IUTAM Symposium on 50 Years of Chaos : Applied and Theoretical
Kyoto, Japan, November 28 - December 2, 2011

Tolerance of Delayed Feedback Control for Maintaining Periodic Rotation

Y. Yokoi and T. Hikihara

Department of Electrical Engineering, Kyoto University, Nishikyo, Kyoto 615-8510, Japan

Abstract

A parametrically excited pendulum is a simple nonlinear dynamical system. The rotation of parametric pendulum [1] exhibits a conversion from the external vibration into its rotational motion. The converted motion is applicable to energy scavenging from vibration of external source. Since the periodic rotation coexists with low energy states and motions, the control is required to maintain the periodic rotation against irregularity, noise, and frequency variation of the vibration. We propose a control method [2] for establishing the periodic rotation of the parametric pendulum based on the delayed feedback control [3]. In the implementation of the control, the delay is fixed at the period of the target motion. For the setting of the delay the frequency variation of vibration causes the mistuned delay in the control scheme. In this paper, the tolerance of the proposed control with mistuned delay is confirmed experimentally.

The experimental setup for parametric pendulum is shown in Fig. 1. The mechanical pendulum consists of the mass $m = 189.1\,\mathrm{g}$ and the length $l = 138.3\,\mathrm{mm}$. The pendulum is supported by a mechanical rig mounted on an electromagnetic shaker. The electromagnetic shaker generates a sinusoidal excitation in the vertical direction which corresponds to the parametric excitation. Fig. 2 shows the block diagram of the control method for establishing the periodic rotation of the mechanical pendulum. The dynamics of the experimental setup is described by

$$\begin{cases} \dfrac{\mathrm{d}\theta}{\mathrm{d}t} = v, \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = -\dfrac{D(\theta, v)}{ml^2} - \dfrac{g + a\cos(2\pi f t + \phi)}{l}\sin\theta + \dfrac{Fu(t)}{ml^2}, \\ u(t) = K\big(\theta(t-\tau) + \Theta - \theta(t)\big), \end{cases} \tag{1}$$

where t denotes the time, θ the angular displacement of pendulum from the downward position, v the angular velocity, and g the gravity acceleration. The vertical excitation is regulated with the amplitude a and the frequency f. The constant ϕ denotes the initial phase of excitation. Since we have no exact model of the damping effect, the damping is described as the function $D(\theta, v)$. The linear viscous coefficient is estimated around $1 \times 10^{-4}\,\mathrm{N{\cdot}m/s}$. The function $u(t)$ denotes the control input with the control gain K, the delay time τ, and the periodicity on Θ. The control input is applied as a torque to the mechanical pendulum by a DC motor through gears with $F = 0.18\,\mathrm{N{\cdot}m/A}$. The required angular displacement θ is measured by an angle sensor. The delayed feedback loop can be implemented as a program in a computer with A/D and D/A converters. Now we target the periodic rotation at which the pendulum rotates once during the excitation period $T := 1/f$. For the target rotation the angular displacement θ exhibits the periodicity $\theta(t) = \theta(t - T) + 2\pi$. Thus we set the delay time $\tau = T$ and the periodicity on $\Theta = 2\pi$ so that the periodic rotation is established.

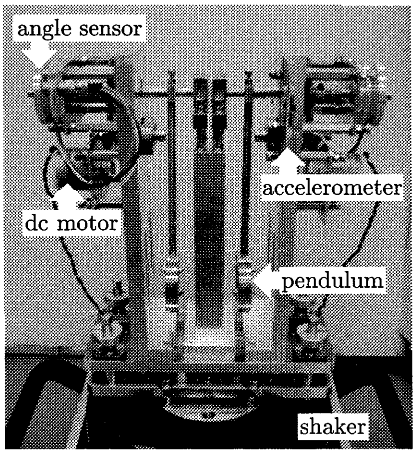

Figure 1: Experimental setup for parametric pendulum.

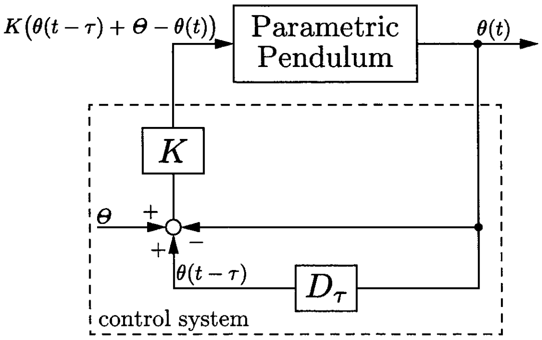

Figure 2: Block diagram of the start-up control with time delay for the periodic rotation inherent in the parametric pendulum.

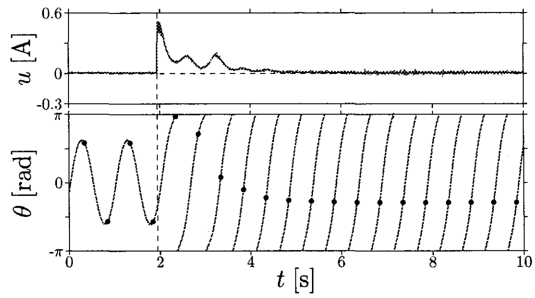

Figure 3: Establishment of the periodic rotation inherent in the vertically excited mechanical pendulum from its periodic oscillation.

