm}^{-1}$ and at temperature 5 K . The $\Delta J=1$ transition in $\mathrm{HD}^{\circ} \mathrm{C}_{60}$ is suppressed because there is an interference of two dipole terms, $A^{010}$ and $A^{101}$ which have opposite signs. The final state consists of $80 \%$ of pure rotational state $J$ $=1, N=0$ and $18 \%$ of the pure translational state $N=1, J$ $=0$, Table V . This mixed final state has matrix elements from the ground state for both $A$ coefficients, $A^{010}$ and $A^{101}$, which nearly cancel each other.

Olthof et al. did not consider induced dipole moments in $\mathrm{CO} @ \mathrm{C}_{60} \cdot{ }^{16}$ Their approach is probably justified as free CO has large rotational and vibrational dipole moments and the induced moments are relatively small. Our study shows that in $\mathrm{HD} @ \mathrm{C}_{60}$ the induced rotational and translational dipole moments are both equally important in rotational transitions and two orders of magnitude larger than the permanent rotational dipole moment of HD.

## VI. SUMMARY

We measured and analyzed the infrared absorption spectra of $\mathrm{D}_{2} @ \mathrm{C}_{60}$ and $\mathrm{HD} @ \mathrm{C}_{60}$ and compared the results to $\mathrm{H}_{2} @ \mathrm{C}_{60}$. The rotation and translation transitions appear in combination with the fundamental vibrational transition, $v=0 \rightarrow 1$. Experimentally, the two heavier hydrogen iso-
topomers are more challenging than $\mathrm{H}_{2}$. Several IR absorption lines were missing in $\mathrm{D}_{2} @ \mathrm{C}_{60}$ and $\mathrm{HD} @ \mathrm{C}_{60}$ spectra, including the fundamental vibrational transition. Heavier mass reduces the IR line intensity through several parameters and in addition, shifts spectra to lower frequency where the background absorption of the $\mathrm{C}_{60}$ powder is stronger and, as a consequence, the signal-to-noise ratio is reduced.

The potential parameters of $\mathrm{H}_{2}$ and $\mathrm{D}_{2}$ show little mass dependence. This is true for the isotropic part of the HD potential ${ }^{v} V_{00}^{002}$ as well. Absence of the inversion symmetry in HD allows additional potential parameters, ${ }^{v} V_{00}^{111}$ and ${ }^{v} V_{00}^{113}$, which mix the rotational levels and $J$ is not a good quantum number any more. This is the effect of the confining potential what prohibits the rotation of the HD molecule about its center of mass, as predicted for $\mathrm{HD} @ \mathrm{C}_{60}$ (Ref. 8) and for another endohedral complex without the center of inversion, $\mathrm{CO} @ \mathrm{C}_{60} .{ }^{16}$

In $\mathrm{HD} @ \mathrm{C}_{60}$ the dipole moment $A^{010}$ for rotational transition $v=0 \rightarrow 1, J=0 \rightarrow 1$ with $\Delta N=0$ is enhanced by two orders of magnitude compared to the free HD molecule value. However, because the final state is not a pure $J=1$ state, the interference with the induced dipole moment term $A^{101}$ renders this IR transition invisible in our measurement.

The accuracy of the potential could be improved further by measuring the IR fundamental vibrational and the $\Delta N=-1$ transitions, and transitions to higher $N$ levels in $\mathrm{HD} @ \mathrm{C}_{60}$ and $\mathrm{D}_{2} @ \mathrm{C}_{60}$ with better signal-to-noise ratio. This could be achieved by using samples in the shape of a single crystal, few mm in size. In addition, a single crystal study would shed some light on the splitting of some $\Lambda<3$ levels and increase the accuracy of the dipole moment parameters as there would be no light scattering such as in the powder sample. However, this would increase the predictive power of our model for higher $N$ values only. The separation and
translation-rotation splitting of the lowest $N$ levels already detected in the experiment would not change, especially in the HD sample where additional information from the INS experiment ${ }^{10}$ was used to compensate for the missing fundamental transition.

## ACKNOWLEDGMENTS

We thank Professor Pekka Pyykko for helpful discussions. This research was supported by the Estonian Ministry of Education and Research Grant No. SF0690029s09, Estonian Science Foundation Grant Nos. ETF7011, ETF8170, JD187. We thank Engineering and Physical Sciences Research Consul (U.K.) (EPSRC(GB)) and the Royal Society (UK) for support.

