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Stabilization of Buckled Beam with Coulomb Friction by High-Frequency Excitation

Hiroshi Yabuno[1]
[1] *Faculty of Science and Technology, Keio University, Hiyoshi, Kouhoku-ku, Yokohama, 223-8522, Japan, yabuno@mech.keio.ac.jp*

Introduction

Dynamics of a hinged-hinged beam is theoretically and experimentally under high-frequency excitation. There have been many studies on the stabilization of buckled beams by the high-frequency excitation [1, 2, 3]. By the way, different from a fixed-fixed beam, the bending moment at the supporting points exits due to Coulomb friction at the rotating parts (for example bearing). Generally, such bending moment is very slight, but affects the buckling phenomenon very much [4] because of very low stiffness. The Coulomb friction produces continuous stable steady states in the regions which surround the branches of pitchfork bifurcation in the case without Coulomb friction. In the present study, we investigate the effect of high-frequency excitation on the buckled beam subjected to Coulomb friction. It is theoretically predicted that the high-frequency excitation shifts the stable region. In contrast with the stabilization of buckled clamped-clamped beam, the deflection cannot be zero, but decreased to infinite small because even if the stable region is shifted, the deflection is again included in the shifted stable region. Furthermore, we experimentally confirm the theoretically predicted behavior under the high-frequency excitation.

Analytical model and governing equation

We set an inertial frame $X-Y$ as shown in Fig. 1. A hinged-hinged beam subjected to the static compressive force P at the end mass m is set on the plate. There is a radial bearing at supporting point S_2 and its rotation produces the bending moment due to Coulomb friction, M. On the other hand, at supporting point S_2 subjected to compressive force, there are both radial and linear bearings. Also at this point, the bending moment M is produced. $v(s,t)$ is the displacement of the inextensible center line at the position s in y-direction. Under periodic excitation of the plate in X-direction as $a\cos Nt$, we perform bifurcation control of the beam and analyze the dynamics of the buckled beam under the effects of the friction at the supporting points. The dimensionless equation of motion for the deflection is expressed as follows:

$$\ddot{v} + Pv'' + v'''' - a\Omega^2 \cos\Omega t\{(1-s)v'' - v'\} - ma\Omega^2 v'' \cos\Omega t = 0 \tag{1}$$

The boundary conditions for the hinged-hinged beam are

$$v\big|_{s=0} = v\big|_{s=1} = 0, v''\big|_{s=0} = M_{s1}, v''\big|_{s=1} = M_{s2}, \tag{2}$$

where dot and prime denote $\partial/\partial t$ and $\partial/\partial s$, respectively. The bending moment of M_{s1} and M_{s2} is assumed the same as M and M is assumed using a set-valued sign function as follows [5]:

$$M = M_{max}\mathrm{Sgn}(\dot{v}'), \tag{3}$$

where

$$\mathrm{Sign}(\dot{v}') = \begin{cases} \mathrm{sgn}(\dot{v}'), & \dot{v}' \neq 0 \\ [-1,1], & \dot{v}' = 0 \end{cases} \tag{4}$$

M_{max} is the maximum static bending moment due to the maximum static friction. $\mathrm{sgn}(\dot{v}')$ stands the sign function: $\mathrm{sgn}(\dot{v}') = 1$ for $\dot{v}'|_{x=0} > 0$ and $\mathrm{sgn}(\dot{v}') = -1$ for $\dot{v}'|_{x=0} < 0$.

Perturbation analysis and modulation equation

We seek a third-order uniform expansion in the form as $v = \epsilon v_1 + \epsilon^2 v_2 + \epsilon^3 v_3$, where ϵ is an order parameter ($|\epsilon| \ll 1$) and we introduce the multiple time scales as follows: $t_0 = t, t_1 = \epsilon t, t_2 = \epsilon^2 t$. For focusing on the dynamics of the critical point of buckling, we set the compressive force P as $P = P_{cr}(1+\sigma)$, where P_{cr} is the critical load and σ is a small detuning parameter; the beam is in the state of post-buckling when $\sigma > 0$. We estimate the order of the parameters as follows: $a = \epsilon\hat{a}, \sigma = \epsilon^2\hat{\sigma}$, and $M = \epsilon^3\hat{M}$. Applying the method of multiple scales [4], we obtain the approximate solution as

$$v = A(t)\Phi(s) + O(\epsilon^2), \tag{5}$$

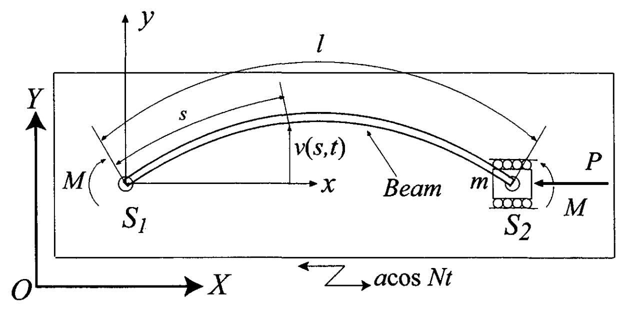

Figure 1: Analytical model for buckled beam with Coulomb friction at the supporting points

where Φ is the first mode shape at the critical point in the case without excitation and Coulomb friction and the equation describing the deflection at the midpoint A is obtained from the solvability condition for v_3 as:

$$\frac{d^2A}{dt^2} + (C_1(\Omega)a^2 - C_2P_{cr}\sigma)A = M, \tag{6}$$

where C_2 is a positive constant and $C_1(\Omega)$ increases with the excitation frequency Ω. Neglecting the effect of Coulomb friction M, the critical point for the buckling is $\sigma = 0(P = P_{cr})$ and the excitation increases the critical point to $C_1(\omega)a^2/(C_2P_{cr})$. However, the Coulomb friction produces the equilibrium region surrounded with the curve in Fig. 2 (a); the states in this region are stable and even large deflection is stable in the neighborhood of the critical point. The excitation can shift this region to as Fig. 2(b) and the deflection in the buckling state is decreased.

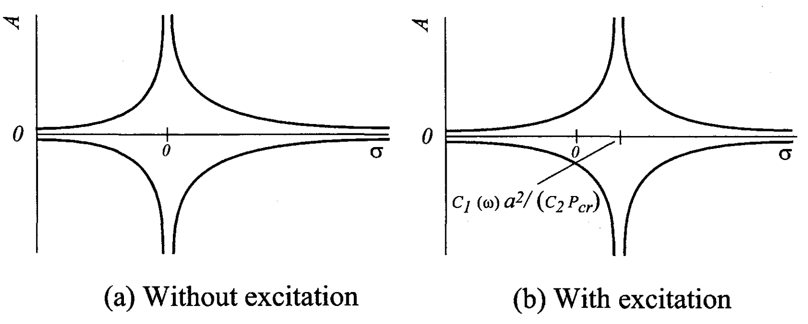

Figure 2: Stabilization of buckled beam under the high-frequency excitation (Linear theory)

Conclusion

In this research, it is theoretically and experimentally investigated that Coulomb friction at the supporting points produces a stable region of the beam in the post-buckling state, and the axial excitation shifts the region and stabilize the buckling state.

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