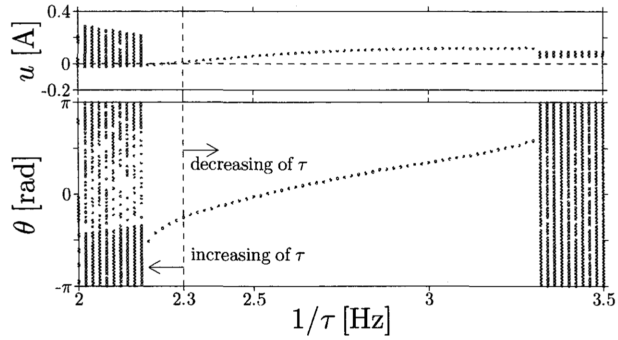

Figure 4: Bifurcation diagram of rotation with respect to the delay time τ in the experimental setup.

Figure 3 shows an example of the control for starting up the periodic rotation inherent in the experimental setup. The vertical excitation is fixed at $a = 1.1\,\mathrm{m/s^2}$ and $f = 2\,\mathrm{Hz}$ so that the periodic rotation coexists with a periodic oscillation. The control parameters are adjusted as $K = 0.072\,\mathrm{A/rad}$, $\tau = T = 1/f = 0.5\,\mathrm{s}$, and $\Theta = 2\pi$. The points in the bottom figure denote the stroboscopic points taken at every excitation period T. The vertical dash line represents the moment of onset of the control. The result shows that the periodic rotation is established from the periodic oscillation. After the establishment of rotation, the control input $u(t)$ disappears. This suggests that the periodic rotation is inherent in the experimental setup at the excitation.

An experimental bifurcation diagram of rotation with respect to the delay time τ is shown in Fig. 4 at $a = 1.2\,\mathrm{m/s^2}$, $f = 2.3\,\mathrm{Hz}$, $K = 0.072\,\mathrm{A/rad}$, and $\Theta = 2\pi$. The diagram is plotted through the stroboscopic observation at every excitation period $T = 1/f = 1/2.3\,\mathrm{s}$. The points represent steady rotations measured by decreasing and increasing the delay time τ from the excitation period T. According to the experimental procedure, we display the bifurcation parameter at the reciprocal of the delay time in Fig. 4. The inherent periodic rotation is maintained with null control input u at the delay time $\tau = T$. The periodic rotation is denoted by the single stroboscopic point in Fig. 4, which implies that the period of the rotation is coincident with the excitation period T. Decreasing the delay time τ shifts the stroboscopic point of angular displacement $\theta(t)$ in the positive direction. The shifted single stroboscopic point corresponds to a periodic rotation that does not exist without the control. The control input $u(t)$ remains and vibrates periodically. Further decrease of the delay time τ induces a bifurcation. At around $\tau = 1/3.3\,\mathrm{s}$ the periodic rotation disappears and a quasiperiodic rotation appears. The quasiperiodic rotation is depicted by a number of the stroboscopic points. Increase of the delay time τ shifts the the stroboscopic point of $\theta(t)$ in the negative direction in a symmetric fashion. By increasing the delay time τ to $1/2.18\,\mathrm{s}$, another bifurcation occurs. We observe quasiperiodic rotations for the longer delay time τ. The bifurcation diagram shows the existence range of periodic rotation in the domain of the delay. That is, the proposed control can track a periodic rotation in a certain range of the delay time τ. The existence of periodic rotation in the domain of delay represents the tolerance of proposed control with mistuned delay. The width corresponds to the tolerable range of incorrect delay. For the periodic rotation the maximum of input torque is much smaller than the maximum torque induced by the gravity. Therefore the periodic rotation can be maintained by sufficiently low energy consumption of the control.

In this abstract, we clarified the tolerance of control method to maintain the periodic rotation of parametric pendulum with mistuned delay. The tolerable range of delay indicates the performance of the proposed control. However, the analysis of the system with delay is complicated because of the infinite dimension of state space. The presentation will report a theoretical estimation of the tolerable range of delay by considering a system without delay.

Acknowledgments

This research was partially supported by the Global COE program of Kyoto University founded by the Ministry of Education, Culture, Sports and Technology of Japan.

References

[1] M. J. Clifford and S. R. Bishop, Approximating the escape zone for the parametrically excited pendulum, *J. Sound Vib.*, vol. 172, pp. 572–576, 1994.

[2] Y. Yokoi and T. Hikihara, Start control of parametric pendulum into periodic rotation, *ISCIE J.* (to be published) (in Japanese).

[3] K. Pyragas, Continuous control of chaos by self-controlling feedback, *Phys. Lett. A*, vol. 170, pp. 412–428, 1992.