Geometry of state space in plane and pipe flows

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A large conceptual gap separates the theory of low-dimensional chaotic dynamics from the infinite-dimensional nonlinear dynamics of turbulence. Recent advances in experimental imaging, computational methods, and dynamical systems theory suggest a way to bridge this gap and make a fundamental breakthrough in our understanding of turbulence. It has recently been discovered that recurrent coherent structures observed in wall-bounded shear flows (such as pipes and boundary layers) result from close passes to weakly unstable invariant solutions of the Navier-Stokes equations. These 3D, fully nonlinear solutions (equilibria, traveling waves, and periodic orbits) structure the state space of turbulent flows and provide a skeleton for analyzing their dynamics. We calculate a hierarchy of invariant solutions for a canonical wall-bounded shear flow and use these solutions (1) to develop a quantitative description of the flow's turbulent dynamics, and (2) to predict, directly from the fundamental equations, physical quantities such as bulk flow rate and mean wall drag. We use a combination of novel and proven numerical and analytical techniques, such as periodic orbit theory, group representation theory, nonlinear search methods and variational solvers, and computational fluid dynamics. All results and numerical software are disseminated through through our group's collaborative e-book ChaosBook.org[1] and open-source CFD software and invariant solution database Channelflow.org[2].

Long-term averages in low-dimensional dynamical systems can be expressed as averages over invariant state-space sets (equilibria, periodic orbits, partially hyperbolic invariant tori). In this technical presentation we focus on this aspect of the theory, and speculate as to how such theory might work for moderately turbulent flows[3, 4, 5, 6, 7, 8].

Acknowledgments

References

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