

STUDIES  
ON  
PERFORMANCE ANALYSIS OF  
FLEXIBLE MANUFACTURING CELLS

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**STUDIES  
ON  
PERFORMANCE ANALYSIS OF  
FLEXIBLE MANUFACTURING CELLS**

by  
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# Preface

In recent years, manufacturing industries of advanced countries have been moving increasingly towards computerized automation systems in efforts not only to respond quickly to market fluctuations and high competition environments but also to improve productivity. In reality, with the result of great advances in the field of computers, automatic control techniques, and artificial intelligence, it became possible to introduce the flexibility into conventional systems. Also the result made possible the development of automatic production systems which coped with the high rate of manufacturing environmental changes. Apparently the advent of a new production system, which is called a Flexible Manufacturing System (FMS), contributed so much to resolving various inherent limitations of conventional production systems. For example, shortening the cycle time of products, reducing the waiting time and work-in-process inventory which was major cost in those systems.

However, although this new production system provided a revolutionarily high technology as a positive step towards a fully unmanned-automated factory, it spawned a lot of new problems in its design and operations. Without solving these problems, it would be very difficult to efficiently implement an FMS. Up to the present, many researchers widely investigated various problems arising in the development and implementation of FMS, and studied the analysis method for its performance measures. Especially, in the half of 1980's, it became one of the most active fields in the application of the queueing theory, which offered good techniques to model an FMS and provided extensive analytical tools for studying the optimal design and efficient operation.

Generally, The exact analysis of FMS with the application of the queueing theory is extremely difficult due to the inherent complexity of FMS, and the exact solution involves an unacceptable amount of computation. Therefore, most of previous studies directly applied the queueing network model to approximately analyze the FMS. It seems nearly impossible to exactly analyze the FMS. We emphasize the importance of analyzing a Flexible Manufacturing Cell (FMC) system prior to the analysis of FMS, because an



FMS consists of several fundamental FMCs that is relatively easy to solve.

The main purpose of this dissertation is to provide FMC models and to exactly analyze the performance measures of these models through the application of the queueing theory. In Chapter 1, the overview of FMS and the outline of this dissertation are provided. In Chapter 2, we consider an FMC system with two waiting storage buffers connected by a conveyor material handling system. It is modelled as the discrete time queueing system and the exact analysis is performed on the special case. The stationary probability generating functions of the number of materials at storage buffers are derived. The average performance measures such as average queue length and average waiting time are provided. In Chapter 3, we extend the FMC model provided in Chapter 2. In Chapter 4, the FMC model with an automatic interchanging device is considered. We analyze this model to investigate the effects of the tool switching time. We consider the tool switching time in an FMC system significantly affects the performance measures.

The results in this dissertation give insights into basic problems in the category of FMS, and show the way to improve the performance of these systems reasonably. Furthermore, the methodology of the performance analysis developed in this dissertation is applicable to the construction of optimal Computerized Integrated Manufacturing (CIM) systems and other automatic manufacturing systems. The author expects that FMC models developed in this dissertation will be widely utilized and will stimulate further study in the factory automation fields.

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## Chapter 1

### Introduction

Up to the present, various new computer-controlled production systems of job shop type were developed and are under developing with the tremendous interest of manufacturing industries. Among new revolutionary systems, the FMS was developed as a typical system which can increase productivity and improve quality control in manufacturing factories. Since FMS was introduced as an high-tech system in the early of 1960's, Various FMSs were installed and operated in many factories. The earliest FMS was installed by Sunstrand in 1964 and a typical FMS is the Kearney & Trecker system. In this chapter, we overview concept, problems and models of FMS.

#### 1.1 Concept of FMS

##### 1.1.1 Definition of FMS

In general, production systems were characterized in terms of the persistence of manufacturing activity and the regularity of material movement through the system. Formerly, the batch production, which was a type of mass production, contributed to enhancing the production efficiency but had always inherent limitations such as high level of work-in-process. Fortunately, it was recognized that these limitations could be resolved with the aid of computer and numerical control techniques. This fact led to the initial stage of the basic concept of FMS as a new manufacturing technology.

