

Typical chaotic dynamics in squid giant axons

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Introduction

Deterministic chaos has been attracting a lot of attentions for the last 50 years. There are many systems in which deterministic chaos could possibly characterize the essential part of underlying dynamics. However, there are few real systems which are demonstrated as deterministic chaos. In this presentation, we show that the dynamics of a squid giant axon is of deterministic chaos.

Dataset used here

The dataset we analyzed in this presentation was obtained by stimulating a squid giant axon by pulses. The membrane potentials were observed stroboscopically at the leading edge of each stimulating pulse. We normalized the dataset so that the values range between 0 and 1. The total length of the dataset was 500. First we cut down the initial transient part. Then we confirmed using the method of Kennel [1] that the dataset is stationary. The used part of the dataset is shown in Fig. 1(a) and (b). This is one of the three datasets analyzed in Ref. [2].

Devaney's chaos

We follow the definition of deterministic chaos by Devaney [3]. The definition consists of three parts: topological transitivity, denseness for the set of periodic orbits, and sensitive dependence on initial conditions. However, his definition is difficult to be confirmed when a time series of a finite length is given.

Recently, we relaxed his definition of deterministic chaos so that these three conditions can be checked using a recurrence plot obtained from a time series of finite length [4]. Since the relaxed conditions are weaker than the original conditions, one can deny the possibility of Devaney's chaos if one of the relaxed properties is not fulfilled.

We applied the method of Ref. [4] to the dataset of giant squid axon. A recurrence plot for the dataset is shown in Fig. 1(c). We found that the dataset satisfies the three relaxed conditions, meaning that the dataset is consistent with deterministic chaos in Devaney's sense.

Pesin's equality

Actually this dataset fulfils the Pesin's equality [5], i.e., the sum of positive Lyapunov exponents is equal to the metric entropy. We estimated the maximal Lyapunov exponent by using the method of Kantz [6] (see Fig. 1(d)). In addition, we estimated the metric entropy by first estimating a generating partition for this dataset by the method of Ref. [7] (see Fig. 1(b) for the estimated generating partition for the dataset) and then estimating the entropy rate by the method of Ref. [8]. Both estimated values agreed well. It means that the squid giant axon has the typical property for deterministic chaos.

Conclusions

In sum, the dynamics of squid giant axon is consistent with Devaney's chaos. It also satisfies Pesin's equality, which is one of characteristics for deterministic chaos. Therefore, the dynamics of squid giant axon is a typical example of deterministic chaos.

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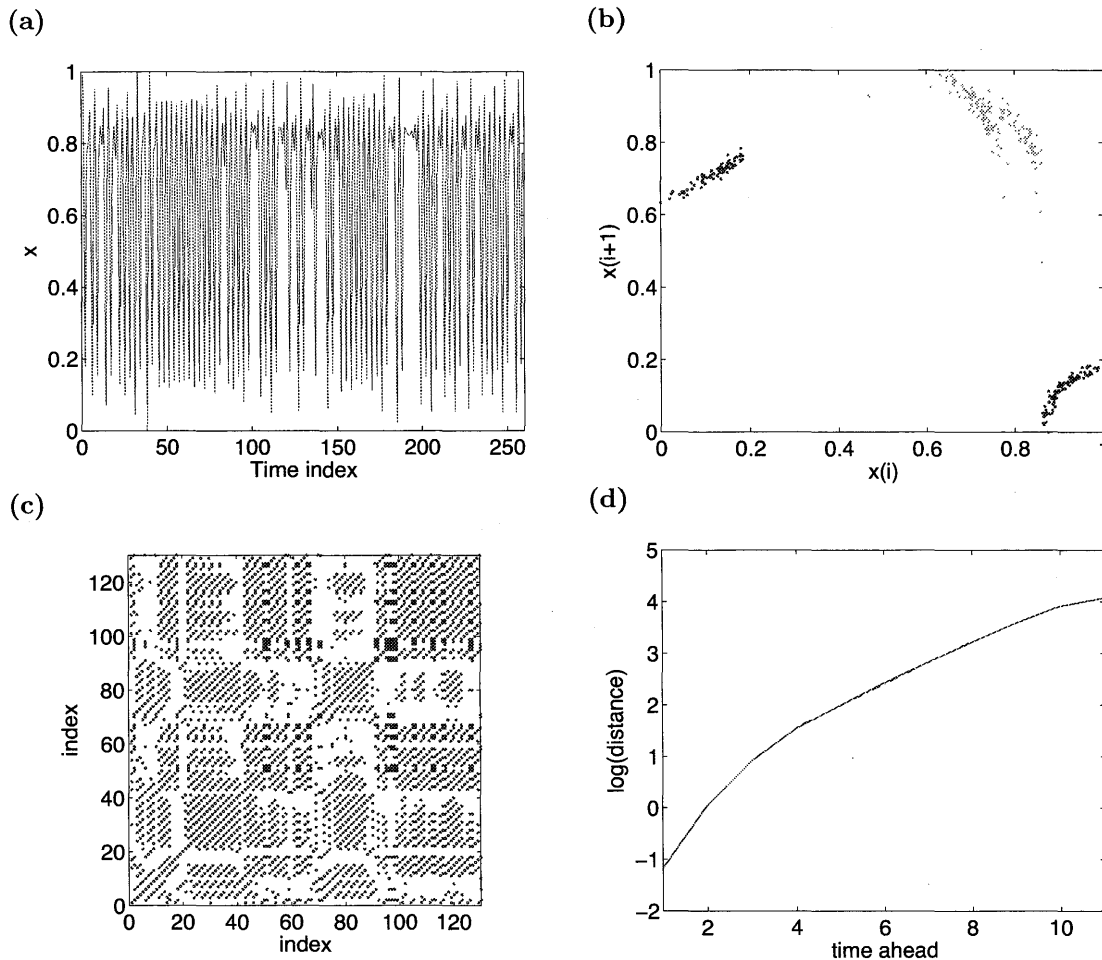


Figure 1: Analysis of giant squid axon used in this presentation. (a) Time series data. (b) Return plot. Shown with different colors is a generating partition for the dataset. (c) Recurrence plot of the dataset. (d) Maximal Lyapunov exponent was estimated as 0.42 bits/observation, which coincided with the metric entropy obtained by the generating partition of (b).

References

- [1] M. B. Kennel, Statistical test for dynamical nonstationarity in observed time-series data, *Phys. Rev. E*, vol. 56, pp. 316–321, 1997.
- [2] Y. Hirata, K. Judd, and K. Aihara, Characterizing chaotic response of a squid axon through generating partitions, *Phys. Lett. A*, vol. 346, pp. 141–147, 2005.
- [3] R. L. Devaney, *An Introduction to Chaotic Dynamical Systems* (Addison-Wesley Publishing Company, 1989).
- [4] Y. Hirata and K. Aihara, Devaney’s chaos on recurrence plots, *Phys. Rev. E*, vol. 82, art. no. 036209, 2010.
- [5] M. Pollicott, *Lectures on ergodic theory and Pesin theory on compact manifolds* (Cambridge University Press, 1993).
- [6] H. Kantz, A robust method to estimate the maximal Lyapunov exponent of a time series, *Phys. Lett. A*, vol. 185, pp. 77–87, 1994.
- [7] Y. Hirata, K. Judd, and D. Kilminster, Estimating a generating partition from observed time series: Symbolic shadowing, *Phys. Rev. E*, vol. 70, art. no. 016215, 2004.
- [8] Y. Hirata and A. I. Mees, Estimating topological entropy via a symbolic data compression technique, *Phys. Rev. E*, vol. 67, art. no. 026205, 2003.