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Chaotic Streamlines in Steady Cavity Flows

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Introduction

When a lid of a cavity moves in a direction parallel to the lid, flows are driven in the cavity. Since the internal flows display a lot of interesting physical phenomena, they have been one of the important subjects of studies in fluid mechanics. The internal flows in the closed system with the simplest geometry are theoretically as well as practically important.

The geometry of a three-dimensional lid-driven rectangular cavity with width H, depth D and span L is presented in Fig.1. The upper wall(y=1) moves in x-direction with constant speed U. Non-dimensional geometrical parameters of the cavity are the aspect ratio $\Gamma=D/H$ and the spanwise aspect ratio $\Lambda=L/H$. The flow parameter is the Reynolds number $Re=U/H\nu$ where ν is the kinematic viscosity.

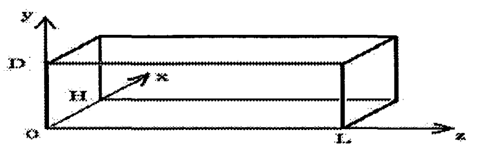


Figure1: Geometry of a lid-driven cavity.

Streamlines and Poincaré sections are availabe for examining characteristic features of three-dimensional cavity flows. Ishii *et al.* [1,2] studied the streameline structure in the steady flow fields in a cubic cavity($\Gamma=\Lambda=1$) for Re from 100 to 400. They showed that Poincaré sections of the streamlines present various structures of invariant curves, resonant islands and chaotic distribution. And we studied [3] the steady flows in a long-span square cavity($\Gamma=1$) with $\Lambda=6.55$. In the present paper we report the results of numerical simulations for the incompressible steady flows in cavities with various values of the aspect ratio and the span aspect ratio.

The Governing Equations

Flows are governed by the three-dimensional incompressible Navier-Stokes equations,

$$\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u}\cdot\nabla\mathbf{u} - \nabla p + \frac{1}{Re}\nabla^2\mathbf{u} \quad (1), \qquad \nabla\cdot\mathbf{u} = 0 \quad (2),$$

where $\mathbf{u}(u,v,w)$ is the velocity and p is the pressure. All quantities are normalized with the cavity width H, the speed of the moving lid U and the constant density ρ.

Streamlines are determined by the following equations: $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$.

In steady flows the particle trajectories coincide with the streamlines. The Poincaré section is a map of intersections of streamlines which traverse a fixed plane. A solenoidal vector field in three deimension is equivalent to a time-dependent, one-dimensional Hamiltonian system, and the Poincaré sections of streamlines correspond to phase diagrams of the Hamiltonian system [4].

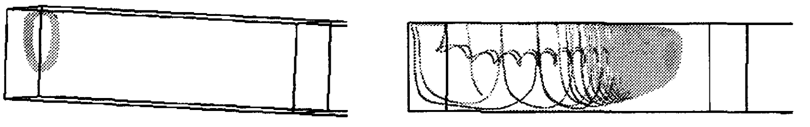
Numerical Method

The governing equations are numerically solved by the Marker-and-Cell (MAC) method. In the MAC method, the Poisson equation for pressure needs to be solved in place of the continuity equation (2).We consider the obtained state as a steady state and stop the time integration of Eq.(1) when the magnitude of the velocity increment for a time step becomes sufficiently small.

The flows in lid-driven cavities have a variety of spatial scales. We therefore have to use numerical schemes with high accuracy and high resolution in the simulation. In the present study the spectral-like Combined Compact Difference (CCD) scheme [5] is adopted to evaluate spatial derivatives. First and second derivatives in the momentum equation and the Poisson equation are evaluated by using the CCD scheme.

Results

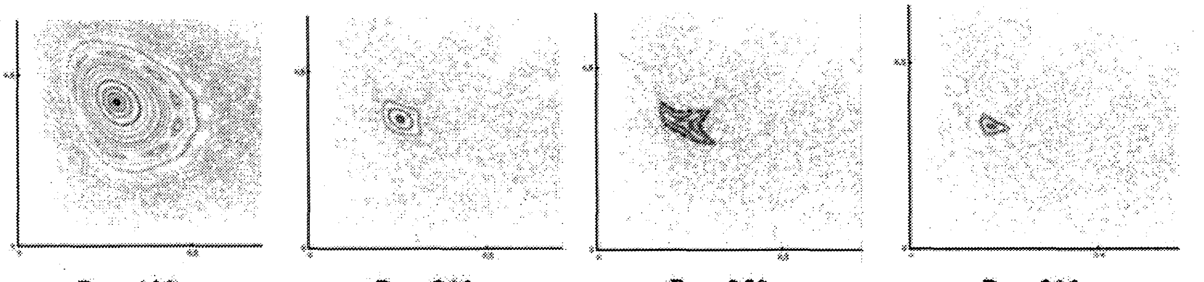
In this section we present the results of a square cavity with Λ=6.55. Figure 2 shows a typical streamline of chaotic motion as well as a streamline which forms a closed curve in the Poincaré section.



(a) A localized streamline (b) A streamline of chaotic motion

Figure2: A localized streamline and a streamline of chaotic motion for Re = 300.

In Fig.3 we present the Poincaré sections in the region near the end-wall for Re = 100, 200, 250 and 300. The left side is the end-wall. At Re = 100, many points form closed curves. The closed curves in the Poincaré section imply that a streamline covers an invariant torus. The map has seven resonant islands. Dots in the outer region surrounding the tori show chaotic motion. In the Poincaré section at Re = 200 and 300, there are fewer closed curves. The figure at Re = 250 is the Poincaré section near the 3:1 resonance. This corresponds well with the phase portrait of a resonant Hamiltonian system [6]. In the case of Λ=6.55 the value of Re at which the 3:1 resonance occurs is smaller for Γ=1 than those for Γ=0.5 and 1.5.



Re = 100 Re = 200 Re = 250 Re = 300

Figure3: Poincaré sections in the region near the end-wall for Re = 100, 200, 250 and 300.

Concluding Remarks

We have studied the variation of the structure of the flow field with the Reynolds number. The Poincaré sections of the resonances are found to correspond well with the phase portrait of a resonant Hamiltonian system.

Acknowledgements

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