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Connection from Single-Phase to Three-Phase Circuit Using Free Oscillation Circuits

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Introduction

Ferroresonance is one of the most important nonlinear phenomena in power systems for over 100 years[1]. Although the power systems are very complex high dimensional systems, the ferroresonance is modeled by simple resonant circuits, e.g. a single-phase RLC series resonant circuit and a three-phase circuit shown in Fig.1, where $\bar{R} = 3R$ and $\bar{C} = C/3$. Single-phase circuits are simplest models which are represented by forced two dimensional differential systems. The detailed analysis is reported in [2, 3] and the chaos in a related system is observed by Y. Ueda [4]. Although the models represent essential phenomena about the ferroresonance, many power systems are three-phase circuits and the single-phase circuits are a model based on the positive sequence equivalent circuit which can be used in linear systems. In order to describe phenomena in the three-phase circuits, we have to expand the 2-dimensional phase space to more than 6-dimensional space [5, 6, 7]. However, it is not easy to survey the 6-dimensional space and the relation between the phase spaces of the single-phase and the three-phase circuits is not clear. This paper proposes a method to connect the single-phase circuit to the three-phase circuit by using free-oscillation circuits. The approach provide more insight to the phenomena in three-phase circuits.

Oscillations in single-phase and three-phase circuit

First, we review the phenomena of fundamental harmonic and 1/3-subharmonic oscillations in the single-phase and the three-phase circuit. When the source voltage is increased in the single-phase circuit, a fundamental harmonic ferroresonance is observed and subsequently the second and the third higher harmonic resonances are generated. The chaotic oscillations are observed in the region of the second higher harmonic resonance. Also the 1/3-subharmonic oscillations in the single-phase circuit have the routes from period doubling bifurcations to chaotic oscillations.

The three-phase circuit has three patterns of oscillations differently from the single-phase circuit: single-phase oscillations, two-phase oscillations, and three-phase oscillations [8]. Fundamental harmonic single-phase and three-phase oscillations are observed although fundamental harmonic two-phase oscillations have not been reported. The three-phase oscillations have the region of the fundamental, the second, and the third higher harmonic resonances, which are similar to those of the single-phase circuit. However, almost periodic oscillations which are not generated in the single-phase circuit are observed in the region of the second higher harmonic resonance[9]. The single-phase oscillations lose the stability by the generation of unstable almost periodic oscillations. Thus, the oscillation patterns and the generation of the almost periodic oscillations characterize the fundamental harmonic oscillations in the three-phase circuit.

There exist all the three patterns for the 1/3-subharmonic oscillations. The bifurcations of single-phase oscillations are similar to those in the single-phase circuit. The three-phase 1/3-subharmonic oscillations are generated as almost periodic oscillations and the two-phase oscillations are generated as mode-locked oscillations in the region of the three-phase oscillations. Thus, the oscillation patterns and the generation of the almost periodic oscillations characterize also the 1/3-subharmonic oscillations in the three-phase circuit.

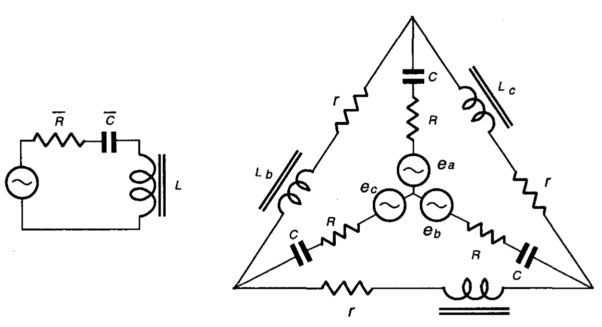

Figure 1: Single-phase and three-phase circuits, where $\bar{R} = 3R$ and $\bar{C} = C/3$.

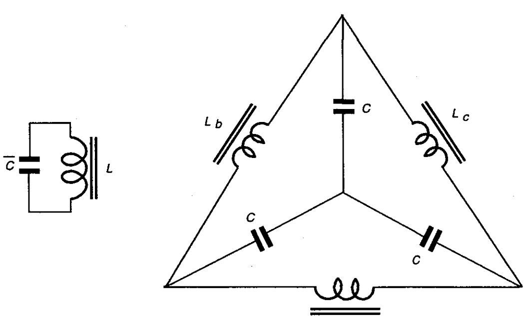

Figure 2: Free oscillation circuits obtained by eliminating resistors and sources.

