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Augmented Lorenz Equations as Physical Model for Chaotic Gas Turbine

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Introduction

The chaotic waterwheel invented by Malkus and Howard [1, 2] is an intriguing machinery that can yield chaotic rotational motion as a manifestation of the strange attractor generated by Lorenz equations [3]. Inspired by the chaotic waterwheel, we have recently developed a chaotic gas turbine whose rotational motion is subject to an augmented version of the Lorenz equations [4, 5]. The augmented Lorenz equations are a nondimensionalized expression of the equations of motion of the gas turbine, being a system of N ($N \to \infty$) subsystems of quadratically coupled ordinary differential equations with three degrees of freedom, in which each subsystem is coupled to each other via the nondimensionalized angular velocity of the rotor of the turbine.

In our recent study [5], we performed a numerical analysis of the augmented Lorenz equations with the reduced Rayleigh number and the Prandtl number specified by the structure of our current gas turbine. The probability density functions of the nth diagonal component Z_n of the matrix variable $\mathbf{Z}$ (briefly described below) were numerically estimated and compared with the histograms of temperature gradient experimentally observed by He and Tong [6] for a turbulent convective cell in water heated from below. Despite a considerable difference in the Prandtl number between our equations ($\sigma = 28.3$) and the working fluid of He and Tong ($\sigma = 5.5$), we have found that our equations are capable of qualitatively reproducing the statistical distribution of the thermal field in the convective cell. In this paper, we conduct a numerical analysis of the augmented Lorenz equations specified by $\sigma = 5.5$ and compare the results with the previous results of He and Tong, discussing how our gas turbine may simulate the convective motion of turbulent heat flows.

Theory

Our gas turbine has a stainless steel disk rotor with a diameter of 40 [mm] sandwiched in three layers of acrylic plates. Its actual structure can be seen in [4]. The circular rotor simulates a convective cell with an aspect ratio of unity. The air inflow impinges the turbine blades near the inlet within $\pm\phi$ about the central horizontal axis of the turbine, driving the rotor to rotate. In our machine, the physical forces driving the Rayleigh-Bénard convection, i.e., buoyancy, thermal dissipation and friction, are simulated by the air pressure on the turbine blades, the leakage of air from the rotor and the frictional force on the axis of the rotor. The augmented Lorenz equations are obtained by the nondimensionalization of the equations of motion of the gas turbine. Details will be shown in [5]. The equations are given as

$$\frac{dX}{d\tau} = \sigma\left[\mathrm{tr}\left((\mathbf{n}^{-1})^2\mathbf{Y}\right) - X\right]\ , \quad \frac{d\mathbf{Y}}{d\tau} = \mathbf{R}X - \mathbf{nZ}X - \mathbf{Y}\ , \quad \frac{d\mathbf{Z}}{d\tau} = \mathbf{nY}X - \mathbf{Z}\ , \tag{1}$$

where $\mathrm{tr}(\cdot)$ denotes diagonal sum, τ is the dimensionless time, $\mathbf{n} = \mathrm{diag}(1\ \cdots\ n\ \cdots\ N)$, the scalar variable X is a nondimensionalized angular velocity of the rotor, and $\mathbf{Y}$ and $\mathbf{Z}$ are $N \times N$ diagonal matrices whose diagonal components represent the nondimensionalized Fourier sinusoidal and cosinusoidal coefficients of the air flow in the turbine, respectively. The $N \times N$ coefficient matrix $\mathbf{R}$ corresponds to scaled Rayleigh numbers and the scalar coefficient σ corresponds to the Prandtl number, defined as

$$\sigma = \frac{v}{I(K+\alpha)}\ , \qquad \mathbf{R} = \frac{2\alpha S r P_{in}}{(K+\alpha)^2 v\pi}\mathbf{n}^2\mathbf{\Phi}\mathbf{W} = R_0\mathbf{n}^2\mathbf{\Phi}\mathbf{W}\ ,$$

where $\mathbf{W} = \mathrm{diag}(\sin\phi\ \cdots\ \sin n\phi\ \cdots\ \sin N\phi)$, $\mathbf{\Phi} = \mathrm{diag}\Big(\phi - \frac{1}{2}\sin 2\phi\ \cdots\ \frac{1}{n-1}\sin(n-1)\phi - \frac{1}{n+1}\sin(n+1)\phi\ \cdots\ \frac{1}{N-1}\sin(N-1)\phi - \frac{1}{N+1}\sin(N+1)\phi\Big)$, K is the air leakage rate, α is the air inflow rate, v is the damping rate for the frictional force, r is the radial position of the center of mass of the rotor, S is the area of the blade, I is the inertial moment of the rotor and P_{in} denotes the pressure of the air inflow into the turbine.

Numerical Analysis and Discussion

We numerically integrate Eqs. (1) using the fourth-order Runge-Kutta method under a time width of $\Delta\tau = 4 \times 10^{-5}$, $N = 1000$, $\phi = 0.36$ [rad], $R_0 = 4000$ and $\sigma = 5.5$. The initial 250 000 data points are discarded

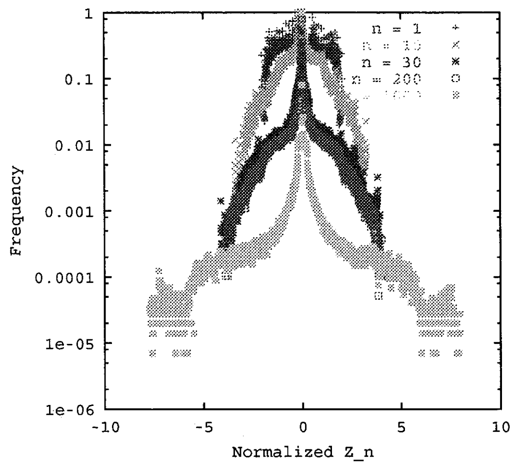

Figure 1: Histogram of normalized Z_n with 1000 partitions. $n = 1$ (+), 10 (×), 30 (∗), 200 (open box) and 1000 (closed box).

to eliminate the transient part of the solutions. Chaotic behaviors are observed for X and for Y_n and Z_n with n running from 1 to N. However, there is a marked difference in Y_n and Z_n between $n = 1$ and $n >> 1$. That is, larger n's generate intermittency in chaos, whereas smaller n's do not. To examine the statistical properties of the behaviors, we estimate the histograms of Z_n as a function of $(Z_n - \bar{Z}_n)/\sigma_n$, where σ_n is the standard deviation of Z_n. The results are shown in Fig. 1. The probability density function of Z_n has in part an exponential tail when n is large, whereas it does not when n is small. The comparison of our results with the previous results of He and Tong leads to a tentative interpretation that Z_1 may simulate the temperature gradient in the outer region of a convective cell, whereas Z_n may simulate those in the inner regions toward the cell center with increasing n. In this context, the motion of our gas turbine may simulate the convective motion of turbulent heat flows.

Acknowledgments

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