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Regular and Chaotic Vibrations of Self-Excited Oscillators Driven by Parametric and External Excitations

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Problem Formulation

Self-excited vibrations can occur in the nature or in systems produced by human beings. They are common in mechanical or electrical engineering. Depending on the model their motion is represented by stable or unstable limit cycles. The dynamics may change if the self-excited system is additionally driven by parametric or external harmonic excitations. Then, interesting dynamical phenomena like quasi-periodicity, frequency looking or transition to chaotic [1] or hyperchaotic motion take place [2], [3]. Self-excitation is very often modelled by van der Pol or Rayleigh terms represented by functions $f_R = \left(-\alpha_1 + \beta_1 \dot{x}_1^2\right) \dot{x}_1$, $f_V = \left(-\alpha_2 + \beta_2 x_2^2\right) \dot{x}_2$, which produce limit cycles in the phase space. It is commonly stated that both Rayleigh and van der Pol models are equivalent and can be transformed one into another. This statement is valid only if the rest terms of the model, e.g. inertia or elasticity, are linear. However, if the model is nonlinear and then we exchange Rayleigh into van der Pol damping, or vice versa, the dynamics of the system is changed as well.

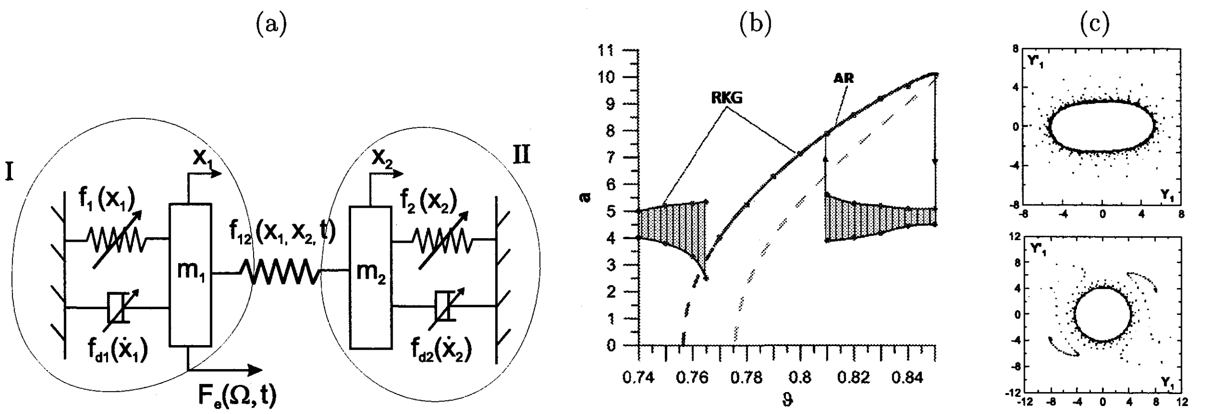

Figure 1: Model of nonlinear coupled self-excited oscillators (a), resonance curve around the first natural frequency (b), Poincaré maps for $\vartheta = 0.76$ and $\vartheta = 0.83$ (c)

The model of the vibrating system considered in this paper is composed of two oscillators with nonlinear springs and nonlinear dampers (Fig.1a). The dampers represent self-excitation of the system and for each oscillator they are considered in two variants as Rayleigh's f_R or/and van der Pol's function f_V. Both oscillator are coupled by a spring with periodically changing stiffness and moreover the system can be excited by external harmonic force. In practice such a model represents for example vibrations of flexible structures under fluid flow. Differential equations of motion take dimensionless form

$$\begin{aligned} m_1\ddot{X}_1 + \delta_1 X_1 + \delta_{12}\left(X_1 - X_2\right) &= \varepsilon\left[-\tilde{f}_{d1}(X_1, \dot{X}_1) - \tilde{\gamma}_1 {X_1}^3 + \tilde{\mu}\cos 2\vartheta\tau\left(X_1 - X_2\right) + \tilde{f}_e\cos\Omega\tau\right] \\ m_2\ddot{X}_2 + \delta_2 X_2 - \delta_{12}\left(X_1 - X_2\right) &= \varepsilon\left[-\tilde{f}_{d2}(X_2, \dot{X}_2) - \tilde{\gamma}_2 {X_2}^3 - \tilde{\mu}\cos 2\vartheta\tau\left(X_1 - X_2\right)\right] \end{aligned} \tag{1}$$

where damping functions f_{d1}, f_{d2} are taken in variants as Rayleigh or van der Pol, ε is a formal small parameter used for grouping of nonlinear terms of the model, parameters μ, ϑ and f_e, Ω mean amplitudes and frequencies of parametric and external excitations, respectively. As it has been found in [2] parametric and/or external excitations may change the system response from quasi-periodic into periodic and then chaotic or hyperchaotic. The scenario is different for Rayleigh and van der Pol self-excitations.

