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Public Policy and the Income-Fertility Relationship

in Economic Development

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Abstract

In the pre-industrial era, there was a positive association between income and fertility across households within societies, but in the modern era, a clear association does not seem to exist, neither positive nor negative. Why the income-fertility relationship within societies changed over time is an unsolved puzzle in the history of economic growth, one that has been raised by Gregory Clark (e.g., *A Farewell to Alms*, 2007). This paper suggests that public policy for children has a key role in solving this puzzle. The interaction between changes in public policy for children and economic development generates changes in the income-fertility relationship across households, as well as hump-shaped dynamics of the average fertility rate over time.

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1 Introduction

It is sometimes argued that early in the development process, a positive association exists between income and fertility across households within societies. Although information on fertility is scarce for the pre-industrial world, there is some evidence that wealth predicted reproductive success at least in some parts of the Western world in the sixteenth through eighteenth centuries. On the other hand, there seems to be no clear association, positive or negative, between income and fertility in modern high-income societies.[1] It is an unsolved puzzle in the history of economic growth why the income-fertility relationship within societies changed over time. This paper is intended to offer a theory solving this puzzle. Our theory suggests that public policy for children has a key role.

In recent years, several growth theorists have attempted to construct a unified theory that captures the long-run growth process from the pre-industrial era to the modern era in a single framework; see Galor (2011) for details of this literature. This paper sheds light on two aspects largely ignored in the literature. One is heterogeneity among agents born in the same period. To consider why the income-fertility relationship within societies changed over time, heterogeneity in income in the same period must be introduced.[2] The other is the role of public policy. The size and type of public policy differ across periods. While contemporary economists, e.g., Malthus (1817), and modern economic historians, e.g., Boyer (1990), have pointed out the importance of public policy in generating the historical fertility transition, modern growth theorists have given it little attention.[3] We construct an overlapping generations model where there is heterogeneity among households in the same period, the number of children and their living standards are privately chosen by each household, and the government provides public services to improve the

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[1] It is widely known that in the modern world, there is a strong negative income-fertility relationship across countries. What we focus on here is not income-fertility relationships across countries, but those within societies.

[2] In the literature on the historical long-run growth process, very few studies incorporate heterogeneity among agents born in the same period. Among the few exceptions are Doepke and Zilibotti (2005) and Galor and Moav (2006), which consider political conflicts over the introduction of child labor regulation and public education, respectively.

[3] Galor and Moav (2006) focus on education reforms in Europe during the nineteenth century and model the rise of public education, but do not consider the endogeneity of fertility.

living standards of children. Solving the model numerically, we examine how well it replicates the data for the UK.

The remainder of this paper is organized as follows. In the rest of this section, we present motivating facts, related existing theories, and a brief description of our model and the British history of public policies for children. Section 2 presents the model and conducts the numerical analysis, showing that our model performs well in reproducing the main features of UK data. Section 3 concludes this paper.

1.1 Motivation

Clark and Hamilton (2004) infer the relationship between income and fertility in England around 1600. Using a sample of wills of male testators from 1585 to 1636, they find a positive association between income and net fertility over a wide range of incomes (see also Clark and Hamilton, 2006; Clark, 2007).[4] Boberg-Fazlic, Sharp and Weisdorf (2011) also find that wealthier families had higher fertility until the 1700s based on data different from those on which the result of Clark and Hamilton (2004) is based.[5] Weir (1995) finds a weakly positive relationship between economic status and fertility in eighteenth-century France. In contrast, there seems to be no association between income and fertility in modern high-income societies: for instance, Dickmann (2003) investigates the income-fertility relationships in the United States, Germany, Canada, the United Kingdom, Sweden, and Finland around 2000 and finds little correlation between income and fertility. Clark and Cummins (2010) examine the wealth-fertility association in England between 1500 and 1890, concluding that the positive association disappeared around 1780.[6] Clark (2005) provides a concise review of this issue, pointing out the difficulty of reconciling

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[4]One might think that using a sample of wills would miss data on the poor. According to Clark and Hamilton (2004) and Clark (2007), however, wills were drawn up by a wide variety of people at that time: 'Higher-income individuals were more likely to leave a will, but there are plenty of wills available for those at the bottom of the economic hierarchy, such as laborers, sailors, shepherds, and husbandmen' (Clark, 2007, p. 86).

[5]Boberg-Fazlic, Sharp and Weisdorf (2011) use data compiled by the Cambridge Group for the History of Population and Social Structure and those in Wrigley, Davies, Oppen, and Schofield (1997). These data have some characteristics different from those of Clark and Hamilton (2004), e.g., they include families that did not leave a will and children who did not appear in the will, and they cover a larger geographical area, though mainly rural areas.

[6]Boberg-Fazlic, Sharp and Weisdorf (2011) also reach a similar result based on different data sets.
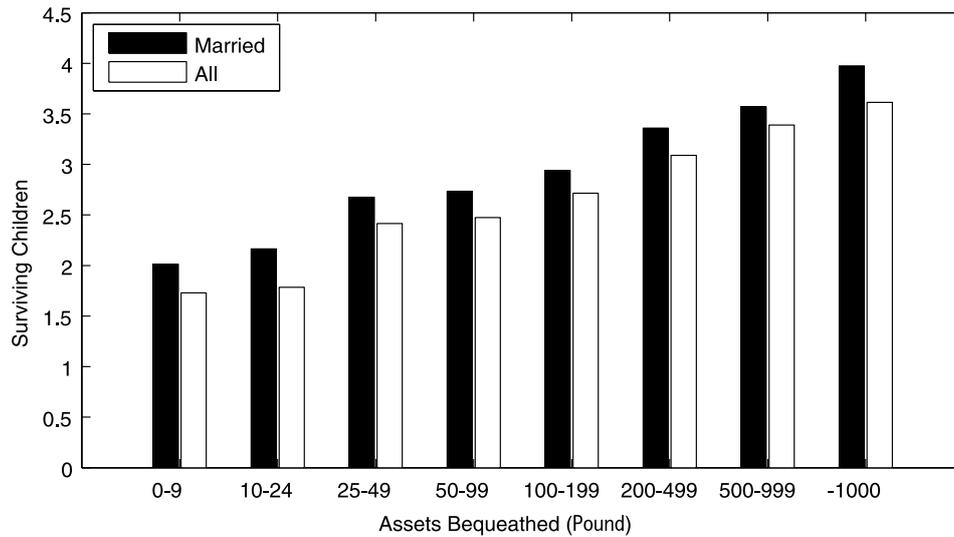
Figure 1: Income-fertility relationship, England, 1585-1638. *Source*: Clark and Hamilton (2006)

the differences in cross-section fertility behavior across the ages.

We focus on England on account of the availability of data. Figures 1, 2, and 3 depict the income-fertility relationship in England around 1600, those in the UK, the US, Germany, and Canada around 2000, and the evolution of the gross and net reproduction rates in England from the sixteenth through twentieth centuries, respectively.[7] Figure 1 indicates the positive association between income and fertility in England around 1600. It follows from Figure 2 that there is no (or weakly hump-shaped) association between income and fertility in modern developed countries. Figure 3 shows the hump-shaped dynamics of average fertility over 400 years. To the best of our knowledge, there is no single framework explaining the patterns presented in these three figures all together. The purpose of this paper is to construct such a framework.

