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Heteroclinic Cycles in Coupled Systems with Applications to Sensor Devices

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Coupled systems of differential equations are often used as models of physical systems. For example they have been used by Hadley *et al.* [5] and Aronson *et al.* [2] to model arrays of Josephson junctions, by Kopell and Ermentrout [1, 6, 7] and Rand *et al.* [12] to model coupled oscillators and central pattern generators (CPGs) in biological systems, by Pecora and Caroll [11] to investigate synchronization of chaotic oscillators, and, more recently, by Susuki, Takatsuji, and Hikihara [13] to study power grid systems. In these works the symmetry of the network is important in determining the patterns of collective behavior that the system can support. One particular pattern of behavior that is commonly found in symmetric coupled systems is cycling behavior, in which solution trajectories can linger around steady states and periodic solutions for increasingly longer periods of time. These type of cycles are formally called *heteroclinic* if the solutions that are part of the cycle are all different. Otherwise the cycles are called *homoclinic.*

In this paper we discuss the existence and stability of heteroclinic cycles in coupled systems and show how they can be exploited to design and fabricate a new generation of highly-sensitive, self-powered, sensor devices. More specifically, we present theoretical and experimental proof of concept that coupling-induced oscillations located near the bifurcation point of a heteroclinic cycle can significantly enhance the sensitivity of an array of magnetic sensors. In particular, we consider arrays made up of fluxgate magnetometers inductively coupled through electronic circuits, see Figure 1.

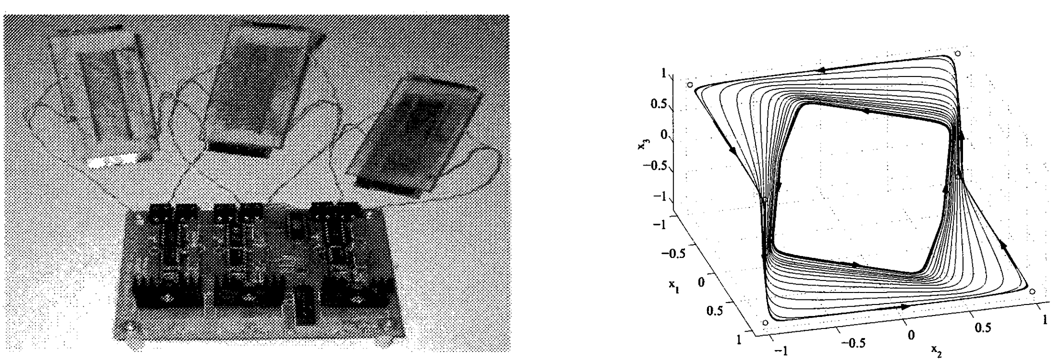

Figure 1: (Left) Prototype of an array of three fluxgates inductively coupled. (Right) Phase-space shows coupling-induced oscillations near a heteroclinic cycle (bold curve).

At the center of this discovery is the phenomenon of coupling-induced oscillations, in which the topology of connections, i.e., which sensors are coupled with each other, and the nonlinearities of materials can be exploited to produce self-oscillations that limit in a heteroclinic cycle. This phenomenon is dictated by symmetry conditions alone. In other words, the ideas and methods are *device-independent*: similar principles can be readily applied to enhance the performance of a wide variety of sensor devices so long as the symmetry conditions are satisfied.

From a mathematical point of view, a heteroclinic cycle is a collection of solution trajectories that connects sequences of equilibria, periodic solutions or chaotic invariant sets via saddle-sink connections. For a more precise description of heteroclinic cycles and their stability, see Melbourne *et al.* [10], Krupa and Melbourne [9], the monograph by Field [3], and the survey article by Krupa [8]. Such behavior is unusual in a general dynamical system. It is, however, a generic feature of dynamical systems that possess symmetry. Indeed, the presence of symmetry can lead to invariant subspaces under which a sequence of saddle-sink connections can be established, resulting in cycling behavior. As time evolves, a typical trajectory would stay for increasingly longer period of time near each solution (which could be either an equilibrium, a periodic orbit or a chaotic invariant set) before it makes a rapid excursion to the next solution. Since saddle-sink connections are robust, these cycles called heteroclinic cycles–are robust under perturbations that preserve the symmetry of the system.

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