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Chaotic Mixing by a Flow in a Curved Pipe

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After a review of development of studies on chaotic motion of fluid particles and fluid mixing by a flow in a curved pipe, numerical results on chaotic motion of fluid particles and fluid mixing by a steady three-dimensional flow through a helix-like circular pipe with periodic variations in curvature and torsion are shown.

Studies of chaotic mixing in a curved pipe

It is widely known that chaotic mixing is an efficient method for the mixing by laminar flows. Chaotic mixing implies the mixing caused by the chaotic motion of fluids that yields the exponential stretching and folding of fluid elements. Chaotic motion of fluid particles, sometimes called the Lagrangian chaos, has been studied extensively on the basis of the theory of dynamical systems, as reviewed in [1] and [2]. There are also several studies of chaotic mixing by two-dimensional time-periodic flows or three-dimensional steady flows, as reviewed in [1], [3] and [4].

As a part of such studies, there are studies on the chaotic motion of fluid particles and the enhancement of mixing and heat transfer in steady laminar flows in a curved pipe composed of many segment pipes of constant curvature and zero torsion caused by connecting them with a twist. Jones et al. [5] investigated the motion of fluid particles by a steady flow in a twisted circular pipe composed of a series of segment pipes curved through 180°. Using Dean's velocity field [6] as the flow in each segment pipe, they showed that chaotic motion of fluid particles is yielded by connecting neighboring segments with a twist, and examined the dependence of this motion on a twisting angle and another parameter characterizing the flow in this pipe. Acharya et al. [7] showed both by numerical simulations using Dean's velocity field and by experiments that chaotic mixing in alternating-axis coils twisted at periodic locations causes a higher rate of heat transfer compared to the mixing in constant-axis coils of no twist. Chaotic mixing by a flow in a twisted pipe was also examined in [8] both experimentally and numerically. They used a coiled tube composed of a succession of 90°-bends connected with neighboring bends with a twist of 90° in experiments, and used Dean's velocity field as the flow in each bend in numerical simulations. They showed that the chaotic motion of fluid particles in this coiled tube contributes to an increase in transverse dispersion. It should be noted that the velocity field used in the numerical examinations of all the above studies is discontinuous on the cross-sections where neighboring segments are connected with a twist. Contrary to it, using numerically obtained continuous velocity fields, Yamagishi et al. [9] examined the chaotic mixing and heat transfer by a steady flow in a twisted pipe composed of a series of 90°-bends for various twisting angles between neighboring bends. They showed the dependence of mixing performance on the twisting angle by using Poincaré sections, Lyapunov exponents, and residence time distributions.

Flow and fluid motion in a helix-like pipe

In this section, the results on the chaotic motion of fluid particles and fluid mixing by a steady continuous flow through a helix-like circular pipe with periodic continuous variations in curvature and torsion are shown [10]. That is, we consider a three-dimensional steady flow in a helix-like pipe with a circular cross-section of radius a. This pipe is twisted around a circular or elliptic cylinder, as shown in Fig. 1. The centerline of this pipe is expressed as $x = A\cos\eta,\ \ y = B\sin\eta,\ \ z = c\eta - d\sin(\eta-\eta_0)$, by using parameter η, where Cartesian coordinates x, y and z are non-dimensionalized by a. Also A, B, c, d and η_0 are non-dimensional constants. A and B are close to semiminor and semimajor axes of the ellipse around which the pipe is twisted. This pipe is periodic with period 2π of η. The difference between z coordinates of this pipe at $\eta = 0$ and $\eta = 2\pi$ is $2\pi c$. Moreover, d characterizes the amplitude of periodic displacement of the pipe in the z direction from a usual helical pipe with $d = 0$. From the above expression of the centerline, the curvature $\kappa(\eta)$ and torsion $\tau(\eta)$ of centerline of this pipe, both periodic with period 2π, can be calculated.

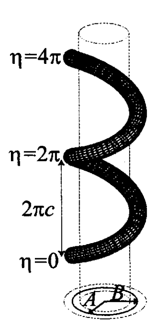

Figure 1: Geometrical configuration of a helix-like pipe.

As a steady velocity field of a viscous incompressible fluid in a curved pipe with constant curvature κ (non-dimensionalized by a) and zero torsion caused by an axial pressure gradient, Dean [6] gave the following approximate solution:

$$u_s(r) = 2(1-r^2),\ u_r(r,\theta) = -\frac{\kappa Re}{72}(1-r^2)^2(4-r^2)\cos\theta,\ u_\theta(r,\theta) = \frac{\kappa Re}{72}(1-r^2)(4-23r^2+7r^4)\sin\theta. \quad (1)$$