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[7] The gross reproduction rate (GRR) and net reproduction rate (NRR) represent the number of daughters born per woman who lived to 50 and the number of those who survived to a childbearing age, respectively. The difference between GRR and NRR mainly reflects the child mortality rate.
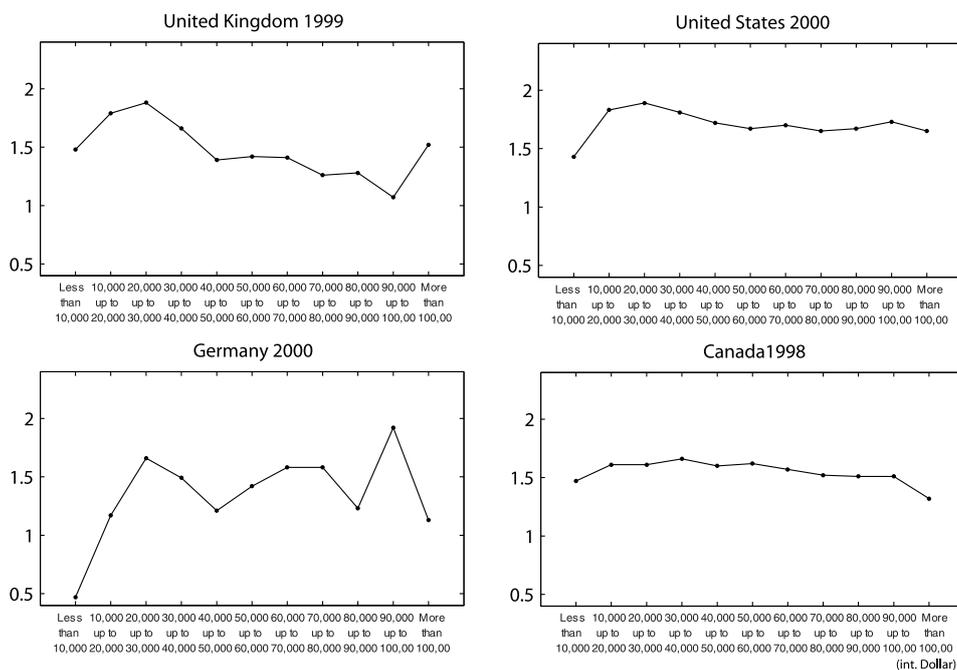
Figure 2: Income-fertility relationship in modern developed countries. *Source*: Dickmann (2003)

1.2   Related theories

Clark and Cummins (2009) point out the difficulty of reconciling the differences in cross-section fertility behavior across the ages based on an existing theory:

> The completely different association of wealth and fertility in the preindustrial compared to the modern world cannot be explained by subsistence constraints, by differences in the quality-quantity trade-off, or by differences in the child mortality rates. The prospects for a unified account of economic growth in both the Malthusian and the Solovian eras thus look decidedly poor.

Consider why existing theories cannot explain the observed pattern of UK fertility mentioned above.

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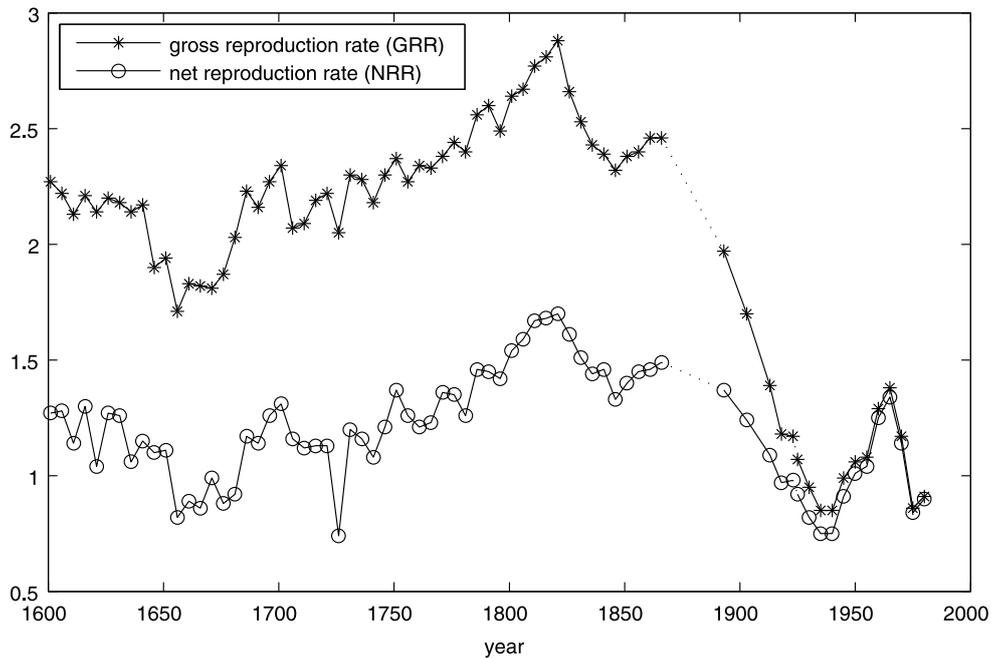
Figure 3: English fertility history, 1600-2000. *Sources*: Office of Population Censuses Surveys (1987) and Wrigley et al. (1997).

Subsistence constraint. To explain the positive relationship between income per capita and population growth in the pre-industrial world, recent studies on the historical long-run growth process, such as Galor and Weil (2000) and Galor and Moav (2002), often assume a subsistence level of consumption below which individuals cannot survive. For sufficiently low levels of income, the subsistence consumption constraint is binding and parents cannot devote a sufficient amount of time to child rearing; higher incomes enable parents to have a larger number of children. In contrast, for sufficiently high levels of income such that the subsistence constraint is no longer binding, any positive association between income and fertility vanishes. This explanation is consistent with the historical evolution of the average fertility rate of each society, but not consistent with the historical evolution of the income-fertility relationship within societies. This is because, based on the model incorporating a subsistence level of consumption, households with income far above subsistence should be liberated from the positive income-fertility association

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even in the pre-industrial era. According to Clark and Hamilton (2004), however, a positive association between income and net fertility was observed for very high-income families who did not worry about subsistence in pre-industrial England.

Human capital investment (quantity-quality tradeoff). It is sometimes argued that in the pre-industrial world, human capital was less important and parents favored quantity over quality in children, whereas the opposite is observed in the modern world. This idea is expressed in Becker, Murphy and Tamura (1990) and Galor and Weil (2000), among others. This explanation appears plausible, but contradicts the fact that children of wealthier parents tended to be better educated than children in poor families, even in the pre-industrial era. Although literacy was largely a cultural skill or a hierarchical symbol and had a limited role in the production process (e.g., Galor, 2011), well-to-do parents not needing their offspring to work and able to afford school fees educated their children to be 'gentlemen' (e.g., Mokyr, 2009). Furthermore, there is evidence that children in wealthier households were more likely to survive into adulthood in pre-industrial England (Clark, 2007), suggesting that wealthier parents provided their children with better nutrition and sanitary conditions; based on data for Prussia, Becker, Cinnirella and Woessmann (2010) find that a tradeoff between quantity and quality of children existed before the German demographic transition. Considering these findings, it seems reasonable to say that parents cared about the quality of life of their children and that there existed a tradeoff between quantity and quality even in the pre-industrial world where schooling was not intended to nurture educated workers.

Some might argue that severe credit constraints prevented parents from improving child quality in the pre-industrial world, and thus higher income led to more children rather than higher quality. For the same reason as the subsistence constraint, however, this explanation is not consistent with the historical evolution of the income-fertility relationship within societies: in the pre-industrial world, there was a positive association between income and net fertility among wealthy households without credit constraints.

Declines in child mortality. Clark (2007) proposes the following hypothesis. The desired number of surviving children for married couple is always two or three, independent of income. In a high-mortality environment, such as in pre-industrial England, the number of children in risk-averse households has to be large to ensure a good chance of a surviving child. The poor in the pre-industrial era could not afford such a precautionary birth because of poor health, nutrition, and economic resources. Thus, the rich had more surviving children than the poor. Fertility differentials by the precautionary motive declined with decreased child mortality, collapsing the positive income-fertility association. In Clark and Cummins (2009), however, Clark himself admits that this hypothesis is flawed: if this hypothesis is correct, fertility differentials must be larger in regions with higher mortality in pre-industrial England, but there is no data supporting this.[8]

Different income sources. It is sometimes argued that high incomes mainly came from non-human assets, such as land and physical capital, in the pre-industrial world, whereas modern high incomes mainly come from human capital. An increase in family income by a rise in the return on nonhuman assets is likely to raise fertility through the income effect, while that by a rise in wages is likely to reduce fertility through the substitution effect (Schultz, 1981, 1994). The difference in the major source of income might explain the difference in the income-fertility relationship. To assess the validity of this hypothesis, however, we must test whether cross-section fertility in both the pre-industrial and modern worlds correlate differently with income between rentiers who live on asset income and workers who live on human capital. Although it is difficult to conduct such a test because of scarce information on fertility by occupation for the pre-industrial world, evidence in pre-industrial China provides a skeptical view on this hypothesis: according to Lee and Campbell (1997), high-wage occupations, such as soldiers, artisans, and officials, had more surviving children than commoners.

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[8] Doepke (2005) argues that precautionary demand should not have a strong effect because deceased children can be replaced through higher subsequent fertility. Galor (2005) also points out the improbability of the precautionary-demand hypothesis from an evolutionary perspective.