Klahorst divided the development of FMS into the following three stages [Huan 86]. Creation stage (1967-73) could be characterized by the employment of dual computers for direct numerical control and traffic/information management. Expansion stage (1974-79) introduced programmable controllers to integrate and update the dual computer control systems. Refinement stage (1980 to the present) could be characterized the development of control systems. The emphasis of the last stage was on the refinement of software



packages, while the emphases of the first two stages were mainly on the necessary hardware to create the FMS.

Definitions of the FMS can be found in many literatures [Brow 83], [Buza 82a], [Dopo 82], [Gerw 81], [Hut 73], [Kusi 85a], [McBe 82], [Popp 82], [Stec 83], [Suri 81], [Torr 82], [Yao 85a]. Generally, it is very difficult to strictly define FMS. There are some uncertainties concerning the conditions under which a manufacturing system may be termed 'flexible'. For example, dedicated and fixed transfer line systems containing only automated storage and retrieval are not FMS [Brow 84]. Furthermore, there are different views on FMS. FMS is considered as the continuation or extension of conventional manufacturing shops respectively by [Pun 85], [Wolp 87], to which computer control, material handling and storage systems are merely added, and their variety simply mimics that of the traditional shops. Their views on FMS evolve through combining the highest productivity of transfer lines with the flexibility of numerical controlled (NC) machines. On the other hand, FMS is described as a new high-technology/high automation production system [Purc 85]. These considerations bring some confusions into the definition of FMS. But, it is clear that the FMS essentially represents not only an computerized component within traditional shops but also the concept of a unmanned-automated factory. Of course, in order to move from the present state of the art to fully unmanned computer-controlled factory, a number of technical problems must be solved [Lern 81]. Here, we present the definition which may provide a basic guidance for FMS, considering the category of a fully unmanned-automated manufacturing, as follows.

The FMS is a versatile automatic production system, which integrates several cellular work stations, material handling equipments, and storage systems into a manufacturing group, operated by computer controlled networks in order to perform flexible production operations of various material types for dynamic routing among work stations in the system.

Essential characteristics of FMS are the linking of various work stations and automated equipments through the material information flow. A typical systematic conceptualization of FMS can be characterized by the following features [Huan 86].

- It is an attempt to solve the production problem of mid-volume and mid-variety materials.
- It is designed to process simultaneously several types of materials in a given mix which will remain the same, at least in the short run.

- It is equipped with rather sophisticated flexible machines which are capable of processing a sequence of different materials with negligible tool switching time.
- Materials are transferred from work station to station by a computerized MHS.
- Materials are generally tied to pallets which facilitate their positioning on machines.
- Some mechanisms are provided for automatically transferring the work-in-process between the transportation system and the machine tool.

As a general rule, the FMS is made up of several installation components to be mixed, according to factory environments such as scale, layout, etc. The primary physical components of FMS are (1) work station module, (2) material handling module, (3) storage module and (4) system control module.

### Work Stations

Work stations, which are the most necessary component for FMS, can be set up to carry out many different functions such as machining, assembly, inspection, loading and unloading [Zisk 83]. Each station consists of one or more flexible machines which are able to automatically process and assembly various material types. A flexible machine has a peripheral equipments such as an automatic tool changer. In order to be conveniently inserted into FMS, work stations must possess the following three characteristics [Torr 82].

- (1) Work stations must be provided with entrance and exit modules not only for the materials to be processed, but also for the devices capable of dealing with loading and unloading without suspending the processing operation.
- (2) Work stations must have a level of intelligence to be self-controlled and self-run, that is, be able to make alternative choices according to the different situations which may arise.
- (3) Work stations must have a reliability level higher than the standard in order to ensure the system has a level of reliability at least equal to that of a single work station.

### Material Handling System

Material handling equipments are normally used as the key integrating cellular manufacturing units into FMS. They play a central role in FMS conveying quickly and reliably



materials from work stations to stations according to its processing requirements, and automatically loading/unloading it. A discussion of material handling equipments for FMS must cover the movement, storage and control of materials [Tomp 83]. In general, the material handling equipment for a line flow type tend to be special purpose in nature, whereas, the material handling equipment for a process flow type is flexible. The basic characteristics which an MHS must have are [Klah 81]; (1) lack of floor obstructions, (2) low cost per unit transport, (3) low cost per foot transport, (4) design configuration flexibility, (5) proven reliability, (6) quietness, (7) expandability and (8) in-process dynamics.