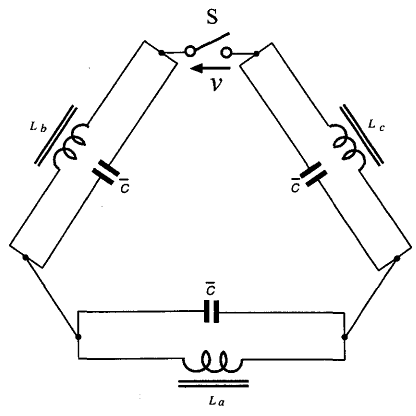

Figure 3: Connection between three free oscillation circuits

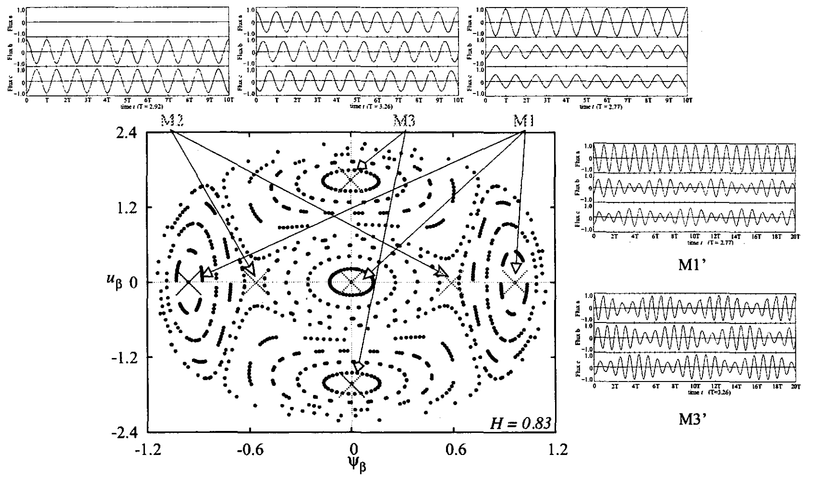

Figure 4: Poincare Map in three LC ladder circuit.

Connection by free oscillation circuit

In order to clarify relations between the single-phase and three-phase circuit, we introduce free oscillation circuits shown in Fig.2, which are derived by eliminating the resistors and sources. The single-phase and three-phase circuits are transformed into a resonator and a three LC ladder circuit. Then, we introduce a connection circuit shown in Fig.3 to connect the two free oscillation circuits. When the switch S is opened, the circuit is three independent resonators and when the switch S is closed, the circuit is an equivalent circuit of the LC ladder circuit. Thus, by closing the switch S, we connect the resonator to the LC ladder circuit. We assume that the switch S is closed when the voltage $v = 0$ in Fig.3.

Let us consider the case of linear circuit. The natural angular frequency of the resonator is $\sqrt{3/LC}$ and those of the ladder circuit are $\sqrt{3/LC}, \sqrt{3/LC}$, and 0. Thus, all free oscillations in the connection circuit are periodic. If a free oscillation is generated in only one resonant circuit, then the closing of S generates an oscillation which corresponds to a single-phase oscillation in the three-phase circuit. If antiphase oscillations are given to the two resonators, then the closing of S generates an oscillation which corresponds to a two-phase oscillation in the three-phase circuit. If three-phase oscillations are given to the three resonators, then the closing of S generates an oscillation which corresponds to a three-phase oscillation in the three-phase circuit.

When the circuit is nonlinear, the angular frequencies of each mode are different and the closing of S generates almost periodic oscillations in general. The Poincare map of the three LC ladder circuit reveals the structure of the phase space(see Fig.4). Because of the cutset of the capacitors, the loop of the inductors, and the fixed energy, Fig.4 represents whole phase space. There exist 3 sorts of periodic oscillations M1, M2, and M3 which corresponds to the single, two, and three-phase oscillations in the three-phase circuit and almost periodic oscillations M1′ and M3′ are observed around M1 and M3. Although the structure of the forced dissipative system is different from Fig.4, the characteristics in the three-phase circuit are explained by this map.

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