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A study on intrinsic localized modes in a macro-mechanical cantilever array with tunable on-site nonlinearity

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A macro-mechanical cantilever array with tunable on-site nonlinearity is proposed for experimental study on dynamics of intrinsic localized mode (ILM). In this paper, the cantilever array is first introduced and the equation of motion is derived. An experimentally excited ILM is shown and compared with the corresponding ILM obtained numerically. In addition, an unstable ILM is also shown with its waveform and Floquet multipliers.

Intrinsic Localized Mode

Intrinsic localized mode (ILM) is a spatially localized and temporary periodic solution in nonlinear homogeneous lattices, which is first discovered by A. J. Sievers and S. Takeno [1]. Before the discovery, it is well known that a localized mode appears on an impurity in linear lattice. Although the *linear* localized mode requires impurities to exist, ILM can appear in nonlinear homogeneous lattice. Therefore, ILM can stand and even travel anywhere in the lattice [2, 3].

We have numerically investigated coexistence, stability, global phase structure, and mechanism how ILM travels for the coupled ordinary differential equations derived from the micro-cantilever array [4, 5]. As results, it was reported that a traveling ILM can be manipulated by the stability change of coexisting ILMs, which is called *capture and release manipulation.* Although the manipulation method should be confirmed experimentally, it seems difficult to adjust the nonlinearity in the micro-cantilever array. In addition, it also difficult to measure individual oscillation of cantilevers in the array. Thus, the macro-mechanical cantilever array is newly proposed for experimental study of ILM [6].

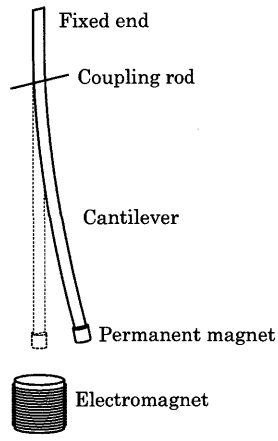


Figure 1: Schematic image of the cantilever.

Macro-mechanical cantilever array

An oscillator of the proposed cantilever array is shown in Fig. 1. The upper end of an elastic beam is fixed. At the another end, a permanent magnet is attached. An electromagnet is placed below the free-end of the cantilever. Because of the magnetic interaction caused between the permanent magnet and the electromagnet, the restoring force of cantilever becomes nonlinear [6]:

$$-\omega_0^2 u + (\chi_0 + \chi_1 I_{\mathrm{EM}}) \frac{u}{(u^2 + d_0^2)^{\frac{3}{2}}}. \qquad (1)$$

Table 1: List of symbols in the equation of motion for the cantilever array.

Symbol	Value	Symbol	Value
ω_0	$2\pi\times35.1$ rad/s	γ	$1.5\ \mathrm{s}^{-1}$
C	$284\ \mathrm{s}^{-2}$	χ_0	$-4.71\times10^{-5}\ \mathrm{m}^3/\mathrm{s}^2$
d_0	3.0 mm	χ_1	$-9.14\times10^{-3}\ \mathrm{m}^3/\mathrm{s}^2\mathrm{A}$
A	$3.0\ \mathrm{m/s}^2$	ω	$2\pi\times36.1$ rad/s

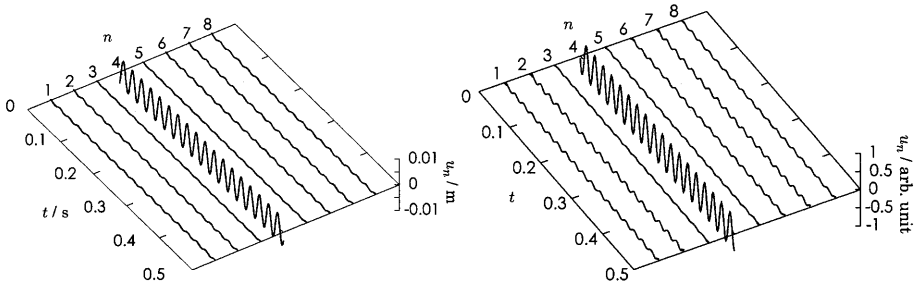
Table 1 lists symbols and their values. In the proposed cantilever array, each cantilever is coupled with the coupling rod. Therefore, the vibration of cantilever array is described as [6]

$$\ddot{u}_n = -\omega_0^2 u_n - \gamma \dot{u}_n + (\chi_0 + \chi_1 I_{\mathrm{EM}}) \frac{u_n}{(u_n^2 + d_0^2)^{\frac{3}{2}}} + A\cos(\omega t) - C(u_n - u_{n+1}) - C(u_n - u_{n-1}), \qquad (2)$$

where γ and C denote the damping coefficient and the coupling coefficient, respectively.

Observation of ILMs and its stability

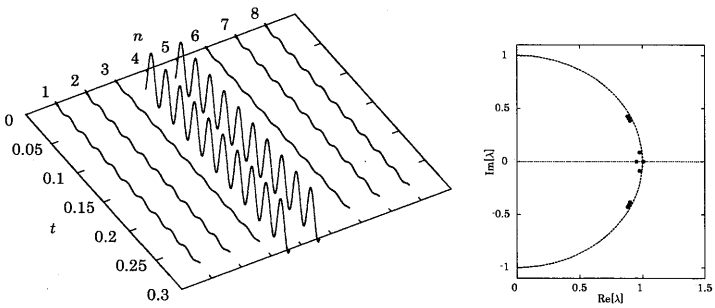
Figure 2(a) shows wave forms of an ILM which is experimentally observed when the array is excited at 36.1 Hz. Only one cantilever oscillates with large amplitude while the others show small oscillation. The mechanical energy is clearly localized in the array. The corresponding ILM in Eq. (2) is also obtained as shown in Fig. 2(b). This implies that Eq. (2) is suitable for investigating ILM in the cantilever array.



(a) Experimental (b) Numerical

Figure 2: Intrinsic localized mode excited at $n = 4$.

The stability of ILM is determined with Floquet multipliers [7]. The ILM shown in Fig. 2(b) is stable because all the Floquet multipliers are inside unit circle. On the other hand, an unstable ILM is also obtained by numerical simulation. Wave form and Floquet multipliers are shown in Figs. 3(a) and 3(b). Two neighboring cantilever oscillate in the same amplitude and phase. Since one of multipliers locates outside unit circle, the ILM is unstable.



(a) Waveform (b) Floquet multipliers

Figure 3: Intrinsic localized mode standing between $n = 4$ and 5.

The center of unstable ILM which has two cantilevers oscillating with large amplitude is between sites, for instance, that of the ILM shown in Fig. 3(a) is between $n = 4$ and 5. Thus, in the array with the parameter listed in Tab. 1, stable ILM and unstable ILM are alternately exist in the array. This situation is similar to the case of nonlinear Klein-Gordon lattices. In our previous work, it has been pointed out that invariant manifolds of unstable ILMs govern the behavior of traveling ILMs [4]. Therefore, unstable manifolds of the unstable ILM is a key to understand the behavior of traveling ILM in the experimental system.

Conclusions

Several ILMs were successfully observed in the newly proposed macro-mechanical cantilever array. In addition, these ILMs were also confirmed by numerical simulation using the equation of motion describing vibration of cantilevers. It implies that the macro-mechanical cantilever array enables us to investigate experimentally the dynamics of ILM. By considering the scaling law, the experimental studies on the macro-mechanical cantilever array will possibly be applied to micro-/nano-mechanical resonators.

Acknowledgments

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