## Tracking chaotic behavior in environmental hydraulics. Measurement models for the temporal coefficients of Proper Orthogonal Decomposition of free-surface flows

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Most environmental flows are chaotic, and thereby difficult to track. At the ocean surface, however, remote sensing and unmanned submarine "gliders" have revolutionized sensing of flows. In coastal regions, shore-based high-frequency radar provides additional data on surface flow fields. Such capability has allowed operational real-time tracking, or "nowcasting", of these nonautonomous chaotic systems. Following this trend to smaller and faster scales, our group hopes to contribute to nowcasting of lacustrine or estuarine flow fields, notably of the larger near-bank eddies. The present contribution tests regression models, or "measurement models", that estimate a large-scale flow field from limited sensor data. Accordingly, the thrust is on empirically characterizing the spatial statistics of such flows more than their dynamics; nonetheless some multitime statistics fall within our scope.

To statistically characterize turbulent flows, the Proper Orthogonal Decomposition (POD) has been advocated as an optimally convergent orthogonal decomposition. Yildirim, Chryssostomidis, and Karniadakis [1] have applied the POD to characterize near-coastal flows, and stated that "simulated ocean dynamics ... is energetically equivalent to the wake dynamics behind a cylinder at low Reynolds number". To optimize sensor deployment for nowcasting of such flows, they proposed "adaptive" glider deployment at the maxima of the lowest-order POD eigenfunctions.

Dynamical models based on low-order truncations of the POD are common in monitoring and control applications, and such models are now being applied to environmental flows. As a prototype for flow through a cityscape, Mokhasi, Rempfer, and Kandala [2] considered the turbulent wake of a surface-mounted cube, simulated by Large Eddy Simulation (LES). Simulated data from surface-mounted pressure "sensors" drove a nonlinear Kalman filter to track the instantaneous flow field. Their "physical model" was a POD-based set of ODEs. To feed back the sensor information, they introduced "measurement models", i.e. regression models that relate sensor measurements to the low-order POD coefficients.

Our group has applied Mokhasi et al.'s models to a complex experimental flow, the turbulent backstep [3]. In some cases, improved performance of the measurement models is achieved by including past information. In those experiments, the "sensor information" was the distribution of wall shear stress along the bottom wall. In the environment, by contrast, velocities at the water surface are more accessible. The current contribution investigates POD-based measurement models that use *surface velocities* to estimate subsurface flowfields, with application to flow over a turbulent backstep, and to the flow simulated by LES in a reach of the Hudson River north of New York City.

For the first test case, the free-surface flow over a backstep has been simulated by LES. The POD is performed on velocity fields in a streamwise-vertical section, downstream of the mean reattachment, where turbulent "boils" from the bottom impinge on the surface. At this location, good estimates of the velocity field are possible, based only on a line distribution of streamwise velocity at the surface. The best estimates were obtained by a nonlinear regression proposed by Mokhasi et al., "KRR-POD", exemplified in Fig 1 at two instants. As measured by the square of differences indicated in (b), the estimate at the first of the two instants is of typical quality. In work underway, we extend to estimating full 3D velocity fields from areal velocity distributions at the surface, as might be obtained by aerial PIV, or high-frequency radar in an estuary.

We have also applied POD to the surface velocity fields computed by LES in a reach of the Hudson River Estuary that is affected by cooling water from a nuclear power station. Like the POD by Yildirim et al.[1], convergence is extremely rapid. As with the backstep, modes 2 and 3 reflect advecting vortices, which in this case are shed quasiperiodically on the West bank. Work to be reported will test various measurement models for the POD coefficients, and discuss implications for a hypothetical refinement of the currently operational NYHOPS nowcasting system.

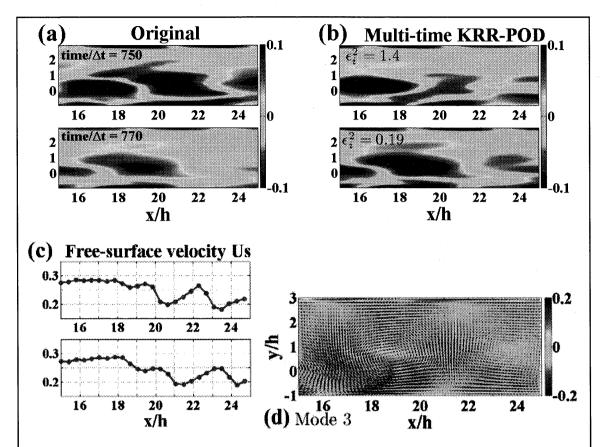


Figure 1. Estimation of sub-surface velocity field from free-surface velocity field downstream of a backstep by multi-time KRR-POD. Horizontal axes are distance downstream of step, as a multiple of step height. Except for (c), vertical axes are normal distance from lower wall. (a) Original spanwise vorticity at the two instants indicated, (b) corresponding spanwise vorticity of the estimated flowfields using (c) instantaneous free-surface velocity distributions. (d) POD mode 3, with arrows showing velocity field and color showing the corresponding vorticity; note the similarity with the vorticity of the second flow field of (a).

## References

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