We do not mean to deny the importance of the above theories for the determination of fertility. In addition, gains in life expectancy (Soares, 2005; Soares and Falcão, 2008), declines in gender gap (Galor and Weil, 1996; Lagerlöf, 2006; Kimura and Yasui, 2010), and declines in child labor (Hazan and Berdugo, 2002; Doepke and Zilibotti, 2005) are also important factors affecting fertility. These many factors must have jointly shaped the long-run fertility transition, although any of them alone is unlikely to provide a foundation for understanding differences in cross-section fertility behavior within a society between the pre-industrial and modern worlds. So, what could have caused the change in the income-fertility relationship? Let us turn, then, to our theory.

1.3   Our mechanism

This paper sheds light on variations in public policy for children in order to explain variations in the income-fertility relationship. Our model is a variant of the model of Kimura and Yasui (2009), where public and private spending for children coexist and fertility is endogenously determined. We extend the model developed by Kimura and Yasui (2009) in two directions: (i) a minimum consumption level of children is introduced and (ii) the static environment is extended to an overlapping generations framework.[9]

Our model is based on conventional microeconomic and macroeconomic theories. First, we follow the spirit of the quantity-quality model of fertility choice à la Becker (1960), Becker and Lewis (1973), Schultz (1973), and Willis (1973). What differentiates our model from the previous literature is the coexistence of public and private spending on children: the government provides public services to improve the living standards of children, and households can privately complement these.[10]   Second, an off-the-shelf overlapping generations model with

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[9]Kimura and Yasui (2009) develop a politico-economic model in which the size of public policy is endogenously determined by majority voting. The model developed here is simplified in that the size of public policy is exogenously given.

[10]In the literature on public provision of private goods, there are two major classes of model: the topping-up model and opting-out model. The former assumes that agents can privately supplement public provision (e.g., Epple and Romano, 1996b; Gouveia, 1997), whereas the latter assumes that agents are not allowed to further supplement public services, but are allowed to opt out in order to purchase private services (e.g., Stiglitz, 1974; Epple and Romano,

population growth is used as the framework of analysis, as in Galor and Weil (1996, 2000). We examine the fertility dynamics in line with the economic development caused by endogenous capital accumulation and exogenous technological progress.[11]

Some key results are as follows. The relationship between income and fertility within a society varies depending on the size of public policy for children. If the level of public provision is sufficiently low, a positive income-fertility association exists; the positive association vanishes if the level of public provision becomes sufficiently high. In other words, fertility positively correlates with income in societies where most of the child-rearing cost is borne by parents, while it does not in societies where some of the burden is relieved by public support. If we interpret the economy with sufficient public support for children and the one without it as a modern society and a pre-industrial one, respectively, then the prediction of our model is consistent with the historical facts exhibited in Figures 1 and 2. Furthermore, whether economic development raises the average fertility rate also depends on the size of public policy for children. If the level of public provision is sufficiently low, rises in wages induced by economic development raise the average fertility rate. In contrast, in the economy with sufficiently large public support, higher wages are more likely to lower the average fertility rate. If public policy for children has become more generous over time, then the prediction of our model is consistent with the hump-shaped fertility transition depicted in Figure 3.

1996*a*; Glomm and Ravikumar, 1998). In other words, the topping-up model treats public and private alternatives as jointly consumable, whereas the opting-out model treats them as mutually exclusive. It is sometimes argued that the topping-up model is especially applicable to the health care debate and the opting-out model to the education debate. However, the assumption that agents can opt out but not top up is only one extreme, as is the one that agents can top up but not opt out. Whatever kind of public service we consider, the reality must be somewhere between these two assumptions. Kimura and Yasui (2009) and de la Croix and Doepke (2009) develop quantity-quality models of fertility choice under a topping-up framework and an opting-out one, respectively.

[11]There are now many papers that quantitatively examine the extent of a model's fitness for actual fertility transitions in an overlapping generations framework (e.g., Greenwood and Seshadri, 2002; Greenwood, Seshadri and Vandenbroucke, 2005; Lagerlöf, 2006; Lord and Rangazas, 2006; Doepke, Hazen and Maoz, 2007; Kimura and Yasui, 2010).

1.4 Historical evidence

In this subsection, we review the history of public policy related to child rearing for Great Britain during the seventeenth through twentieth centuries.

Historically, British public policies for children can be largely divided into two categories: those under the Poor Law and those under the welfare-state regime of the twentieth century. The nationwide Poor Law dates to 1597, when the English Parliament passed a law requiring each parish in England and Wales to take responsibility for relieving its poor inhabitants. Under the Poor Law, child allowances and apprenticeships played a major role in child-related public policies. The administration of child allowance policies differed across parishes, but most employed similar systems in which a fixed allowance was provided to each child at and beyond a fixed number of children, commonly three or four, at which aid began. Around 1800, when relief generosity was reaching its peak, the child allowance became large enough to cover the subsistence needs of an additional child for agricultural laborers (Boyer, 1990). The parish apprenticeship system contributed funding for the care of children apprenticed to masters. Although some argue that pauper apprentices were in a sorry plight under the parish apprenticeship system, 'at least in some cases it saved youngsters from a life of begging or vagrancy and provided them with gainful employment in respectable work environments' (Mokyr, 2009, p.332). Two major trends in the history of the Poor Laws were the shift toward increased generosity for the able-bodied, which began around 1750, and the subsequent decline in generosity that began in 1834 with the passage of the Poor Law Amendment Act.[12] The period of rises in fertility of the poor relative to the rich, the eighteenth century, coincides with the period of increases in relief expenditure. Furthermore, it may not be a coincidence that the UK fertility rate turned downward in the first half of the nineteenth century, when generosity declined (Figure 3).

The middle of the nineteenth century is sometimes referred to as 'the age of laissez-faire', and there were declines in public assistance for the poor during this time. However, this was

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[12]Thomas Malthus and his followers accused the Poor Law of increasing fertility and their agitation was finally successful in 1834 with the reform of the Poor Law. Although whether the Poor Law actually had this effect on fertility is still a matter of some debate, the estimation of Boyer (1990) indicates the positive effect.

also a period when government intervention in public health and housing increased. The Public Health Act of 1848 empowered local authorities to establish boards of health to manage sewer and drainage systems and water supplies. Especially after 1870, increases in urban public health expenditure led to reductions in death rates from cholera, typhus, typhoid, and infant diarrhoea, and improved the living standards of the working class. Furthermore, the British Parliament attempted to construct working-class housing through the Housing of the Working Classes Act of 1890. These public policies partially offset the effects of declines in relief generosity triggered by the passage of the Poor Law Amendment Act of 1834, improving the living environment of children.

During the twentieth century, under the welfare-state regime, the size of public provision for children expanded at a much higher rate than before. Although the British welfare state is generally considered to have been created in the 1940s, public expenditure on social services, including those for children, began to increase in the early twentieth century. Between 1906 and 1914, Parliament passed several pieces of social welfare legislation which are collectively known as the 'Liberal welfare reforms': beginning in 1912, government grants were provided for the medical treatment of children. In an interwar period, the Housing Acts of 1919, 1923, and 1924 provided subsidies for the construction of low-cost housing. In the 1940s, adopting the major proposals of the Beveridge Report, which was published in 1942, Labour government enacted a flurry of legislation, including the Family Allowance Act in 1945, the National Insurance and National Health Service Acts in 1946, the National Assistance and Children Act in 1948, and the Housing Act in 1949, establishing the foundations of the post-war welfare state. From the standpoint of improving the living standards of children, of particular note are acts related to healthcare. The centrally funded National Health Service made medical attention available to everyone free of charge.

2 Model

Consider an overlapping generations model in which agents live for three periods: childhood, adulthood, and old age. In childhood, they do not make any decisions and consume fixed quantities of goods and time from their parents for survival. Furthermore, they can receive additional goods, which are not related to subsistence and which can be interpreted as goods improving their living standards, from their parents and the government. In adulthood, they raise children and supply labor to the market; they decide the number of children to have, as well as the expenditure on their children. For simplicity, suppose that they consume nothing in adulthood. In old age, they only consume savings from the previous period.

