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Inner Angles Made of Consecutive Three Points on a Circle for Chaotic and Random Series

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Introduction

These days, many properties of chaos are found. However, researches on methods to judge whether the obtained series are chaos or pure random still attracts many researchers' interests. Recently, an inner angle of triangle on a circle made of consecutive three points is found to show the difference between chaotic series and random series[1].

In the paper, we investigate differences among several chaotic series and random series by investigating the inner angle[1] whose details are mentioned below. We focus on the chaotic series obtained by the Bernoulli map, tent map, and Chebyshev map. Also, the uniform random number series and normal number series are focused on and compared.

Inner angle of triangle on a circle made of consecutive three points

The inner angle which we focus on here is defined as follows. After obtaining the series data $\{X_i\}$ which are normalized and upset within $[0,1]$, we plot points consecutively on the circle whose circumferential length is unity one. A length of segment in the circle starting at point O' in Fig. 1 corresponds to the series data. For example, the data 0 and 0.25 are plotted on the points O' and A in Fig. 1, respectively. Then, we make a triangle by using consecutive three points, namely $\triangle X_{i-1}X_iX_{i+1}$. The angle which we focus on here is the inner angle at the middle point of consecutive three points, namely $\alpha = \angle X_i$. Figure 1 shows an example. We investigate an averaged value of this angle. Here, we set an angle between the segments $\mathrm{O}X_{i-1}$ and $\mathrm{O}X_{i+1}$ as θ. This value takes a positive value in a counterclockwise direction from $\mathrm{O}X_{i-1}$ to $\mathrm{O}X_{i+1}$. By using the angle θ, the angle α can be represented as follows:

$$\alpha = \begin{cases} (2\pi - \theta)/2, & X_{i-1} \to X_i \to X_{i+1} \\ \theta/2, & X_{i-1} \to X_{i+1} \to X_i \end{cases} \tag{1}$$

where these expressions about α consist in the case these points are in order $X_{i-1} \to X_i \to X_{i+1}$ and $X_{i-1} \to X_{i+1} \to X_i$ counterclockwise, respectively. Considering the order of the points X_{i-1}, X_i and X_{i+1} and these magnitudes, these cases can be classified into six cases. Tables 1 and 2 show the values of θ and α in these six cases. In the numerical simulation, the relation among the consecutive three points are classified into these six cases. Then, using the formulas in tables, the inner angle is calculated.

Table 1: $X_{i-1} \to X_i \to X_{i+1}$

	θ	α
$X_{i-1} < X_i < X_{i+1}$	$2\pi(X_{i+1} - X_{i-1})$	$\pi(1 + X_{i-1} - X_{i+1})$
$X_{i+1} < X_{i-1} < X_i$	$2\pi(X_{i+1} + 1 - X_{i-1})$	$\pi(X_{i-1} - X_{i+1})$
$X_i < X_{i+1} < X_{i-1}$	$2\pi(X_{i+1} + 1 - X_{i-1})$	$\pi(X_{i-1} - X_{i+1})$

Table 2: $X_{i-1} \to X_{i+1} \to X_i$

	θ	α
$X_{i-1} < X_{i+1} < X_i$	$2\pi(X_{i+1} - X_{i-1})$	$\pi(X_{i+1} - X_{i-1})$
$X_i < X_{i-1} < X_{i+1}$	$2\pi(X_{i+1} - X_{i-1})$	$\pi(X_{i+1} - X_{i-1})$
$X_{i+1} < X_i < X_{i-1}$	$2\pi(X_{i+1} + 1 - X_{i-1})$	$\pi(1 - X_{i-1} + X_{i+1})$

It is already known analytically that the expected inner angles $\langle\alpha\rangle$ using the chaotic series generated by the Bernoulli map and the uniform random number series are obtained as

$$\langle\alpha\rangle = \begin{cases} \pi(2 + 1/p)/6, & \text{Bernoulli map} \\ \pi/3, & \text{uniform random} \end{cases} \tag{2}$$