[^1]${ }^{7}$ M. Xu, F. Sebastianelli, Z. Bačić, R. Lawler, and N. J. Turro, J. Chem. Phys. 128, 011101 (2008).
${ }^{8}$ M. Xu, F. Sebastianelli, Z. Bačić, R. Lawler, and N. J. Turro, J. Chem. Phys. 129, 064313 (2008).
${ }^{9}$ M. Ge, U. Nagel, D. Hüvonen, T. Rõõm, S. Mamone, M. H. Levitt, M. Carravetta, Y. Murata, K. Komatsu, J. Y.-C. Chen, and N. J. Turro, J. Chem. Phys. 134, 054507 (2011).
${ }^{10}$ A. J. Horsewill, S. Rols, M. R. Johnson, Y. Murata, M. Murata, K. Komatsu, M. Carravetta, S. Mamone, M. H. Levitt, J. Y.-C. Chen, J. A. Johnson, X. Lei, and N. J. Turro, Phys. Rev. B 82, 081410 (2010).
${ }^{11}$ M. Xu, F. Sebastianelli, B. R. Gibbons, Z. Bačić, R. Lawler, and N. J. Turro, J. Chem. Phys. 130, 224306 (2009).
${ }^{12}$ S. Mamone, J. Y.-C. Chen, R. Bhattacharyya, M. H. Levitt, R. G. Lawler, A. J. Horsewill, T. Rõõm, Z. Bačić, and N. J. Turro, Coord. Chem. Rev. 255, 938 (2011).
${ }^{13}$ J. M. Brown and A. Carrington, Rotational Spectroscopy of Diatomic Molecules (Cambridge University Press, Cambridge, England, 2003).
${ }^{14}$ N. J. Turro, J. Y. C. Chen, M. Sartori, E.; Ruzzi, A. A. Marti, R. G. Lawler, S. Jockusch, J. López-Gejo, K. Komatsu, and Y. Murata, Acc. Chem. Res. 43, 335 (2010).
${ }^{15}$ N. J. Turro, A. A. Marti, J. Y.-C. Chen, S. Jockusch, R. G. Lawler, M. Ruzzi, E. Sartori, S.-C. Chuang, K. Komatsu, and Y. Murata, J. Am. Chem. Soc. 130, 10506 (2008).
${ }^{16}$ E. H.T. Olthof, A. van der Avoird, and P. E. S. Wormer, J. Chem. Phys. 104, 832 (1996).
${ }^{17}$ S. L. Altmann and P. Herzig, Point-Group Theory Tables (Oxford University Press, Oxford, 1994).
${ }^{18}$ K. P. Huber and G. Herzberg, Constants of Diatomic Molecules. IV. Molecular Spectra and Molecular Structure (Reinhold, New York, 1979).
${ }^{19}$ A. D. Buckingham, Trans. Faraday Soc. 56, 753 (1960).
${ }^{20}$ N. H. Rich, J. W. C. Johns, and A. R. W. McKellar, J Mol. Spectrosc. 95, 432 (1982).


[^0]:    ${ }^{a}$ Electronic mail: toomas.room@kbfi.ee.

[^1]:    ${ }^{1}$ K. Komatsu, M. Murata, and Y. Murata, Science 307, 238 (2005).
    ${ }^{2}$ M. Murata, Y. Murata, and K. Komatsu, J. Am. Chem. Soc. 128, 8024 (2006).
    ${ }^{3}$ S. Flügge, Practical Quantum Mechanics (Springer-Verlag, Berlin, 1971), Vol. 1.
    ${ }^{4}$ S. Mamone, M. Ge, D. Hüvonen, U. Nagel, A. Danquigny, F. Cuda, M. C. Grossel, Y. Murata, K. Komatsu, M. H. Levitt, T. Rõõm, and M. Carravetta, J. Chem. Phys. 130, 081103 (2009).
    ${ }^{5}$ R. J. Cross, J. Phys. Chem. A 105, 6943 (2001).
    ${ }^{6}$ T. Yildirim and A. B. Harris, Phys. Rev. B 66, 214301 (2002)