Regular Oscillations

Regular dynamics of the system is found assuming that the differential equations of motion (1) are weakly nonlinear. The original equations are transformed from generalised X_1, X_2 to *quasi-normal coordinates* Y_1, Y_2

$$\begin{aligned} M_1\ddot{Y}_1 + \omega_{01}^2 Y_1 &= \varepsilon\left(u_{11}\tilde{F}_1 + u_{21}\tilde{F}_2\right) \\ M_2\ddot{Y}_2 + \omega_{02}^2 Y_2 &= \varepsilon\left(u_{12}\tilde{F}_1 + u_{22}\tilde{F}_2\right) \end{aligned} \tag{2}$$

where u_{ij} are vibration modes coefficients, M_1, M_2 modal masses, and F_1, F_2 nonlinear functions related to the first and the second oscillator. If $\varepsilon = 0$ then the system (2) is uncoupled and the corresponding modes are linear. The analytical solutions and bifurcation points are found around the natural frequencies by multiple scale of time method taking into account that ε is small and positive. Typical resonance curve near the first natural frequency is presented in Fig.1(b). The response is manifested by quasi-periodic oscillations (vertical lines in Fig.1 b). Quasi-periodic response occurs out of the resonance zones. Approaching the resonance the frequency locking is observed (solid blue line). However, quasi-periodic and periodic oscillations may co-exist for the same parametric excitation frequency (Poincaré maps in Fig.1 c). This regular dynamics may change if small external force is added to the system inside the frequency locking zones [2].

Chaotic Oscillations

Analytical results presented has been found for a weakly nonlinear system. The analysis is completed by numerical and also experimental investigations on an analogue computer taking into account change of the parameters in wide ranges. Amplitude of parametric excitation μ is selected as one of the important bifurcation

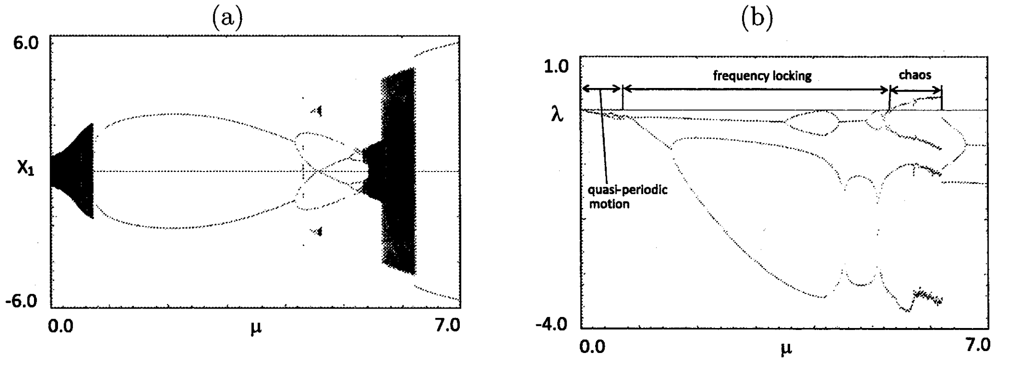

Figure 2: Transition to chaos, (a) bifurcation diagram of X_1 coordinate versus parametric excitation and (b) equivalent Lyapunov exponents, mixed Rayleigh van der Pol model

parameters. Figure 2 presents how parametric excitation influence the system dynamics. We can see the transition from quasi-periodicity (black region) to subharmonic response (double solid line) after the inverse second kind of Hopf bifurcation, and about $\mu \approx 4$ the period doubling and then cascade of period doubling leading to chaos take place. Further increase leads to subharmonic response after *the boundary crisis bifurcation.* These all regions are clearly visible on Lyapunov diagram (Fig.2 b).

Conclusions

Analysis of nonlinear systems dynamics with self-excitations show that models with van der Pol term present higher tendency in going to chaos. It has been observed that model with Rayleigh terms produce less and narrower chaotic zones. The small external force, increasing from zero, transits smoothly the system from chaos, via quasi-periodicity, to periodicity. However, this force change qualitatively and quantitatively regular dynamics near the resonance regions.

Acknowledgements

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References

[1] Y. Ueda, *The Road to Chaos–II* (Aerial Press, Inc., 2001).

[2] J. Warminski, Synchronisation Effects and Chaos in van der Pol-Mathieu Oscillator, *Journal of Theoretical and Applied Mechanics*, 4(39), pp.861–884, 2001.

[3] A. Tondl, A. Ecker, H., Cancelling of Self-Excited Vibrations by Means of Parametric Excitation, *Proc. of the 1999 ASME Design Engineering Technical Conferences*, DETC99/VIB-8071, Las Vegas, Nevada, USA (1999).