2.1 Household behavior

The preferences of household i of generation t (born in $t-1$) are defined over consumption in old age, c_{t+1}^i, the number of children, n_t^i, and spending on each child, e_t^i. The preferences are represented by

$$\gamma\left(\phi\ln e_t^i+\ln n_t^i\right)+(1-\gamma)\ln c_{t+1}^i,$$

where $\gamma\in(0,1)$ and $\phi\in(0,1)$ denote the relative weights given to children and spending on them, respectively.[13] Spending on each child, e_t^i, is used for non-subsistence goods, which improve the living standards of children. Thus, the quantity of e_t^i also measures the living standards of each child. Households are differentiated by their labor endowment, i.e., the quantity of physical endowment that they can supply to the labor market per unit of time. labor endowment levels across households are distributed according to the cumulative distribution function $F\left(\cdot\right)$, which is exogenous and constant over time. We assume that the support of $F\left(\cdot\right)$ is $\mathbb{R}_+$ and that $F\left(\cdot\right)$ is strictly increasing and twice continuously differentiable.

Non-subsistence goods for children can be obtained through private purchase and govern-

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[13]The assumption $\phi\in(0,1)$ guarantees that the second-order condition for maximization is satisfied in the household's problem. If $\phi\geq 1$, households obtain infinite utility by reducing the number of children close to zero and increasing spending on them close to infinity.

ment provision. All children receive the same quantity of public provision in this economy.[14] Households are allowed to supplement the publicly provided quantity, and the quantity of private purchase may differ across households. The quantity of non-subsistence goods provided for each child of household i of generation t, e_t^i, is given by

$$e_t^i = v_t^i + g_t, \tag{1}$$

where v_t^i and g_t denote the quantities of private purchase and public provision per child, respectively. The publicly provided quantity cannot be traded and thus v_t^i must be non-negative.

In adulthood, each household is endowed with a unit of time that can be devoted to child-rearing and labor market activities. Raising one child takes fraction $z \in (0,1)$ of a household's time. Let $\varepsilon > 0$ be the cost in terms of goods of raising one child; we can interpret ε as the subsistence consumption level per child. The presence of ε implies that some part of the child-rearing cost is independent of the parent's wage, which would be reasonable given that there is a minimum required level of nutrition. The government collects taxes from all households at a rate τ_t. We assume that spending for children is tax deductible. This assumption simplifies the analysis because it implies that taxation does not distort the household's choice.[15] Each household allocates the after-tax income to savings for future consumption. The budget constraint for household i of generation t with labor endowment h^i is then given by

$$s_t^i = (1 - \tau_t) \left[\left(1 - z n_t^i\right) w_t h^i - \left(v_t^i + \varepsilon\right) n_t^i \right], \tag{2}$$

where w_t is the wage rate, i.e., the return on a unit of labor endowment, at time t. The consump-

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[14]Although we assume that the government provides private goods, we are aware that government provision of public goods, e.g., sanitation, water, and sewer, has also played an important role in improving the living environment of children. Since these public goods do not exhibit fully the feature of non-rivalry, it can be interpreted that goods provided by the government in this model include them. Furthermore, it should be noted that adding public goods to our model is not difficult and does not changes the main result qualitatively.

[15]This assumption is employed for the sake of brevity, and is not crucial for our main results. de la Croix and Doepke (2009) also assume the tax deductibility of education spending for children.

tion in old age is

$$c_{t+1}^i = (1 + r_{t+1}) s_t^i, \qquad (3)$$

where r_{t+1} is the net return on a unit of physical capital at time $t+1$.

Given policy variables, τ_t and g_t, each household makes plans for the number of children and the level of private spending for children. Household i of generation t solves the following maximization problem:

$$\max_{v_t^i, n_t^i, s_t^i} \gamma \left(\phi \ln e_t^i + \ln n_t^i \right) + (1 - \gamma) \ln c_{t+1}^i, \qquad (4)$$

s.t.   equations (1), (2), and (3),

$$v_t^i \geq 0 \text{ and } n_t^i \geq 0, \qquad (5)$$

and given $\tau_t \in (0,1)$ and $g_t \geq 0$.

The solution to this problem can either be interior or at a corner; there is a threshold level of labor endowment, $\hat{h}_t$, below which households choose a corner solution of not supplementing the publicly provided quantity. The threshold may differ across time because the wage rate, w_t, and the quantity of government provision, g_t, may differ across time. The threshold is

$$\hat{h}_t = \frac{g_t - \phi \varepsilon}{\phi z w_t}. \qquad (6)$$

It follows that $\partial \hat{h}_t / \partial g_t > 0$ and $\partial \hat{h}_t / \partial w_t < 0$. The larger the size of the government support, the smaller the fraction of households with a positive supplement. The higher the wage rate of the economy, the larger the fraction of households with a positive supplement.

If the household has a sufficiently large labor endowment such that $h^i > \hat{h}_t$, it chooses an interior solution for the optimal private supplement level; the first-order conditions imply

$$v_t^i = \frac{\phi z w_t h^i + \phi \varepsilon - g_t}{1 - \phi} \text{ and } n_t^i = \frac{\gamma (1 - \phi) w_t h^i}{z w_t h^i + \varepsilon - g_t}. \qquad (7)$$

If the household has a sufficiently small labor endowment such that $h^i \leq \hat{h}_t$, the optimal private supplement level is zero; the first-order condition implies

$$v_t^i = 0 \quad \text{and} \quad n_i = \frac{\gamma w_t h^i}{z w_t h^i + \varepsilon}. \tag{8}$$

Using (6), (7), and (8), we obtain the following comparative static results regarding the relationship between labor endowment and fertility.

Proposition 1. *(i) In the case of $g_t \leq \phi \varepsilon$, all households choose an interior solution. Fertility increases as labor endowment increases across the support of the distribution. (ii) In the case of $\phi \varepsilon < g_t \leq \varepsilon$, households with $h^i \leq \hat{h}_t$ choose a corner solution and those with $h^i > \hat{h}_t$ choose an interior solution. Fertility increases as labor endowment increases across the support of the distribution. (iii) In the case of $g_t > \varepsilon$, households with $h^i \leq \hat{h}_t$ choose a corner solution and those with $h^i > \hat{h}_t$ choose an interior solution. Fertility increases as labor endowment increases for $h^i \leq \hat{h}_t$ and decreases for $h^i > \hat{h}_t$.*

Proof. See Appendix.

This proposition states that the relationship between income and fertility within a society varies depending on the size of public policy for children.[16] In societies where the social support for children is insufficient ($g_t \leq \varepsilon$), high-income households have higher levels of fertility. On the other hand, in societies where it is sufficiently provided ($g_t > \varepsilon$), there is no monotonic relationship between income and fertility: a hump-shaped pattern is observed. Figure 4 depicts the income-fertility relationship of each case.

Let us elaborate the mechanism behind this. An increase in the labor endowment has two opposing effects on fertility, a positive income effect and a negative substitution effect. First, consider the case of households with no supplement. Under the assumption of logarithmic utility, if parental time is the only cost for raising children and labor income is the only source of

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[16]It can be easily verified that the disposable income of household i at time t is $(1 - \tau_t)(1 - \gamma) w_t h^i$, that is, the disposable income is linear with respect to the labor endowment. Therefore, the statement of Proposition 1 regarding the relationship between labor endowment and fertility also holds for the relationship between income and fertility.
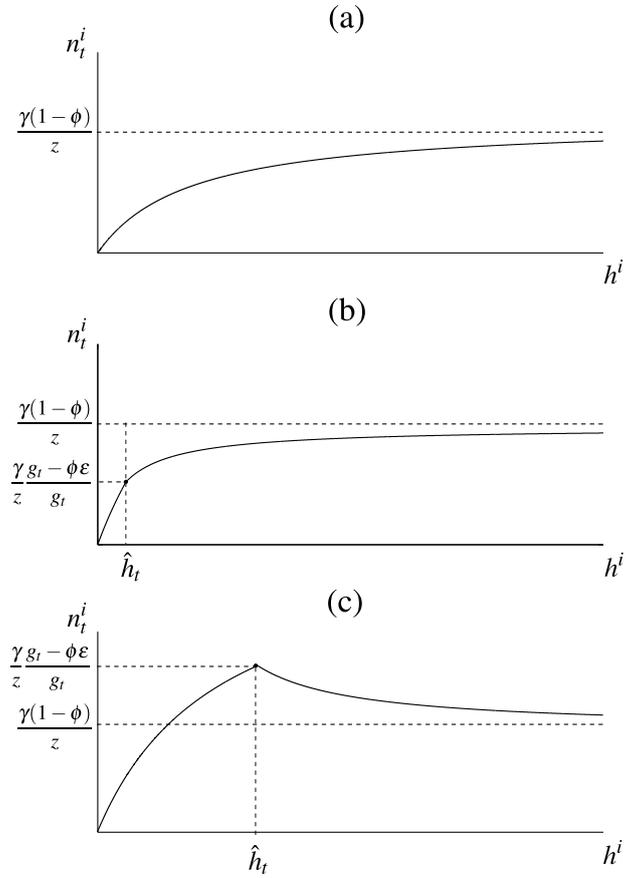
Figure 4: Relationship between labor endowment and fertility. (a) The case of $g_t \leq \phi \varepsilon$. (b) The case of $\phi \varepsilon < g_t \leq \varepsilon$. (c) The case of $g_t > \varepsilon$.

