

# Noise-induced Phenomena in Two Strongly Pulse-coupled Spiking Neuron Models

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## Introduction

Over the past three decades, chaos in neurons has been investigated in detail [1]-[3]. For instance, chaotic responses have been observed in the Onchidium giant neurons as non-/autonomous systems [1]-[2] and the Squid giant axons as a non-autonomous system [3] at single cell level. Furthermore, chaos in neural systems has been studied in both experiments and simulations at network level [4].

Bifurcations of such biological chaotic systems are quite different. Near saddle-node bifurcation and periodic doubling bifurcation have been found in physiological experiments (*ex.* [1]-[2]), such bifurcation structures can be analyzed by the discrete return map. More complex structures are predicted through numerical simulations at single cell level [3] and physical experiments at network level, which are essentially related with the piecewise smooth dynamical systems recently established [5]-[6].

In this study, we focus on the bifurcation structure of the system of two strongly pulse-coupled resonate-and-fire neuron models [7]-[9] in order to consider the nonlinear effects of the noise of chaotic dynamical systems on the discrete return map. We found a novel type of noise-induced phenomena in the system with specific noise in chaotic regions. Based on the results, we consider the relationship between the noise-induced phenomena and physical and mathematical structures of the discrete map with piecewise nonlinearity as a piecewise smooth dynamical system.

## Model

### Resonate-and-Fire Neuron

We here consider the resonate-and-fire neuron model as a spiking neuron model [7]. This type of the spiking neuron model is a complex extension of the integrate-and-fire neuron model, and it has second-order membrane dynamics with a firing threshold and a reset value corresponding to after hyperpolarization. Due to such dynamics, the neuron model exhibits subthreshold oscillation of the membrane potential, and its response is sensitive to the timing, strength, and direction of pulse inputs, resulting in the frequency preference and post-inhibitory rebound observed in biological neurons [7].

### Pulse-coupled System

We consider a system of two pulse-coupled resonate-and-fire neuron models that exhibits in-phase and anti-phase synchronization [8] and out-of-phase burst synchronization and chaotic phenomena [9] depending on the location of the firing reset value in the complex plane [9]. The global stability of the system can be analyzed by the firing time difference map (FTDM) constructed from the 1D discrete maps with respect to the firing time difference [8]-[9]. The FTDM indicates that the system can be regarded as a piecewise smooth dynamical system [5].

## Results and Discussion

### Burst Synchronization and Chaotic Phenomena

In the system of two strongly pulse-coupled resonate-and-fire neuron model, 1:1 in-phase, anti-phase, and N:N out-of-phase synchronization and chaotic phenomena have been observed by changing the coupling strength as a bifurcation parameter [8]. We can observe discontinuity-induced bifurcation [4] as well as saddle-node bifurcation in the system.

### Noise-induced Phenomena

Noise-induced nonlinear phenomena, such as stochastic resonance, coherence resonance, noise shaping, common noise-induced phase synchronization, and noise-induced order and chaos [10]-[12] are ubiquitously found in biological systems. Their constructive roles in information processing are attractive for engineered systems. We focused on noise-induced order and chaos in the system with specific noise and consider the robustness of the chaotic and synchronization phenomena. When we added Gaussian colored noise on the threshold, the chaotic phenomena were suppressed. In contrast, the chaotic phenomena were robust against additive noise on the orbit of each neuron model of the system. It should be noted that we consider the effects of the additive Gaussian noise on the continuous dynamical system not directly the discrete map as different from other previous works [10]-[11]. These results show the nontrivial and nonlinear effects of the noise in the neural system as a piecewise smooth dynamical system.

## Concluding Remarks

We have investigated the relationships between the noise-induced phenomena and the physical and mathematical structures of the system of two coupled spiking neuron models. We will analyze the system as the piecewise smooth dynamical system in detail in near future.

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