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Time-Reversibility, Instability and Thermodynamics in N-body Systems interacting with Long-Range Potentials

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Chaos is considered to be one of the origins of irreversibility appearing in macroscopic systems, such as N-body systems [1]. Accordingly, stability of the system has been extensively examined numerically, especially in an N-body system interacting with short-range potentials. However, numerical irreversibility due to round-off errors may behave as if it were a physical irreversibility, although it is not a physical one [2]. The influence of round-off errors should be a more serious problem in an N-body system interacting with 'long-range potentials', e.g., self-gravitating systems [3]. For instance, in a typical star-rich cluster with a million stars, each star feels enough of the granularity of the gravitational field of the other stars that the consequent perturbations lead to a total loss of memory of the initial conditions of its orbit [4]. However, in N-body simulations, numerical fluctuations due to round-off errors could behave as if they were the physical perturbations. Therefore, we have investigated numerical irreversibility and instability of the self-gravitating system, through the Loschmidt reversibility paradox based on a velocity inversion technique [5]. Consequently, the memory loss time, when the simulated trajectory completely forgets its initial conditions, increases approximately linearly with the Lyapunov time.

Because of long-range attractive potentials, such a self-gravitating system exhibits several peculiar features, such as gravothermal catastrophe, negative specific heat and nonextensive statistical mechanics [6, 7]. In particular, the negative specific heat causes thermodynamic instability during dynamical evolutions of the system and, therefore, it has been investigated theoretically and numerically from a thermodynamic viewpoint [8, 9]. However, velocity distributions and velocity relaxations have not yet been extensively discussed in long-term nonequilibrium processes except for a few studies [10, 11], although they should play an important role in the thermodynamic properties and irreversibility of the system [12]. For example, Iguchi *et al.* proposed universal non-Gaussian velocity distributions for a spherical collapse in a violent gravitational process of a collisionless stage ($t < \tau_r$) [10], while Ispolatov *et al.* discussed Gaussian velocity distributions in core-halo states in a collisional stage ($t \gg \tau_r$) [11]. (Here τ_r represents the relaxation time, which is driven by the two-body encounter [6]). These works may suggest that long-range attractive interacting systems should finally relax towards a Boltzmann-like state thorough a collapse process [12].

In the present study, to clarify irreversibility and thermodynamic properties of self-gravitating N-body systems, we numerically examine long-term evolution of those systems, from an early relaxation to a collapse, especially focusing on velocity relaxations [12]. For this purpose, we observe a cold collapse process under a restriction of constant mass and energy. To simulate the dynamical evolution, we consider a typical small N-body system ($N = 125$) enclosed in a spherical container with adiabatic walls [11, 12]. In our units, the relaxation time τ_r of the system is evaluated as $\tau_r \approx 0.5$ [2]. To examine whether the simulated velocity distribution function is Gaussian, a q-Gaussian distribution function is defined as $f_q(v) = A[1 - B(1-q)v^2]^{1/(1-q)}$, where q is the Tsallis entropic parameter [7]. (A and B correspond to a normalization parameter and an inverse of temperature, respectively.) Moreover, to examine an overview of the velocity distributions, we employ the normalized ratio of velocity moments VM, i.e., a ratio of velocity moments $< {v_i}^2 >^2 / < {v_i}^4 >$ is normalized by a specific value corresponding to a Gaussian distribution [12]. (Here v_i and $< X >$ represent the velocity speed of the i-th particle and the mean of X at time t, respectively.) When the velocity or speed distribution is Gaussian, q or VM approaches 1, respectively.

In this abstract, we observe time evolutions of typical properties of the cold collapse process. As shown in Fig. 1, VM and q deviate from 1 in an early relaxation ($t < 1$). Accordingly, the velocity distribution is non-Gaussian (VM < 1 and $q > 1$) in this stage. Thereafter, VM and q further deviate

