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Nonlinear phenomena in hysteretic systems

Walter Lacarbonara, Fabrizio Vestroni
Dept. of Structural and Geotech. Engineering, Sapienza University of Rome, via Eudossiana 18, Rome

Introduction and background

Hysteresis is a pervasive phenomenon in mechanics (e.g., elastoplasticity, pseudoelasticity, hysteretic friction in materials and systems are only a few examples) as well as in magnetism or ferroelectricity. The constitutive modeling of materials, including smart materials such as shape-memory alloys, electroactive foams, etc., must account for the description of hysteresis as a substantial feature of their physical nonlinear behaviors. The constitutive laws, often based on the introduction of internal variables devoted to the representation of the memory effects, may feature softening, hardening, pinching, thermomechanical coupling. Hysteresis thus delivers a highly nonlinear signature to the associated dynamic responses. Besides the theoretical interest towards nonlinear dynamics of such systems, there is a parallel technological interest in exploiting advantageously hysteresis in engineering applications such as in vibration absorbers or isolators.

In the 70s the analytical study of the dynamics of one-dof hysteretic systems led to important observations such as that of Iwan [1] according to which in a system with double bilinear hysteresis, jumps occur in the softening-type responses giving rise to multi-stability. This work initiated a rich series of studies addressing nonlinear dynamics of hysteretic systems which highlighted new phenomena besides multi-stability, such as bifurcations, quasi-periodicity, and chaos. In the context of these studies, Vestroni and co-workers showed [2, 3] that stiffness-degrading and stiffness-strength degrading systems exhibiting pinching do possess jumps at the saddle-node bifurcations, hence multiple coexisting solutions.

The use of continuation techniques [4] highlighted the fact that systems with softening Bouc-Wen hysteresis can exhibit multiple coexisting periodic attractors, quasi-periodic solutions through Neimark-Sacker bifurcations, phase-locked solutions, and chaotic attractors through torus breakdown.

Shape-memory alloy (SMA) materials [5, 6] exhibit hysteresis due to the phase transformations with recovery of the original shape. SMAs have been attracting interest in various applications in which continuous SMA elements are used in the form of wires, bars, rods often subject to dynamic excitations. The remarkable properties of SMAs motivate different applications in fields such as bioengineering, aerospace and civil engineering; examples are actuators for vibration and buckling control of flexible structures, fasteners, seals, connectors and clamps, thermally-actuated switches, self-deployable structures, stabilizing mechanisms, micromanipulators and robotics actuators. SMA wires embedded in composite materials are also being used to modify appropriately their mechanical characteristics.

In a one-dof thermomechanical oscillator with a restoring force typical of a SMA material [7], the authors found that chaos emerges through a period-doubling cascade both in the primary-resonance and superharmonic-resonance regions. The temperature effects were taken into account in a thermomechanical framework where the fast loading rates associated with nearly adiabatic conditions led to quasiperiodic responses via secondary Hopf bifurcations. Other studies highlighted the occurrence of chaos in a von Mises (two-bar) SMA truss [8] or the transition from chaos to hyperchaos in a system of two coupled SMA oscillators [9]. These transitions were studied mostly by calculating the Lyapunov exponents. On the other hand, in [10] the method of Wandering Trajectories was employed to detect the sensitivity to initial conditions of the orbits of a thermomechanical SMA oscillator. In continuous SMA systems, which exhibit the constitutive nonlinearity in the prevailing operating regimes, nonlinear multi-mode responses are affected by highly nonlinear transfer of energy between the modes due to internal resonances. The nonlinear modal couplings are responsible for a rich variety of nonlinear dynamical behaviors and transitions to chaos.

Bifurcation analysis and phenomena

In previous studies of the authors [4, 7], a pseudo-arclength continuation strategy was devised by employing the Poincaré map for detecting the periodic solutions constructed via a finite-difference approach in state space. The continuation method together with the Floquet theory enabled the unfolding of the bifurcations of single-dof hysteretic systems governed by three or four variables (the displacement, the velocity, an internal variable and the temperature). After determining the ranges where the fundamental periodic response loses the stability though various bifurcations, the numerical construction of bifurcation diagrams unfolded very rich scenarios of transition to chaos highlighting the important role of the nonlinear hysteretic material behavior. An example of such diagrams for a SMA oscillator excited via a (superharmonic) resonance of order one-third is shown in Fig. 1 along with the phase portrait and restoring force of a typical chaotic attractor. The bifurcation diagram shows a full bubble structure culminating into chaos, within which a boundary crisis was found, as well as fold bifurcations leading to period-five solutions and subsequent period-doubling cascades.

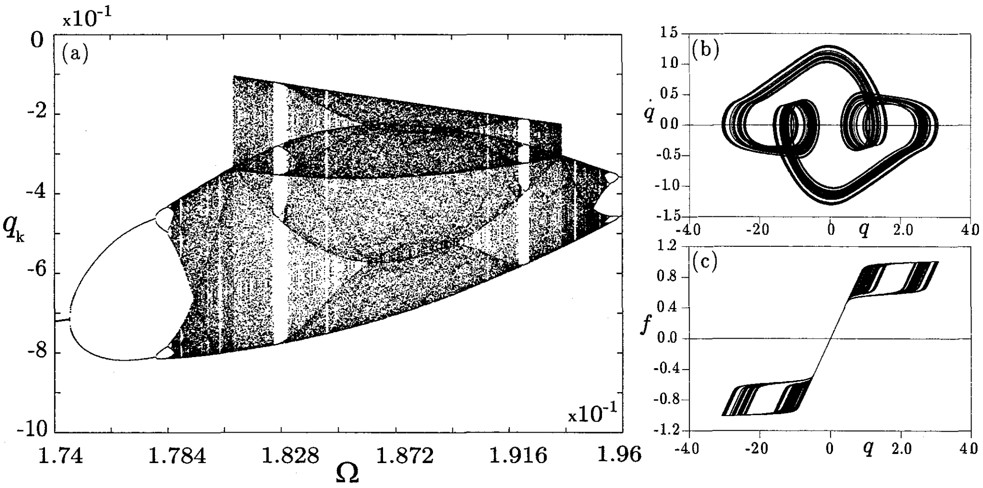

Figure 1: (a) Bifurcation diagram of the SMA oscillator subject to a superharmonic resonance of order one-third. (b) phase portrait and (c) hysteresis loops of the pseudoelastic force.

Conclusions

The bifurcation scenarios leading to chaos have been extensively studied in the literature in the context of deterministic nonlinear systems with emphasis on geometric nonlinearities. Far less attention has been placed on the role of material nonlinearities which become prominent for new classes of smart materials employed as dynamic actuators, vibration absorbers, energy harvesting systems. In such systems, nonlinear couplings arising from internal resonances due to the material nonlinearities are responsible for bifurcations leading to attractors of various kinds, including quasi-periodic attractors, phase-locked solutions, and chaotic attractors. The diverse transitions to chaos are documented in this study with a wealth of new observations.

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