income, then fertility is independent of the labor endowment because the two effects cancel each other out. In this model, however, goods as well as time are required for child rearing, and thus time is not the only cost even for parents who do not privately supplement public provision. It follows that the income effect dominates the substitution one for households with no supplement: their fertility increases as the labor endowment increases. The case of households with a positive supplement is more complicated because a quantity-quality interaction exists. Households choosing an interior solution equate the marginal rate of substitution between quantity, i.e., the number of children, and quality, i.e., spending on each child, to their relative price. Variations in

the labor endowment change fertility by affecting this interaction. The presence of g_t mitigates the changes of the marginal utility of quality, $\gamma\phi/\left(v_t^i + g_t\right)$, whereas the presence of ε mitigates the effect of changes in labor endowment on the relative price, $\left(zw_t h^i + v_t^i + \varepsilon\right)/n_t^i$. When the labor endowment increases, g_t and ε have opposing effects on the demand for the number of children. If g_t is sufficiently large relative to ε, the increase in labor endowment induces parents to substitute quality for quantity.

2.2 Temporary Competitive Equilibrium

Thus far, we have analysed the household decision problem, taking policy variables τ_t and g_t and factor prices w_t and r_{t+1} as given. Here, using the government budget constraint and the equilibrium condition for the factor markets, we characterize the temporary competitive equilibrium where the capital stock, K_t, and the number of households, N_t, are given, that is, the quantity of capital stock per household, $k_t \equiv K_t/N_t$, is given. Note that if we choose one of the two policy variables, τ_t and g_t, as an exogenous variable, then the other is endogenously determined by the government budget constraint. We treat g_t as a policy variable that is exogenously given.

Public provision is financed by a proportional tax on income, τ_t, after the deduction of spending on children. The government's budget constraint is

$$\tau_t \bar{m}_t N_t = g_t \bar{n}_t N_t, \tag{9}$$

where $\bar{n}_t$ and $\bar{m}_t$ denote the average number of children and the average tax base, respectively.[17]

It follows from (6), (7), and (8) that the average fertility rate is

$$\bar{n}_t = \int_0^\infty n_t(h)\,dF(h) = \int_0^{\hat{h}_t} \frac{\gamma w_t h}{zw_t h + \varepsilon}\,dF(h) + \int_{\hat{h}_t}^\infty \frac{\gamma(1-\phi)w_t h}{zw_t h + \varepsilon - g_t}\,dF(h), \tag{10}$$

where the first and second terms of the RHS represent the fertility rates of households with no supplement and those with a positive supplement, respectively.

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[17]Note that g_t must be sufficiently small so that $g_t \bar{n}_t/\bar{m}_t \leq 1$ because τ_t must not be more than 1.

It follows from (6), (7), and (8) that the average tax base is

$$\bar{m}_t = \int_0^\infty \left\{ [1 - zn_t(h)] w_t h - [v_t(h) + \varepsilon] n_t(h) \right\} dF(h)$$

$$= (1 - \gamma) w_t \int_0^\infty h \, dF(h). \tag{11}$$

Given w_t, the average tax base is determined exclusively by exogenous parameters.

Production occurs according to a Cobb-Douglas production technology. The output produced at time t is given by

$$Y_t = A_t K_t^\alpha H_t^{1-\alpha},$$

where K_t and H_t are, respectively, the quantities of capital and labor (measured in efficiency units) employed in production, A_t is the level of technology, and $\alpha \in (0, 1)$ is the capital share of income. The factor markets are competitive, and thus both capital and labor earn their marginal products as follows:

$$w_t = (1 - \alpha) A_t \left(\frac{K_t}{H_t} \right)^\alpha, \tag{12}$$

and

$$r_t + \delta = \alpha A_t \left(\frac{K_t}{H_t} \right)^{\alpha - 1},$$

where $\delta \in [0, 1]$ is the depreciation rate of physical capital.

Noting that $H_t = N_t \int_0^\infty h [1 - zn_t(h)] dF(h)$ and using (6), (7), (8) and (12), we obtain the equilibrium condition for the labor market:

$$w_t = (1 - \alpha) A_t \left[\frac{k_t}{\int_0^{\hat{h}_t} h \frac{(1-\gamma)zw_t h + \varepsilon}{zw_t h + \varepsilon} dF(h) + \int_{\hat{h}_t}^\infty h \frac{(1-\gamma+\gamma\phi)zw_t h + \varepsilon - g_t}{zw_t h + \varepsilon - g_t} dF(h)} \right]^\alpha$$

$$\equiv \Psi_t(w_t). \tag{13}$$

Given g_t and k_t, a competitive equilibrium is characterized by $\{w_t, \hat{h}_t, \bar{n}_t, \bar{m}_t, \tau_t\}$ satisfying (6), (9), (10), (11), and (13) simultaneously. We obtain the following proposition related to the

competitive equilibrium.

Proposition 2. *Given g_t and k_t, there exists a unique competitive equilibrium. (a) An increase in capital stock per household causes the wage rate to rise. (b) An increase in government spending per child causes the wage rate to rise.*

Proof. *Existence and uniqueness.* The equilibrium wage rate is given by a fixed point of $\Psi_t(\cdot)$. Note that $\Psi_t(0) > 0$ and $\lim_{w_t \to \infty} \Psi'_t(w_t) = 0$. If $g_t < \varepsilon$, then $\Psi'_t(\cdot) > 0$ and $\Psi''_t(\cdot) < 0$. If $g_t \geq \varepsilon$, then the sign of $\Psi'_t(\cdot)$ is not uniquely determined and it follows from $\Psi''_t(\cdot)|_{g_t = \varepsilon} < 0$ and $\partial \Psi''_t(\cdot) / \partial g_t < 0$ that $\Psi''_t(\cdot) < 0$. Since $\Psi_t(0) > 0$, $\lim_{w_t \to \infty} \Psi'_t(w_t) = 0$, and $\Psi''_t(\cdot) < 0$, the fixed point of $\Psi_t(\cdot)$ uniquely exists.

(a) The increase of k_t shifts $\Psi_t(\cdot)$ upward. (b) The increase of g_t shifts $\Psi_t(\cdot)$ upward.

The positive (resp. negative) slope of $\Psi_t(\cdot)$ represents strategic complementarity (resp. substitutability) among households. If $g_t < \varepsilon$, no households decrease the number of children in response to the rise of wages. In this case, the best response of a household to an increase in fertility of all others is to increase the number of children because higher fertility means smaller labor supply and higher wage. Therefore, $\Psi'_t(\cdot) > 0$. Although strategic complementarity generally leads to the possibility of multiple equilibria, the uniqueness is guaranteed in our model because the fertility behavior is not so elastic with respect to changes in wages that $\Psi''_t(\cdot) < 0$. If $g_t \geq \varepsilon$, households with no supplement increase the number of children in response to the rise of wages, while those with a positive supplement decrease it. The slope of $\Psi_t(\cdot)$ depends on which effect is dominant. Even if the former dominates the latter and thus $\Psi'_t(\cdot) > 0$, the fertility behavior of households with no supplement is not so elastic, i.e., $\Psi''_t(\cdot) < 0$, that the uniqueness is still guaranteed.

The increase of g_t raises fertility by inducing households with a positive supplement to substitute quantity for quality. The rise in fertility means a decrease in the labor supply, thereby raising the wage rate.