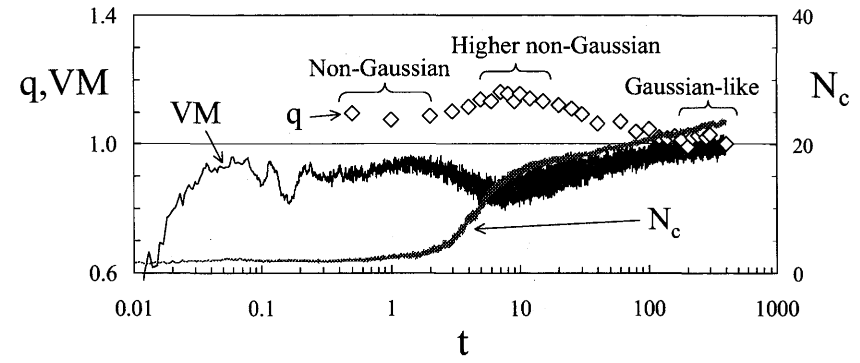

Figure 1: Time evolutions of the properties of the cold collapse process [12]. To mimic a cold collapse, the initial kinetic energy is set to be negligible values smaller than the order of 1% of the total energy. The total energy is $\varepsilon = -0.8$. The results are averaged over at least 30 simulations. To determine q, the q-Gaussian distribution function $f_q(v)$ is fitted with the simulated velocity distribution function.

from 1. In fact, the number N_c of core particles starts to increase at $t \approx$ 1–2. This suggests that the velocity distribution undergoes higher non-Gaussian distributions, especially when the core forms rapidly in the collapse process. However, the velocity distribution gradually relaxes toward a Gaussian-like distribution (VM, $q \sim 1$), after the core forms sufficiently ($t \gtrsim 200$). We found that the velocity distribution does not monotonically relax towards a Gaussian-like distribution. We clearly show such a transition of the velocity distribution, based not only on the Tsallis entropic parameter q but also on the normalized ratio of velocity moments VM. Of course, we have not yet clarified irreversibility in the strong nonequilibrium process of self-gravitating systems. However, our studies open up a new theoretical and numerical approach for examining irreversibility, instability and thermodynamics appearing in long-range interacting systems.

References

[1] W. G. Hoover, *Time reversibility, Computer simulation, and Chaos* (World Scientific Publishing Co., 1999).

[2] N. Komatsu, T. Abe, Numerical irreversibility in time-reversible molecular dynamics simulation, *Physica D*, vol. 195, pp. 391–397, 2004; Noise-driven numerical irreversibility in molecular dynamics technique, *Comput. Phys. Commun.*, vol. 171, pp. 187–196, 2005;

[3] R. H. Miller, Irreversibility in small stellar dynamical systems, *Astrophys. J.*, vol. 140, pp. 250–256, 1964.

[4] D. C. Heggie, P. Hut, *The gravitational million-body problem* (Cambridge University press, 2003).

[5] N. Komatsu, T. Kiwata, S. Kimura, Numerical irreversibility in self-gravitating small N-body systems. (II). Influence of instability affected by softening parameters, *Physica A*, vol. 388, pp. 639–650, 2009.

[6] J. Binney, S. Tremaine, *Galactic Dynamics* (Princeton University Press, 1987).

[7] C. Tsallis, *Introduction to Nonextensive Statistical Mechanics: Approaching a Complex World* (Springer, 2009).

[8] A. Taruya, M. Sakagami, Long-term evolution of stellar self-gravitating systems away from thermal equilibrium: connection with nonextensive statistics, *Phys. Rev. Lett.*, vol. 90, 181101, 2003.

[9] N. Komatsu, S. Kimura, T. Kiwata, Negative specific heat in self-gravitating N-body systems enclosed in a spherical container with reflecting walls, *Phys. Rev. E*, vol. 80, 041107, 2009; Thermodynamic properties of an evaporation process in self-gravitating N-body systems, *Phys. Rev. E*, vol. 82, 021118, 2010.

[10] O. Iguchi *et al.*, Universal non-Gaussian velocity distribution in violent gravitational processes, *Phys. Rev. E*, vol. 71, 016102, 2005.

[11] I. Ispolatov, M. Karttunen, Collapses and explosions in self-gravitating systems, *Phys. Rev. E*, vol. 68, 036117, 2003.

[12] N. Komatsu, T. Kiwata, S. Kimura, Relaxation of non-Gaussian velocity distributions in collapses of long-range attractive interacting systems, Abstracts of the 2nd international symposium on Multi-scale Simulations of Biological and Soft Materials (MSBSM2011), pp. 22, Kyoto, Japan, Sep., 2011; Transition of velocity distributions in collapses of self-gravitating N-body systems, *submitted to Phys. Rev. E.*