IUTAM Symposium on 50 Years of Chaos : Applied and Theoretical
Kyoto, Japan, November 28 - December 2, 2011

Controlling Nonlinear Dynamics of Systems Liable to Unstable Interactive Buckling

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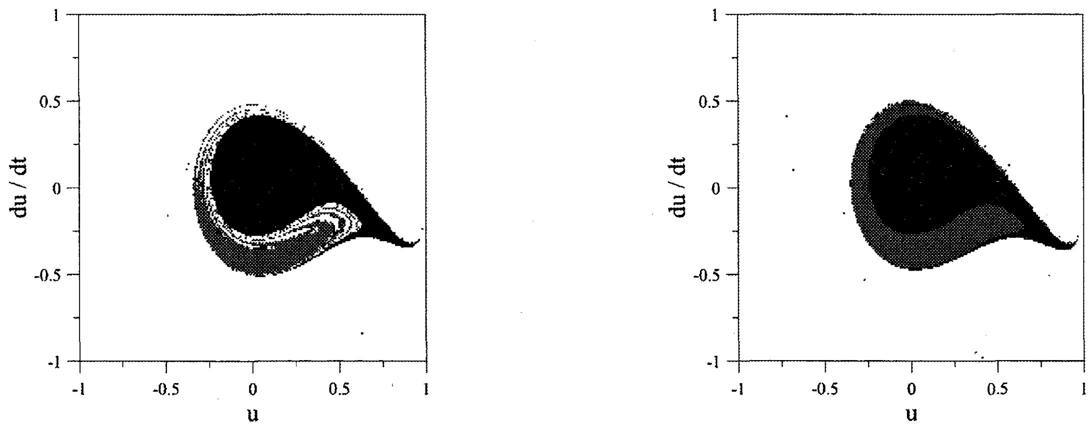
Since its early development at the beginning of the Nineties, chaos control has been a very active research area with important methodological and phenomenological achievements, while at the same time being partially revisited and extended in its very meaning along the seminal idea of exploiting the chaotic behavior of systems to control their dynamics, an idea which in fact was already embedded in the most promising original studies. An important matter in this framework is whether the dynamical phenomenon to be controlled by whatever chaos control technique has to be intended as chaos in a strict sense or, rather, as any kind of *complex behavior* of a dynamical system, which may have different aspects according to whether a theoretical or a practical viewpoint is adopted [1]. In the former, dynamical systems oriented, perspective, one can think of, e.g., any global bifurcation event possibly entailing complex behavior of the system, or of the relevant escape from a safe subset (a potential well or a basin of attraction) in parameter control space; in the latter, application-oriented, perspective, the kind and meaning of the complex phenomenon of interest is dictated by the practical goal to be attained.

This paper aims at highlighting the meaningfulness of chaos control, in its extended notion, as regards the design of discrete and continuous systems liable to unstable buckling, which is an important research area in structural mechanics and engineering; in particular systems where the interaction of different stable buckling modes with equal or nearly equal critical loads leads to coupled unstable post-buckling paths. Among them, Augusti's model has been used in the stability literature for many decades as a conspicuous model of such behaviour [2]. These structures are sensitive to initial imperfections or disturbances and may loose stability at load levels well below the critical load of the perfect system. For load levels lower than the maximum stable static load, these systems are characterized by a multi-well potential function and may escape from the pre-buckling well under a finite perturbation. The safety of the desired pre-buckling configuration can be measured through the associated basin of attraction. Recent studies have shown that this safe basin decreases swiftly and becomes zero at the critical load [3, 4]. The boundary of this safe basin is defined by the saddles lying on the contour of the safe pre-buckling well. For the undamped case, the boundary of the safe region in phase-space is defined by the homoclinic or heteroclinic orbits connecting these saddles. The maximum stable load is in this case an unsafe upper bound of the load carrying capacity of the system. When, in addition to static load, these systems are subjected to dynamic loads, they usually display a complex dynamics. Their safety can be evaluated by analyzing the evolution and stratification of the basins of attraction of the safe solutions as a function of the load parameters. This process is also controlled by the evolution of the stable and unstable manifolds of the saddles associated with the boundary of safe region. This leads to the determination of erosion profiles, which enlighten the loss of safety of the structure due to penetration of eroding fractal tongues into the safe basin, up to final escape. These tongues are due to global bifurcations and may be controlled by the elimination of homoclinic (or heteroclinic) intersections.

In this paper a method for controlling nonlinear dynamics and chaos, recently developed [5] and tested on various mechanical systems [1, 6], is implemented and applied to two prominent examples of structures displaying unstable interactive buckling, namely the perfect and imperfect Augusti's model and a simplified model of a guyed tower [4]. The method consists in the optimal elimination of homoclinic (or heteroclinic) intersection by properly adding superharmonic terms to a given harmonic excitation. By means of the solution of an appropriate optimization problem, it is possible to select the amplitudes and the phases of the added superharmonics in such a way that the manifolds distance is as large as possible. Figure 1 illustrates the beneficial influence of the control method on the safe basin of the perfect guyed tower model. While the basin of the uncontrolled model is already highly eroded, the controlled model exhibits a much broader safe area (the non-white one). To check the performance of the control method, several integrity profiles have been built with and without control, and they are compared with each other in Figure 2, where the variation of the global integrity measure (GIM) and of the integrity factor (IF) [1, 2] are shown as a function of the forcing amplitude [4]. The results show that the control systematically permits to shift toward higher excitation amplitudes the sudden fall of integrity which ends up with the escape phenomenon. This shows the effectiveness of control in increasing the "practical" safety and performance of the system in a dynamic environment, and represents an important result of the present work.



(a) Original model (b) Controlled model

Fig 1: Basins of attraction of the uncontrolled and controlled guyed tower at the same load level F=0.03 [4].

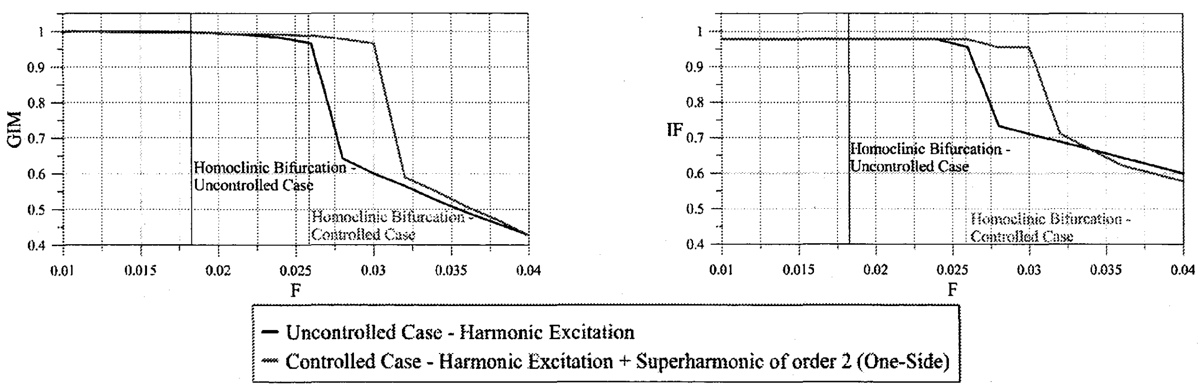


Fig. 2: Comparison of the global integrity measure (GIM) and of the integrity factor (IF) as a function of the forcing amplitude for the guyed tower considering the controlled and uncontrolled cases [4].

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