Here r and θ are polar cross-sectional coordinates non-dimensionalized by a, in which $\theta = 0$ is the direction toward the center of curvature, and s is a non-dimensional axial coordinate. Also, u_s, u_r and u_θ are s, r and θ components of fluid velocity non-dimensionalized by mean axial velocity U. Reynolds number Re is expressed as $Re = aU/\nu$ where ν is kinematic viscosity. Velocity components $u_r(r,\theta)$ and $u_\theta(r,\theta)$ yield streamlines of a cross-sectional flow composed of two symmetric vortices. Velocity field (1) is a good approximation of exact flow if κ is small and Re is not so high. Although the helix-like pipe we consider has a periodically-varying non-zero curvature and torsion, we use eq. (1) with $\kappa = \kappa(\eta)$ as an approximate velocity field at each cross-sectional location η under the assumption that κ and τ are small and their variations are slow. The cross-sectional motion of fluid particles associated with their axial movement is expected to be approximately governed by

$$\frac{\mathrm{d}r}{\mathrm{d}\eta} = s'(\eta)\frac{u_r(r,\theta)}{u_s(r)}, \quad r\frac{\mathrm{d}\theta}{\mathrm{d}\eta} = s'(\eta)\left[\frac{u_\theta(r,\theta)}{u_s(r)} - r\tau(\eta)\right], \tag{2a, b}$$

where $(r(\eta), \theta(\eta))$ are polar coordinates of fluid particles in which $\theta = 0$ is the direction toward the center of curvature at the cross-section of η, and $s'(\eta)$ expresses the derivative of arclength along the centerline of the pipe with respect to η. Here the last term on the right-hand side of eq. (2b) is necessary in order to take the effect of torsion into account. For a usual helical pipe of constant κ and τ, no chaotic motion of fluid particles is expected because eq. (2) is an antonomous system with respect to two variables.

By solving eq. (2) numerically, we examine the chaotic motion of fluid particles and fluid mixing. In this examination, Poincaré sections of the locations of fluid particles on the (R, θ) cross-sectional plane at $\eta = 2n\pi$, (n : integer) are mainly used. Here $R = r\sqrt{2 - r^2}$ is used in place of r so that the Poincaré map of cross-sectional motion of fluid particles associated with their axial movement by $\eta = 2\pi$ is area-preserving. It is found that the chaotic region in Poincaré sections is relatively large for Re around 40 and that there is an intermediate range of Re for which high mixing efficiency is expected. Next, the dependence of Poincaré sections on the ratio of A and B is examined for fixed value of circumferential length of the ellipse around which the pipe is twisted. Larger chaotic region is observed for a thinner ellipse. Therefore, the pipe twisted around a thinner elliptic cylinder is expected to be more efficient for mixing. From the examination of dependences of Poincaré sections on other constants in the expression of the centerline, we also find that larger chaotic region is observed if the pipe is twisted around a cylinder of smaller radius, and that if $A < B$, the case of $\eta_0 = 0$ is expected to be the most efficient for mixing. Moreover, from the examination of motions of initially-separated two kinds of many fluid particles in a few periods of η, we confirm that the chaotic motion is important for the efficient mixing of fluids in a small number of periods. For example, the twisted pipe considered in the present study is much more efficient for mixing in a few periods than a usual helical pipe of constant κ and τ that causes no chaotic motion. Finally, we find that most of the dependences of Poincaré sections and mixing efficiency on the constants in the expression of the centerline can be explained by the variation of $\lambda(\eta) = 12\tau(\eta)/(\kappa(\eta)Re)$ in one period of η. That is, if $\lambda(\eta)$ varies with larger amplitude within the range $0 < \lambda < 1$, larger chaotic region is expected.

References

[1] J. M. Ottino, Mixing, chaotic advection, and turbulence, *Annu. Rev. Fluid Mech.*, **22**, pp. 207–253, 1990.

[2] H. Aref, The development of chaotic advection, *Phys. Fluids*, **14**, pp. 1315–1325, 2002.

[3] J. M. Ottino, *The Kinematics of Mixing: Stretching, Chaos, and Transport* (Cambridge University Press, Cambridge, 1989).

[4] M. Funakoshi, Chaotic mixing and mixing efficiency in a short time, *Fluid Dyn. Res.*, **40**, pp. 1–33, 2008.

[5] S. W. Jones, O. M. Thomas, and H. Aref, Chaotic advection by laminar flow in a twisted pipe, *J. Fluid Mech.*, **209**, pp. 335–357, 1989.

[6] W. R. Dean, Note on the motion of fluid in a curved pipe, *Phil. Mag.*, **4**, pp. 208–223, 1927.

[7] N. Acharya, M. Sen, and H. -C. Chang, Heat transfer enhancement in coiled tubes by chaotic mixing, *Int. J. Heat Mass Transfer*, **35**, pp. 2475–2489, 1992.

[8] C. Castelain, D. Berger, P. Legentilhomme, A. Mokrani, and H. Peerhossaini, Experimental and numerical characterisation of mixing in a steady spatially chaotic flow by means of residence time distribution measurements, *Int. J. Heat Mass Transfer*, **43**, pp. 3687–3700, 2000.

[9] A. Yamagishi, T. Inaba, Y. Yamaguchi, Chaotic analysis of mixing enhancement in steady laminar flows through multiple pipe bends, *Int. J. Heat Mass Transfer*, **50**, pp. 1238–1247, 2007.

[10] B. Jang and M. Funakoshi, Chaotic mixing in a helix-like pipe with periodic variations in curvature and torsion, *Fluid Dyn. Res.*, **42**, 035506, 2010.