Proposition 3. *If $g_t \leq \varepsilon$, rises in the wage rate raise the fertility rate. If $g_t > \varepsilon$, the effect of wage rises on the fertility rate is ambiguous, and higher wage rates are more likely to make the effect negative.*

Proof. The results immediately come from the first and second derivatives of (10) with respect to w_t:

$$\frac{\partial \bar{n}_t}{\partial w_t} \begin{cases} > 0 \text{ if } g_t \leq \varepsilon, \\ \lesseqgtr 0 \text{ if } g_t > \varepsilon, \end{cases} \quad \text{and} \quad \frac{\partial^2 \bar{n}_t}{\partial w_t^2} < 0.$$

This proposition states that the relationship between the wage rate and the average fertility rate varies depending on the size of public policy for children. When the size is sufficiently small, the average fertility rate rises as the wage rate rises. When the size is sufficiently large, in contrast, the relationship could be negative. Furthermore, as the wage rate becomes higher, the relationship is more likely to be negative. The intuition is simple. If public policy is sufficiently large, then both households with no supplement and those with a positive supplement exist. Higher wage rates induce the former to raise fertility and the latter to reduce fertility, that is, the positive and negative effects are mixed. Since the proportion of the latter becomes large as the wage rate rises, higher wage rates are more likely to make the negative effect dominant.

Recall that the historical pattern of UK fertility has two distinct features. First, the income-fertility relationship across households changed over time, from positive in the pre-industrial era to hump-shaped in the modern era. Second, the average fertility rate exhibits hump-shaped dynamics with an upward trend in the seventeenth and eighteenth centuries and a downward trend in the nineteenth and twentieth centuries. Can our model explain this pattern? While the answer to this question comes down to a quantitative matter, the analysis above suggests that theoretically speaking, the answer is yes. Suppose that modern England is a society where the quantity of public provision for children is sufficiently large and that pre-industrial England is not. Based on our model, this supposition immediately leads to the observed change in the income-fertility relationship (Proposition 1). Furthermore, rises in wages associated with economic growth could

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initially raise and subsequently lower fertility, that is, could generate hump-shaped fertility dynamics (Proposition 3). To carry the inquiry further, we derive the dynamic system and solve it numerically in the next subsection.

2.3 Intertemporal Competitive Equilibrium

2.3.1 Dynamic System

Given an initial stock of per-household capital, k_0, the sequence of per-household capital stock, $\{k_t\}_{t=0}^{\infty}$, is endogenously determined in the model. Here, we characterize the intertemporal competitive equilibrium where k_t endogenously evolves. Given k_t, as shown above, w_t is uniquely determined. Thus, we focus on the sequence of wage rates, $\{w_t\}_{t=0}^{\infty}$, rather than the sequence of per-household capital stock, $\{k_t\}_{t=0}^{\infty}$.

The stock of capital at time $t+1$, K_{t+1}, is determined by the aggregate supply of savings (after-tax income) at time t: $K_{t+1} = N_t(1-\tau_t)\bar{m}_t$. The number of households at time $t+1$ is $N_{t+1} = N_t\bar{n}_t$. Therefore, we obtain the dynamic equation of per-household capital stock:

$$k_{t+1} = \frac{(1-\tau_t)\bar{m}_t}{\bar{n}_t} = \frac{\bar{m}_t}{\bar{n}_t} - g_t, \tag{14}$$

where the second equality comes from the use of the government budget constraint.

Using (13) and (14), we obtain the difference equation which determines the transition of the wage rate:

$$w_{t+1} = (1-\alpha)A_{t+1}\left[\frac{\frac{\bar{m}_t}{\bar{n}_t} - g_t}{\int_0^{\hat{h}_{t+1}} h\frac{(1-\gamma)zw_{t+1}h+\varepsilon}{zw_{t+1}h+\varepsilon}dF(h) + \int_{\hat{h}_{t+1}}^{\infty} h\frac{(1-\gamma+\gamma\phi)zw_{t+1}h+\varepsilon-g_{t+1}}{zw_{t+1}h+\varepsilon-g_{t+1}}dF(h)}\right]^{\alpha}, \tag{15}$$

where $\bar{n}_t$ and $\bar{m}_t$ are given by (10) and (11), respectively. If an initial level of per-household capital, k_0, and time paths for the government provision per child, $\{g_t\}_{t=0}^{\infty}$, and the technology level, $\{A_t\}_{t=0}^{\infty}$, are given, this economy evolves according to (15).

The dynamic system given by (10), (11), and (15) is quite complex by itself. Furthermore,

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it is difficult to take account of time-varying g_t and A_t, which might be of key importance in the real world, using pencil-and-paper techniques alone. In what follows, we solve the model numerically and examine how well it can account for UK data.

2.3.2 Simulation

Take the length of a period in the model to be 20 years so that an individual lives for 20 years as a child, for 20 years as an adult, and for 20 years as an elderly person. There will be 20 model periods between 1600 and 1980.

We choose values for the parameters governing tastes and technology. The parameter values are chosen on the basis of one of the following two criteria: (i) the parameter values themselves should be reasonable, and (ii) the values of the endogenous variable that follow from those parameter values should be reasonable. The values of α (physical capital share) and δ (depreciation rate of physical capital), the parameters governing $F(\cdot)$ (distribution function of labor endowment), the series of the time cost of a child, $\{z_t\}_{t \in \mathscr{T}}$, where $\mathscr{T} \equiv \{1600, 1620, ..., 1980\}$, and the growth rates of the quantity of public provision per child, g_t, are chosen on the basis of the former criterion.[18] On the other hand, the series of TFP, $\{A_t\}_{t \in \mathscr{T}}$, and the values of γ (weight of children in the utility function), ϕ (weight of the living standards of children in the utility function), g_{1600} (public provision per child in 1600), and ε (subsistence needs per child) are chosen on the basis of the latter criterion.

We set $\alpha = 0.30$ because it is well known that the capital share of income is roughly 30%. The depreciation rate of physical capital, δ, is set to 1.0 for simplification because the variables on which this paper focuses, such as fertility and spending on children, are not affected at all by the choice of δ.

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[18]The parameter z represents the time cost of having a child. Assuming a constant value of z over time is problematic, especially when simulating the model over long periods. There are many factors affecting the time cost: for instance, the prevalence of appliances and frozen foods reduce parental time needed for child rearing; on the other hand, the introduction of child labour laws and the prolongation of the period of compulsory education imply an increase in the time cost because they prolong the period over which children are dependent on their parents. Here, we allow z to vary over time.

We assume that the labor-endowment distribution follows a lognormal distribution $F(\mu, \sigma^2)$, where μ and σ^2 are the mean and variance of the underlying normal distribution, respectively. The Gini coefficient of this distribution is $G = 2N(\sigma/\sqrt{2}) - 1$, where N is the standard normal distribution function. We set $\sigma = 0.74$, corresponding to a Gini coefficient of 0.4.[19]

Next, consider the sequence of the time cost of having a child, $\{z_t\}_{t \in \mathcal{T}}$. Although it is not easy to determine which values are realistic, Haveman and Wolfe (1995) provide guideline evidence: they estimate total expenditures on children in the US in 1992 to 14.5% of GDP. Following previous studies in this literature (e.g., de la Croix and Doepke, 2003; Lagerlöf, 2006), we use their results and set $z_{1980} = 0.14$. We further take the effects of decreases in child mortality and the prolongation of education period into consideration. The reason we consider child mortality is that the time cost of raising a surviving child declines as more newborns survive to adulthood because parental time is also used for children not surviving to adulthood; the reason we consider education period is that the time cost for having a child only accrues as long as the child is a dependent family member, and children in school are commonly dependent on their parents. We assume that all children aged 0–4 are dependent, whereas children aged 5–19 are dependent only if they attend school. Furthermore, assume that parents with children aged 0–4 must bear the infant mortality risk. The time cost function in period t incorporating such ideas is given by $COST_t = 0.25 \cdot 1/(1 - IMR_t) + 0.75 \cdot SER_t$, where IMR_t and SER_t are the infant mortality rate and the school enrollment rate, respectively.[20][21] We calculate $\{z_t\}_{t \in \mathcal{T}}$ so that z_{1980} is equal to 0.14, that is, $z_t = 0.14 \cdot COST_t/COST_{1980}$. As a result, we get $\{z_t\}_{t \in \mathcal{T}} = \{0.073, 0.073, 0.073, 0.073, 0.073, 0.073, 0.073, 0.073, 0.073, 0.073, 0.073, 0.073, 0.073, 0.072, 0.090, 0.104, 0.107, 0.106, 0.123, 0.140\}$.