The selection of an MHS is very important since it affects directly almost every benefit of FMS. There are many types of material handling equipments related to FMS. Typical material handling equipments are

- (1) conveyors for synchronous transport of materials and pallets
- (2) industrial robots for especially loading/unloading in a cellular manufacturing
- (3) tote pan for a basic interface with storage and work stations
- (4) shuttle car for higher levels of production
- (5) guided vehicles with induction track for random delivery.

They are very universally employed as an automatic MHS in modern manufacturing plants, perhaps more than any other type. For more detail of this topics see [Goeb 83], [Klah 81], [Tomp 83], [Zisk 83].

### Storage System

Storage systems automatically store materials to be transferred by transport devices and make control easier as a large number of different materials are always available, making it easy to change the production schedule [Wolp 87]. There are two categories of storage systems; (1) storage buffer for work-in-process, (2) automated storage and retrieval systems (AS/RS) or automated warehouses for materials and finished products.

*Buffer storage* provides an area for temporarily storing work-in-process, and is used as an integrated part of a station as well as the overall MHS. The dynamic routing taken by different kind of materials in FMS bring the blocking at machines, because a material to have been processed at a work station is unable to move on to its next station which

is still busy. Therefore, buffer storage is always required for an incoming material. The main objectives of buffer storage are

- (1) control the work-in-process through the system
- (2) increasing the work station utilization
- (3) reduction of the effect such as breakdowns, variability in operation times, and the blocking of machines due to the diversity of material routing.

*Automated warehouse* provides storage locations where stored materials must be easily accessible and recovered rapidly, for one or more types of pallets on which materials are moved and handled under computer supervision. Furthermore, automated warehouse provides tight and accurate control in FMS by controlling the movement of material, and by reporting information on such movements to the data processing network of the factory. The major system objectives of automated warehouse are [Rank 83]

- (1) computer control of materials, providing the status and location of materials for all cellular systems within FMS control architecture via a network
- (2) reduction of material and finished product inventory
- (3) improvement of security and reduction of product losses from handling damage and location errors
- (4) increasing productivity.

### Computer Control System

The FMS is a highly complex system that requires sophisticated computer hardware and software to monitor and control its processing operations. Hence, it is necessary to apply distributed control theory and data base concepts in order to provide the required flexibility and high level of intelligence in the control of FMS. In general, computer control of FMS is intended for (1) the flexible machine, (2) the material handling equipment, (3) the material movement within the system and (4) the information on system performance [Youn 81]. The FMS is largely controlled by a three level of computer control hierarchy [Lern 81], [Klah 81].

- (1) The top level controller monitors the overall system status, and coordinates system production as a whole, coping with any changes. Management information, real-time routing, and decisions exist at this level of control.



- (2) The second level control supervises the operation of work stations communicating with the bottom level. This level includes programmable logic and minicomputers.
- (3) The bottom level control is the most basic units of control, normally providing by the computerized numerical control units attached to each machine. This level of control is logic oriented and directly controls the operation of machines.

Under these control hierarchy, a typical computer system controls all operations of FMS, performing some or all of the important functions [Buza 86a], [Zisk 83]. Typically, major functions are as follows: (1) Directing the routing of materials through the system to maximize work station utilization and production. (2) Tracking the status of all materials in progress in order to sequence different loads to selected work stations, based on variable routing instructions. (3) Passing the instructions for the processing operations to each station. (4) Providing added necessary control to machines at stations. (5) Monitoring the correct performance of operations and signals' problems which are requiring attention.

Major benefits of FMS come from reducing the uncertainty which is associated with conventional manufacturing system, and eliminating the causes of high finished inventory levels [Prim 88]. The FMS possesses many advantages such as the simultaneous production of several types of materials in small batch sizes, the increase of work station throughput, while attaining the inherent adaptability of traditional job shop operations [Hut 73]. In general, the FMS has the following advantages [Huan 86], [Zisk 83].

#### *Dynamic loading of materials*

Routing material to the proper machines by computer in FMS gives considerable flexibility to deal with changes in the production schedule.

#### *Reduction of lead times*

Set-up time typically constitutes a significant portion of the production lead time in a traditional job shop. In FMS, the production lead time can be drastically reduced since both tooling and material set-up are preset off-line.

