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A Short, Truncated, and Partial History of Chaos

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Abstract

Starting with the story of Henri Poincaré's correction of a serious mistake in his prize-winning paper on the three-body problem of classical mechanics [1] and his subsequent discovery of deterministic chaos in the form of homoclinic tangles (cf. [2, Volume 3]), I shall sketch some key ideas and developments in dynamical systems theory. I shall emphasize problems in periodically-forced nonlinear oscillators, including the work of van der Pol and van den Mark [3], G.D. Birkhoff [4, 5, 6], M.L. Cartright and J.E. Littlewood [7], and N. Levinson [8], culminating in S. Smale's construction of the "horseshoe"in 1959-60 [9, 10]. I will also explain how V.K. Melnikov and V.I. Arnold [11, 12] provided rather general pertubative methods that provide proofs of the existence of homoclinic tangles such as those recognized by Poincaré and Smale.

It is perhaps no coincidence that I truncate this history shortly after the independent discoveries of chaos and strange attractors in 1961 by Y. Ueda [13] and E.N. Lorenz [14, 15]: discoveries that did not immediately attract the attention of mathematicians, although in [16], Ueda and his colleagues provided elegant illustrations of homoclinic tangles. Moreover, stopping in the 1960's allows me to escape submersion in the tsunami of activity, papers and books that followed this period, and gives me time to speculate on the rôles that chance, conservatism, publication delays, and general ignorance of work in other fields played in this history, and (along with fashionable fads) the rôles that they still play on the development of science. If time permits, I will also make some remarks on the influence of "chaos theory" on the study of turbulence in fluids.

Extended versions of parts of this story and comments on more recent developments can be found in [17, 18]. For a more detailed account of Poincaré's work on dynamics, see [19], and for reflections on the sociological and cultural context of nonlinear dynamics and chaos, see [20].

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