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[19] The assumption that σ is constant over time is just for simplification. The result is not sensitive to variation in σ as long as the Gini coefficient is in an appropriate range.

[20] The data for the infant mortality rate are taken from Flora, Kraus and Pfenning (1983) and the Office for National Statistics. Since we have no comparable data prior to 1840, IMR_t for $t = 1600, 1620, ..., 1820$ is set to equal to IMR_{1840}. The data for the school enrollment rate are taken from Mitchell (1998) and the Office for National Statistics.

[21] Some might argue that schooling is not exogenous for parents, but their choice. However, the introduction and enlargement of compulsory education, which were to a large extent exogenous for each household, had a key role in raising British school enrollment after the latter half of the nineteenth century. Galor and Moav (2006) provide a concise review of the history of British education reform.

We allow for the exogenous growth of TFP, A_t, and exogenous changes in the quantity of public provision per child, g_t. The initial level of TFP is normalized to unity, $A_{1600} = 1$, and we choose the series of TFP growth so as to match per-capita output growth in the model to per-capita GDP growth in UK data (Maddison, 2003). The growth rates of g_t between 1700 and 1940 are taken from the data on relief expenditure per capita (Boyer, 2002);[22] those between 1940 and 1980 are taken from the data on per-GDP social security expenditure (Flora, Kraus and Pfenning, 1983) and GDP (Maddison, 2003);[23] since there are no data on appropriate measures between 1600 and 1680, we compute the average growth rate of relief expenditure per capita between 1696 and 1749 from Boyer (2002) and apply it to those periods. As a result, we get $\{g_t/g_{1600}\}_{t \in \mathcal{T}} = \{1.00, 1.26, 1.58, 1.99, 2.51, 3.21, 4.16, 5.12, 6.45, 8.45, 11.64, 13.89, 7.95, 7.32, 8.22, 10.26, 9.34, 21.32, 36.49, 96.78\}$. We find some features in the transition of g_t: an upward trend between 1600 and 1820, a sharp drop in 1840, and a drastic rise in 1940.

The rest of parameters, $\{\gamma, \phi, \varepsilon, g_{1600}\}$, are chosen to minimize the sum of the squared difference between actual fertility and predicted fertility under the following three constraints.[24] (i) According to Boyer (1990), subsistence per week for a family of four was 10s.-12s. and that for a family of six was 12s.-14s. in 1795. We can infer from this that the subsistence level per child was about 1s. per week. Boyer (1990) also suggests that weekly wages for agricultural laborers, who were at the bottom of the income distribution, were 8s.-10s. in 1795. It follows that in that period, the subsistence level per child was about one tenth of the earned income for the poverty group. We choose the parameter set such that ε is in the interval $[0.09 \cdot w_{1800} h (1 - z_{1800} n), 0.11 \cdot w_{1800} h (1 - z_{1800} n)]$ for households at the fifth percentile of the labor-endowment distribution. (ii) According to Boyer (1990), the child allowance became large enough to cover the subsistence needs of an additional child for agricultural laborers around 1800. Thus, the levels of g_{1600} are chosen so that g_{1780} is equal to the subsistence needs

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[22] For periods whose data are not available, we compute them by a linear spline.

[23] Since there are no data for 1980, we use the data for 1975 as a substitute.

[24] We use the net reproduction rate (NRR) as the actual fertility rate. The data on the NRR are taken from Office of Population Censuses Surveys (1987).
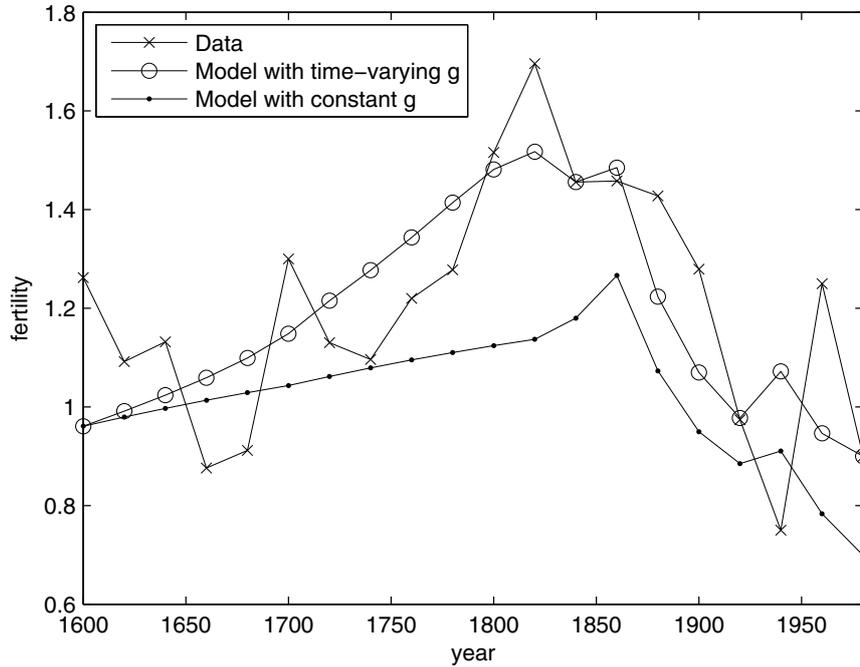
Figure 5: Simulation. Evolution of fertility rate.

per child. (iii) The predicted fertility rate in 1980 is equal to the actual fertility rate in 1980. Following the procedures above, we obtain $\gamma = 0.14$, $\phi = 0.29$, $\varepsilon = 3.78$, and $g_{1600} = 0.45$.

Suppose that the economy is initially in the steady state where the parameter values are specified in the way described above. Imagine starting the economy in 1600. Figures 5 and 6 present the simulations against the actual data for fertility dynamics and the simulated transition of income-fertility relationship, respectively. The model well captures the main features of historical pattern of UK fertility: (i) the average fertility rate exhibits the hump-shaped dynamics, with an upward trend in the seventeenth and eighteenth centuries and a downward trend in the nineteenth and twentieth centuries, and (ii) income-fertility relationship across households changed over time, from positive in the pre-industrial era to hump-shaped in the modern era.[25]

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[25]Figures 5 and 6 also present, for comparison, the simulation results in the case where g_t is fixed to the initial value for all $t \in \mathscr{T}$. We can understand that changes in g_t over time have an important role in generating not only changes in the income-fertility relationship (Figure 6), but also large variations in the fertility rate over the last 400

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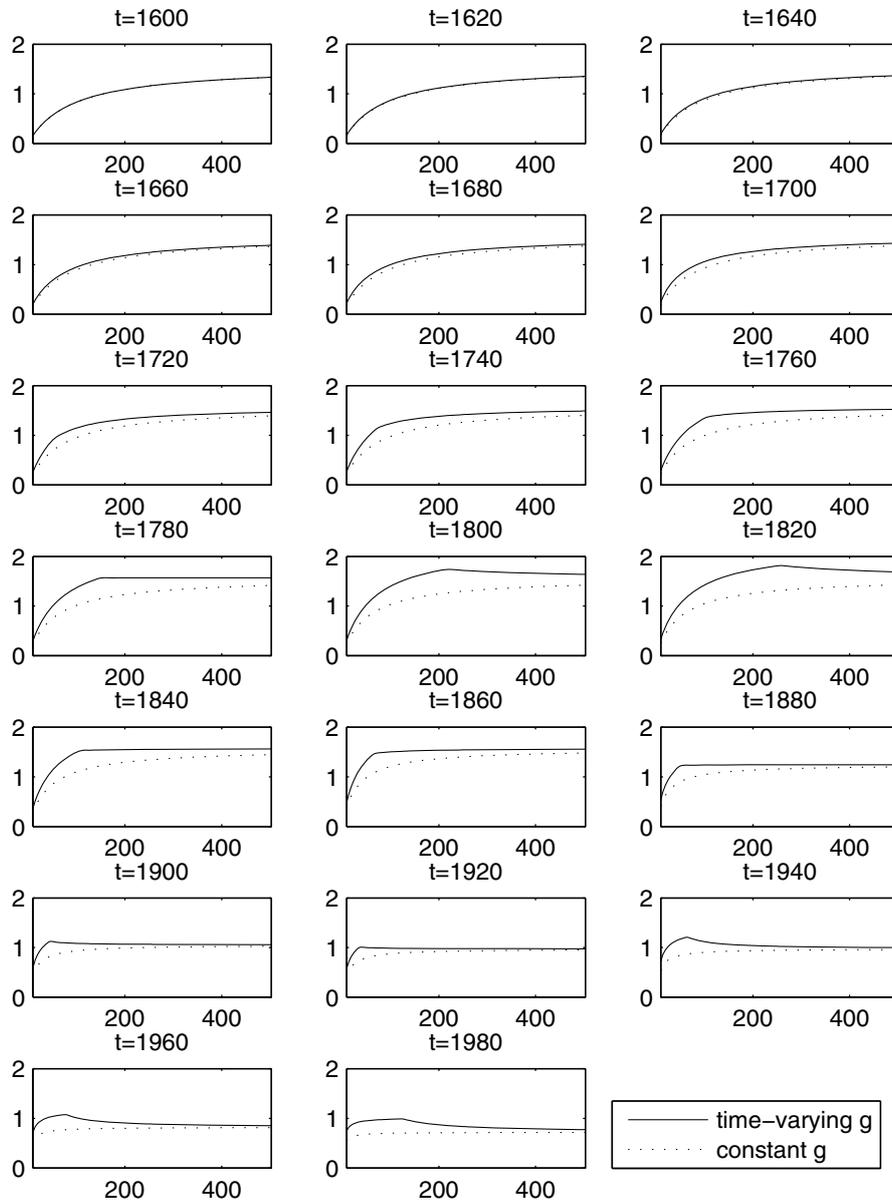
Figure 6: Simulation. Evolution of the income-fertility relationship.

