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# Dynamics of Large Ensembles of Coupled Active and Inactive Oscillators

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Large ensembles of coupled nonlinear oscillators appear in a variety of contexts of science and technology to play a crucial role. For example, many organs of animals such as brains, hearts, and gastrointestinal tracts can be considered as a large ensemble of coupled oscillators, each functioning to support life through properly coordinated synchronization of constituent oscillators[1]. Besides synchronization, coupled oscillators exhibit many other interesting behaviors including clustering, spatiotemporal chaos and so on, as revealed by quite a few theoretical and experimental studies done so far (see e.g. [2]).

In reality, however, any system cannot escape from more or less damages caused by aging, accidents, diseases and so forth. Investigating effects of such damages should be indispensable to establish a full-fledged theory of coupled-oscillator dynamics. As an important case of damaged systems, one may suppose that some elements of the system lose their self-oscillatory nature, becoming damped oscillators. In this case, one encounters an ensemble comprising both normal and damped oscillators. The study of such dynamical systems may also be significant in its own right, because there is an important example (and probably many other) of coupled oscillators having this type of architecture, which is circadian clocks governing daily activities of mammals; recent findings indicate that these physiological clocks include fairly many neurons which do not fire spontaneously[3].

Some years ago, we started to tackle the dynamics of such heterogeneous ensembles of coupled oscillators for the first time, to our knowledge[4]. The purpose of this presentation is to give a brief review on our earlier results, which are mostly about globally coupled systems, and then report on recent results concerning a locally coupled system. Hereafter, normal oscillators, i.e. self-sustained oscillators which may be periodic or chaotic, will be refered to as **active** oscillators, while non-self-oscillatory units, namely damped oscillators in a generalized sense, will be called **inactive** oscillators.

### Globally coupled systems

The general form of equations treated here[4, 5, 6] is given by

$$\frac{d\mathbf{x}_j}{dt} = \mathbf{F}_j(\mathbf{x}_j) + \frac{K}{N}\sum_{k=1}^{N} D\cdot(\mathbf{x}_k - \mathbf{x}_j) \tag{1}$$

for $j = 1, \ldots, N(\gg 1)$, where $K$ is the coupling strength and $D$ is a constant matrix. For simplicity, the uncoupled dynamics represented by $\mathbf{F}_j$ are set to be the same for all active oscillators, $j = 1, \ldots, N(1-p)$, and also for all inactive oscillators, $j = N(1-p)+1, \ldots, N$, where $p$ is the ratio of inactive elements. We examined effects of increasing $p$, which we call "aging", in terms of the $(K, p)$ phase diagram for building blocks such as the Stuart-Landau oscillator, the Rössler oscillator, and the Brusselator. For the case of coupled Rössler oscillators, active oscillators were set to be not only periodic, but also chaotic. Main results are as follow[4, 5, 6]: (1) For $K$ greater than a threshold value, $K_c$, a transition takes place from a dynamic state to a steady state as the parameter $p$ exceeds a critical value, $p_c$, which depends on $K$; we call such a transition an *aging transition* (AT). The critical ratio $p_c$ is an important quantity because it measures the robustness of the system's dynamic activity against the increase of defects or aging. For $K \leq K_c$, the system remains to be dynamic until $p$ reaches unity. (2) An order parameter can be introduced as $M \equiv \langle(\mathbf{X} - \langle\mathbf{X}\rangle)^2\rangle$, where $\mathbf{X}$ is the system's centroid and the brackets stand for long time average. This order parameter obeys universal scaling laws near an AT and also near the critical point $(K, p) = (K_c, 1)$. (3) If the nonisochronicity of active oscillators is strong enough, then there appears a horn-like region in the phase diagram where active oscillators split into a number of clusters. The mechanism of this phenomenon was elucidated[6, 7].

### Locally coupled systems

Globally coupled systems are an idealistic limit of systems with long-range coupling. For describing the behavior of real systems, one needs to study systems with other modes of coupling architecture as well. A large ring of Stuart-Landau oscillators with nearest neighbor interactions as expressed below has been studied as a first step[8]:

$$\frac{dz_j}{dt} = (\alpha_j + i\Omega)z_j - (1+ic_2)|z_j|^2 z_j + K(1+ic_1)(z_{j+1} - 2z_j + z_{j-1}) \tag{2}$$

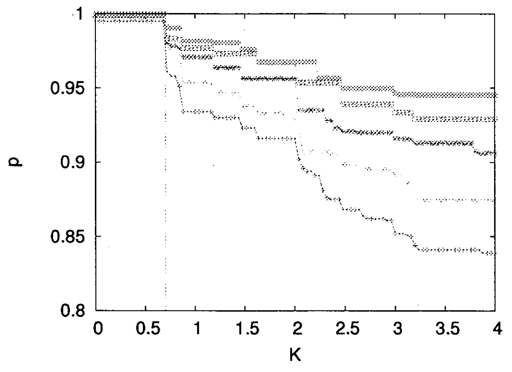


Figure 1: Aging transition boundaries in Eq.(2) for $N = 100, 200, 400, 800, 1600$ from the lowest to the highest. The vertical line shows $K_c$ (theory). From Ref.[8].

for $j$ as in Eq.(1), where $z_j$ is the complex amplitude of the $j$th oscillator with $z_0 = z_N, z_{N+1} = z_1$ and $\Omega, c_k$ are real parameters. The attribute of each oscillator is determined by $\alpha_j$, which is positive ($a > 0$) for active sites, while negative ($-b < 0$) for inactive sites. For $K = 0$, each active element is an identical limit-cycle oscillator with amplitude $\sqrt{a}$. The progress of "aging" was made to occur in a random way in the sense that at each step of increasing $p$, a new inactive site was randomly selected from among active ones. Therefore, simulation results were averaged over many realizations of the aging process. Main results are the following two[8]: (1) The AT boundary disappears for $N \to \infty$ in such a way as $1 - p_c \propto N^{-\gamma}$, where the exponent $\gamma$ depends on parameters (see Fig. 1). (2) Under a certain condition, the quenched disorder of the system created by the random aging process can counterintuitively enhance the system's phase coherence. These results will be given theoretical explanations.

### A concluding remark

The dynamics of large ensembles of coupled active and inactive oscillators is not only important to check the robustness of the behaviors of ordinary ensembles of coupled oscillators, but also provides a rich variety of novel phenomena worth extensive studies.

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