Chaos in Thermo-visco-elastic Systems Subject to Laser Irradiation

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A self-excited nonlinear dynamical system is one that in the absence of external modulated forcing, will undergo bounded periodic limit-cycle oscillations beyond a stability threshold of an equilibrium state. Thermally driven limit-cycle oscillations have been shown to occur in mechanical systems that span multiple spatial scales. A large scale example is a space structure which absorbs solar radiation that can either increase or decrease as the structure bends towards or away from the incoming radiation. This consists of a feedback loop that can change the equilibrium configuration or can lead to self-excited bending vibrations. Additional examples include limit-cycle oscillations of five cm long aluminum coated glass cantilevers [1], and recently, various nano-resonators in the shape of disks, domes, paddles and wires [2]. The advantages of self-excited nano-electro-mechanical-systems include a dramatic improvement of the quality factor via parametric amplification, stability enhancement through the use of feedback, and incorporation of a single optical configuration for both drive and motion sensing. To date, these systems have been modeled by single-degree-of-freedom resonators coupled to a lumped-mass thermal description. However, while their analysis qualitatively reveals the onset of limit cycle oscillations, the analytically determined thresholds differ from measurements by a factor of two [2]. Furthermore, these systems have been shown experimentally to exhibit complex vibrations that alternate between several continuous vibration modes which cannot be explained by lumped-mass models [1].

Thus, in order to resolve the spatio-temporal complexity of the thermo-elastic system response near primary, secondary and internal resonances, we formulate an initial-boundary-value problem that consistently includes both nonlinear viscoelastic and thermal fields [3]. We determine the coupled thermo-elastic field basis functions and construct a low-order nonlinear modal dynamical system for experimental conditions (Fig. 1) defined by Hane in 1996 [1].

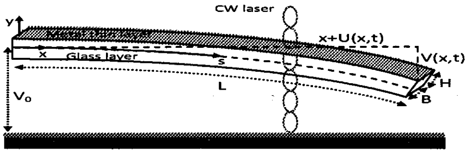

Figure 1: Definition sketch of the laser irradiation initial-boundary-value problem.

The resulting dynamical system truncated to cubic order, consistently incorporates the coupled thermo-visco-elastic equations [3] with the geometric stiffness and gyroscopic nonlinearities of a micro-cantilever developed for finite amplitude dynamics in atomic force microscopy [4]. The influence of the laser is embedded within the thermal field equation as the time-averaged absorption of a standing wave captured within a bi-material (the cantilever) and the mirror, creating a Fabri-Pero interferometer. Stability analysis of the thermo-elastic dynamical system equilibrium configuration reveals existence of a complex bifurcation structure (Fig. 2) which includes coexisting bi-stable solutions and flutter thresholds that correspond to saddle-node and Hopf bifurcations, respectively.

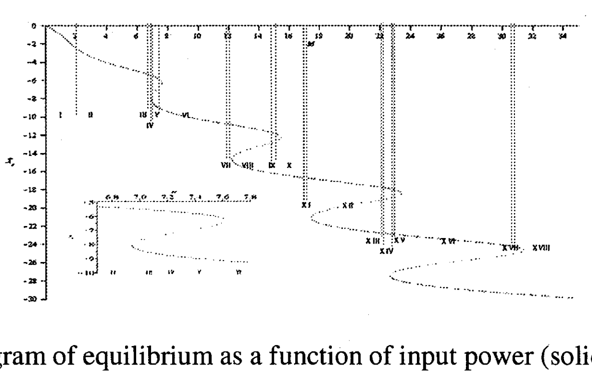

Figure 2: Bifurcation diagram of equilibrium as a function of input power (solid-stable, dashed-unstable).

A numerical analysis of system response exhibits free vibration decay (Fig. 3 left) below the Hopf threshold in region I of Figure 2, self-excited vibrations (Fig. 3 center) for the low power input documented by [1] in region II of Figure 2, and possible irregular chaotic jumps (Fig. 3 right) between coexisting bi-stable solutions in region V of Figure 2.

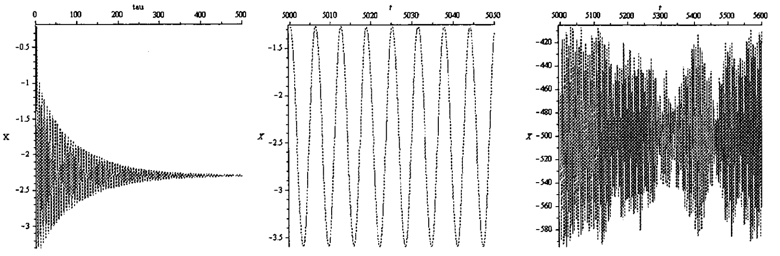

Figure 3: Cantilever tip time-series response: free decay (left) below the first Hopf threshold, periodic limit-cycle motion above the first Hopf threshold (center), and non-stationary response above the first bi-stable transition (right).

Investigation of system periodicity via sampling of the non-dimensional displacement (X) and temperature (Z) response intersection with the zero velocity plane (Y=0), yields a bifurcation diagram of Poincare' points for various values of input power (Fig. 4 left). The bifurcation structure reveals a period-doubling mechanism (M~15) which culminates with a strange attractor (M~15.5) which is then destroyed via a reverse bifurcation (M~16).

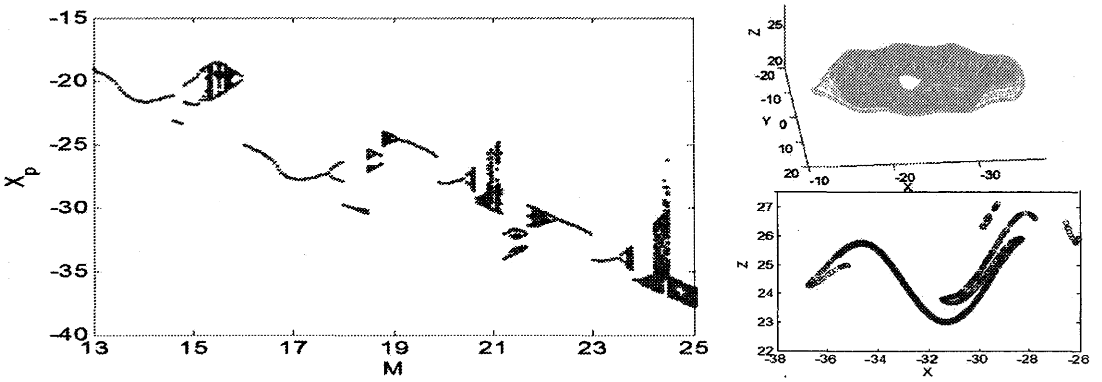

Figure 4: A bifurcation diagram (left) depicting the displacement Poincare' points (Xp) for increasing laser intensity (M) spanning regions VII to XVI in Figure 2. A three dimensional chaotic state-space (upper right) and fractal Poincare' map projection (lower right) for a selected intensity in region XVI of Figure 2 (M=24.4).

An example chaotic strange attractor (M=24.4) is depicted (Fig. 4 upper right) via its three dimensional state-space [Z(X,Y)] and (Fig. 4 lower right) Poincare' map projection [Z(X)] which exhibits a distinct fractal behavior that includes both stretch and fold properties.

This numerical investigation enables a quantitative description of a complex bifurcation structure that includes coexisting equilibrium solutions, self-excited periodic oscillations, and chaotic structural response of the thermo-visco-elastic dynamical system that is subject to laser irradiation.

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