Let us elaborate on the driving force behind the simulated fertility transition in Figure 5. First, the upward trend of the fertility rate in the seventeenth and eighteenth centuries is mainly attributed to the income effect of rising wages. When the size of public policy is sufficiently small, fertility is increasing in income per unit of labor supply (Proposition 3). Thus, rises in the wage rate induced by technological progress and capital accumulation raise the fertility rate of the economy. Second, the drop of the fertility rate in the mid-nineteenth century is triggered by the reversal of public policy for children, i.e., a sudden decline of g_t in the thirteenth model period, which corresponds to 1840 in the real world; the passage of the Poor Law Amendment Act in 1834 was behind this reversal of public policy. Third, there are two main causes for the downward trend of the fertility rate since the second half of the nineteenth century: the upward trend of child-rearing time, z_t, caused by the enlargement of compulsory education, and the substitution effect of rising wages caused by technological progress and capital accumulation. Finally, a baby boom is attributed to a sudden rise of g_t, while a subsequent baby bust is attributed to the substitution effect of rising wages.[26]

Figure 6 displays the changes in the income-fertility relationship in the model. Fertility monotonically increases with income between 1600 and 1780. Subsequently, the association between income and fertility becomes hump-shaped. Clark and Cummins (2010) and Boberg-Fazlic, Sharp and Weisdorf (2011) suggest that the positive association between income and fertility disappeared by the end of the eighteenth century. The simulated model well captures this timing of the disappearance. Although we do not have data on the income-fertility relationship over all the periods of the nineteenth and twentieth centuries, Clark and Cummins (2010) and Dickmann (2003) identify hump-shaped relationships between the 1850s and 1880s and around 2000, respectively. Our result is consistent with their findings.

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years (Figure 5).

[26] The predicted baby boom occurs one period earlier than actual. This might be because our model does not take into account the effects of two world wars. Doepke, Hazen and Maoz (2007) investigate the effects of World War II on the baby boom.

3 Conclusion

The income-fertility relationship within societies has changed over time. In the pre-industrial era, there was a positive association between income and fertility across households within societies, whereas a clear association does not seem to exist, positive or negative, in the modern era. Over the same period, the average fertility rate exhibits hump-shaped dynamics: an upward trend in the seventeenth and eighteenth centuries and a downward trend in the nineteenth and twentieth centuries. This paper offers a theoretical framework for explaining such phenomena. The result indicates the importance of public policy: the income-fertility relationship varies depending on the size of public support for children. If the level of public provision is sufficiently low, a positive income-fertility association exists; the positive association disappears if the level of public provision becomes sufficiently high. Furthermore, we show that the interaction between changes in public policy for children and economic development can reconcile the macro evidence (the hump-shaped dynamics of average fertility) with the micro evidence (the change of the income-fertility relationship). Our quantitative analysis suggests that a major portion of the British fertility transition after the seventeenth century and the change of the income-fertility relationship can be accounted for by changes in the size of public policy for children, TFP growth, capital accumulation, and the establishment of compulsory education.

The aim of this paper is to offer a theory explaining changes in the cross-section fertility relationship without conflicting with time-series macroeconomic fertility behavior, not to identify the trigger for the demographic transition and sustained economic growth. Thus, some important aspects of the history of long-run growth are omitted. First, except for capital accumulation, this paper treats the driving forces behind the fertility transition, such as TFP growth, as exogenous factors. Of course, taking the endogeneity and the interaction with fertility into account is important to provide a comprehensive explanation on the long-run growth from the pre-industrial era to the modern era. Second, the determinants of fertility are likely to be more complex than those embodied in our model: for instance, gains in life expectancy, human capital accumula-

tion, changes in gender gap, and contraceptive availability are also important factors affecting fertility; these many factors, including those on which this paper focuses, must have jointly shaped the long-run fertility transition. Inevitably, there are still many features that cannot be explained in terms of our model. Nevertheless, our model generates an observed pattern, which has been thought of as an unsolved puzzle, in a single framework. Our model may provide an important clue in the search for a unified theory accounting for the history of long-run economic growth.

Acknowledgments

Appendix

Proof of Proposition 1

(i) In the case of $g_t \leq \phi \varepsilon$, the threshold labor endowment $\hat{h}_t$ is non-positive. Then, all the households choose an interior solution. Differentiating n_t^i in (7) with respect to h^i, we obtain

$$\frac{\partial n_t^i}{\partial h^i} = \frac{\gamma(1-\phi)\,w_t\,(\varepsilon - g_t)}{(zw_t h^i + \varepsilon - g_t)^2} \begin{cases} \geq 0 & \text{if } g_t \leq \varepsilon, \\ < 0 & \text{if } g_t > \varepsilon. \end{cases} \qquad (16)$$

The inequality $g_t < \varepsilon$ holds if $g_t \leq \phi \varepsilon$. In this case, therefore, n_t^i is increasing in h^i for any $h^i \in \mathbb{R}_+$.

(ii) In the case of $\phi \varepsilon < g_t \leq \varepsilon$, the threshold labor endowment $\hat{h}_t$ is positive. Then, some

households choose a corner solution. Differentiating n_t^i in (8) with respect to h^i, we obtain

$$\frac{\partial n_t^i}{\partial h^i} = \frac{\gamma w_t \varepsilon}{(z w_t h^i + \varepsilon)^2} > 0. \tag{17}$$

It follows that n_t^i is increasing in h^i for the households choosing a corner solution. On the other hand, n_t^i is also increasing in h^i for the households choosing an interior solution because $g_t \le \varepsilon$. In this case, therefore, n_t^i is increasing in h^i for any $h^i \in \mathbb{R}_+$.

(iii) In the case of $g_t > \varepsilon$, the threshold labor endowment $\hat{h}_t$ is positive. Then, some households choose a corner solution. It follows from (17) that n_t^i is increasing in h^i for the households choosing a corner solution. On the other hand, it follows from (16) that n_t^i is decreasing in h^i for the households choosing an interior solution because $g_t > \varepsilon$. Therefore, n_t^i is non-monotonic with respect to h^i.

Fertility for households that choose a positive supplement in the case of $h^i \to \infty$ and in the case of $h^i \to \hat{h}_t$ are given by $\lim_{h^i \to \infty} n_t^i = \gamma(1 - \phi)/z$ and $\lim_{h^i \to \hat{h}_t} n_t^i = \gamma(g_t - \phi\varepsilon)/(zg_t)$, respectively. Fertility for households that do not supplement the public provision in the case of $h^i \to 0$ and in the case of $h^i \to \hat{h}_t$ are given by $\lim_{h^i \to 0} n_t^i = 0$ and $\lim_{h^i \to \hat{h}_t} n_t^i = \gamma(g_t - \phi\varepsilon)/(zg_t)$, respectively.

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