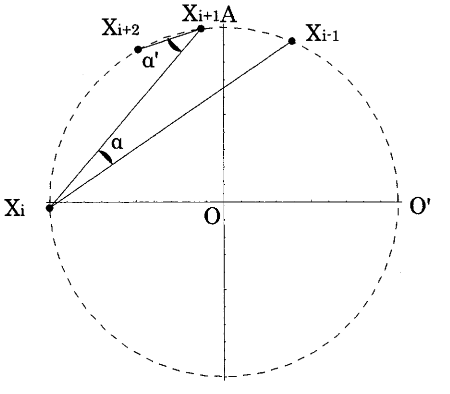

Figure 1: An inner angle which we focus on here is obtained like this figure. In focusing on the consecutive three points, namely X_{i-1}, X_i and X_{i+1}, the objective inner angle in this step is α. The inner angle α' in next step is obtained by the next consecutive points, namely X_i, X_{i+1} and X_{i+2}.

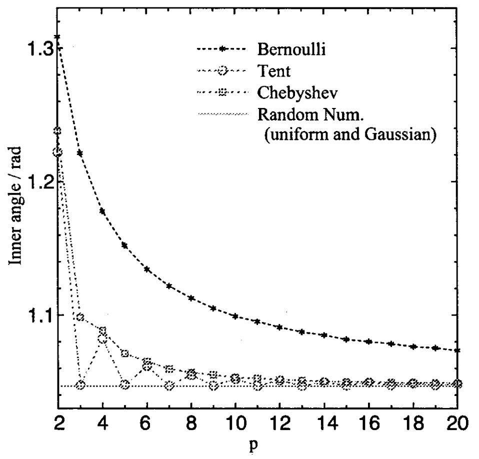

Figure 2: Inner angle as functions of p

respectively [1]. Here, the quantity p is the order of Bernoulli map defined in Eq. (3).

Chaotic and random series

Here, the Bernoulli map B, tent map T and Chebyshev map C are used to generate chaotic series. As is well known, these maps are defined as follows:

$$B_p(x) = px - \lfloor px \rfloor, \tag{3}$$

$$T_p(x) = \begin{cases} px - \lfloor px \rfloor, & \mathrm{Mod}(\lfloor px \rfloor, 2) = 0 \\ \lceil px \rceil - px, & \text{otherwise} \end{cases} \tag{4}$$

$$C_p(\cos x) = \cos(px), \tag{5}$$

where $p \geq 2$ is the order. Using these maps, the chaotic series $\{X_{p,j}\}$ are obtained by $X_{p,j+1} = F_p(X_{p,j})$, where F_p denotes the above maps. For investigating the inner angle obtained by plotting the series on a circle whose circumferential length is unity 1, these series are normalized and upset to be set within $[0, 1]$. The series by the Bernoulli and tent maps are used directly because these are in $[0, 1)$. In the case of the series by Chebyshev map, we apply the procedure $(1 + X_{p,j})/2$ to the series $\{X_{p,j}\}$.

We also focus on the uniform random number series and normal one. These series are also normalized and upset. The uniform random number are generated within $[0, 1]$ uniform distribution. On the other hand, since the distribution for the normal random number is Gaussian, normalization is necessary. After obtaining the normal random number series $\{X_j\}$, the series are divided by the maximum value in $\{|X_j|\}$.

Using these procedure mentioned above, the averaged inner angles for these series were calculated numerically. In the numerical simulation, obtaining averaged inner angles for these series, we used 10 samples of series for each of series whose length are 1000000. Figure 2 shows the averaged inner angle as functions of the order p for the chaotic series and pure random ones. Since the inner angles for the random number series are independent of p, the averaged values obtained numerically is used. The averaged inner angle for the uniform random number series and normal one were 1.046825 and 1.047239, respectively. It is found that the angles for the random number series are almost the same each other in spite of the difference in the distribution. In addition, the angles for the chaotic series approach the one for the random number as p increases.

Conclusions

In the paper, the differences about the inner angle among the several chaotic series and random number ones have been investigated. The angles for the chaotic series are obviously different from the ones for the random number series. Although the invariant measure of the Bernoulli map is the same as the one of the tent map, the averaged inner angles are different each other. In addition, in spite of the different distribution of the uniform and normal random number, the angles are almost the same. The property about the inner angle can be expected to have a potential for judging whether the series is chaotic or pure random.

References

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