Time-Reversibility, Instability and Thermodynamics in \(N\)-body Systems interacting with Long-Range Potentials

Nobuyoshi Komatsu

Department of Mechanical Systems Engineering, Kanazawa University, Kakuma-machi, Kanazawa, Ishikawa 920-1192, Japan, komatsu@t.kanazawa-u.ac.jp

Chaos is considered to be one of the origins of irreversibility appearing in macroscopic systems, such as \(N\)-body systems [1]. Accordingly, stability of the system has been extensively examined numerically, especially in an \(N\)-body system interacting with short-range potentials. However, numerical irreversibility due to round-off errors may behave as if it were a physical irreversibility, although it is not a physical one [2]. The influence of round-off errors should be a more serious problem in an \(N\)-body system interacting with 'long-range potentials', e.g., self-gravitating systems [3]. For instance, in a typical star-rich cluster with a million stars, each star feels enough of the granularity of the gravitational field of the other stars that the consequent perturbations lead to a total loss of memory of the initial conditions of its orbit [4]. However, in \(N\)-body simulations, numerical fluctuations due to round-off errors could behave as if they were the physical perturbations. Therefore, we have investigated numerical irreversibility and instability of the self-gravitating system, through the Loschmidt reversibility paradox based on a velocity inversion technique [5]. Consequently, the memory loss time, when the simulated trajectory completely forgets its initial conditions, increases approximately linearly with the Lyapunov time.

Because of long-range attractive potentials, such a self-gravitating system exhibits several peculiar features, such as gravothermal catastrophe, negative specific heat and nonextensive statistical mechanics [6, 7]. In particular, the negative specific heat causes thermodynamic instability during dynamical evolutions of the system and, therefore, it has been investigated theoretically and numerically from a thermodynamic viewpoint [8, 9]. However, velocity distributions and velocity relaxations have not yet been extensively discussed in long-term nonequilibrium processes except for a few studies [10, 11], although they should play an important role in the thermodynamic properties and irreversibility of the system [12]. For example, Iguchi et al. proposed universal non-Gaussian velocity distributions for a spherical collapse in a violent gravitational process of a collisionless stage \((t < \tau_r)\) [10], while Ispolatov et al. discussed Gaussian velocity distributions in core-halo states in a collisional stage \((t \gg \tau_r)\) [11]. (Here \(\tau_r\) represents the relaxation time, which is driven by the two-body encounter [6]). These works may suggest that long-range attractive interacting systems should finally relax towards a Boltzmann-like state thorough a collapse process [12].

In the present study, to clarify irreversibility and thermodynamic properties of self-gravitating \(N\)-body systems, we numerically examine long-term evolution of those systems, from an early relaxation to a collapse, especially focusing on velocity relaxations [12]. For this purpose, we observe a cold collapse process under a restriction of constant mass and energy. To simulate the dynamical evolution, we consider a typical small \(N\)-body system \((N = 125)\) enclosed in a spherical container with adiabatic walls [11, 12]. In our units, the relaxation time \(\tau_r\) of the system is evaluated as \(\tau_r \approx 0.5\) [2]. To examine whether the simulated velocity distribution function is Gaussian, a \(q\)-Gaussian distribution function is defined as \(f_q(v) = A[1 - B(1 - q)v^2]^{1/(1-q)}\), where \(q\) is the Tsallis entropic parameter [7]. \((A\) and \(B\) correspond to a normalization parameter and an inverse of temperature, respectively.) Moreover, to examine an overview of the velocity distributions, we employ the normalized ratio of velocity moments \(VM\), i.e., a ratio of velocity moments \(\langle v_i^2 \rangle / \langle v_i^4 \rangle\) is normalized by a specific value corresponding to a Gaussian distribution [12]. (Here \(v_i\) and \(\langle X \rangle\) represent the velocity speed of the \(i\)-th particle and the mean of \(X\) at time \(t\), respectively.) When the velocity or speed distribution is Gaussian, \(q\) or \(VM\) approaches 1, respectively.

In this abstract, we observe time evolutions of typical properties of the cold collapse process. As shown in Fig. 1, \(VM\) and \(q\) deviate from 1 in an early relaxation \((t < 1)\). Accordingly, the velocity distribution is non-Gaussian \((VM < 1\) and \(q > 1)\) in this stage. Thereafter, \(VM\) and \(q\) further deviate
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Figure 1: Time evolutions of the properties of the cold collapse process [12]. To mimic a cold collapse, the initial kinetic energy is set to be negligible values smaller than the order of 1% of the total energy. The total energy is \( \varepsilon = -0.8 \). The results are averaged over at least 30 simulations. To determine \( q \), the \( q \)-Gaussian distribution function \( f_q(v) \) is fitted with the simulated velocity distribution function.

from 1. In fact, the number \( N_C \) of core particles starts to increase at \( t \approx 1-2 \). This suggests that the velocity distribution undergoes higher non-Gaussian distributions, especially when the core forms rapidly in the collapse process. However, the velocity distribution gradually relaxes toward a Gaussian-like distribution \( (VM,q \approx 1) \), after the core forms sufficiently \( (t \approx 200) \). We found that the velocity distribution does not monotonically relax towards a Gaussian-like distribution. We clearly show such a transition of the velocity distribution, based not only on the Tsallis entropic parameter \( q \) but also on the normalized ratio of velocity moments \( VM \). Of course, we have not yet clarified irreversibility in the strong nonequilibrium process of self-gravitating systems. However, our studies open up a new theoretical and numerical approach for examining irreversibility, instability and thermodynamics appearing in long-range interacting systems.

References