#### *Reduction of inventory*

As a result of a shorter lead time in FMS, the work-in-process inventory which is closely related to the production lead time is significantly lower than in conventional manufacturing shop. Moreover, as the operation approaches "just-in-time production", the warehousing of materials and finished products is greatly diminished.

#### *Increase of work station utilization*

The FMS can be achieve a much higher work station utilization due to such features as reduced set-up times, efficient material handling, and simultaneous material processing.

#### *Improvement of product quality*

Production methods have important influences on product quality. The consistently precise interfacing of materials and automated equipment that is attainable through the application of automated MHS makes better product quality.

#### *Optimization of facility layout*

The FMS can be expanded in modular fashion to meet high product demand. This feature reduces the risk associated with inaccurate market forecast.

#### *Reduction of direct and indirect labour*

Because both machining and material handling are completely operated under computer control, neither machine operators nor material handlers are needed in FMS.

#### *Ease of engineering changes*

The FMS allows the incorporation of engineering changes without critical production loss and major retooling works. This feature is especially valuable when new products are introduced frequently.

Of course, either direct or indirect, there are many other advantages to be obtained by the implementation of FMS.

### 1.1.2 Flexibilities and Types of FMS

#### 1) Flexibilities in FMS

The FMS can be considered as a manufacturing system in which various flexibilities are given to conventional production system. Therefore, the major objective of FMS is to achieve several flexibilities necessary for efficient simultaneous production of various type materials in small to medium volume. As a general rule, the flexibility can be defined as the ability to effectively cope with changing circumstances [Capu 83]. From this viewpoint, it is considered that the flexibility is not only a fundamental new direction for manufacturing systems but also an essential feature as the necessary, and perhaps even sufficient conditions for FMS. In FMS, the flexibility is not a single factor but a



complex concept due to the dependence upon the capabilities of its components, their interconnections, and the operation and control mode [Bara 88]. In general, the more the FMS is larger, the more flexibility is required.

By some papers, the flexibility can be divided into different categories at the base of various criterion. For example, Mandelbaum observed that flexibilities can be described in two different contexts [Buza 82b], [Gupt 89] that is, *action* and *state* flexibility. Action flexibility is related to situations where decisions are made sequentially without knowing what the future will bring. State flexibility is related to situations where a given system is able to continue operating effectively in many new circumstances. Although both types of flexibility are desirable in manufacturing systems, in the context of FMS, the emphasis on 'flexible' implies state flexibility. Based on the time span classification, flexibilities can be divided into a *short term* and a *long term* perspective [Bara 88], [Warn 82]. Other way of examining flexibility is to represent an *actual* flexibility capable of overcoming concrete changes and a *potential* flexibility for coping with an undefined changes.

There are a number of papers which discussed the importance and types of flexibility [Bara 88], [Bril 89], [Cart 86], [Falk 86], [Fraz 86], [Groo 81], [Gust 84], [Kula 88], [Kuma 87], [Mand 89, 90], [Mura 85], [Whar 88]. In their literature, many different opinions are suggested for the types and definitions of flexibility. However, there are noticeable similarities among the proposed definitions.

Gupt and Goyal [Gupt 89] gave a very comprehensive survey on various types of flexibility for FMS. They synthesized the literature dealing with concepts and measures of flexibility, focused on manufacturing flexibility, in order to discuss the ways of measuring flexibility with a quantitative and qualitative view.

Zelenović [Zeke 82] identified and discussed two types of flexibility as a condition for FMS. *Design adequacy* flexibility is closely related to the degree of utilization of system parameters, and *adaptation* flexibility can be defined as the amount of time needed for the system to adapt to a change in the job.

Buzacott [Buza 82b] considered that any attempt to evaluate the flexibility of a manufacturing system must begin with the consideration of the nature of changes and disturbances with which the system should be able to cope. Hence, he defined two types of flexibility in order to clarify the effect of internal changes such as machine breakdowns and external changes such as processing requirements. *Job* flexibility is the ability of the system to cope with changes in the jobs to be processed by the system. *Machine* flexibility is the ability of the system to cope with changes and disturbances at machines and work

stations.

Kumar [Kuma 86b] discussed *loading* flexibility and *operations* flexibility for measuring flexibilities of FMS with the application of an information theoretic approach. Loading flexibility arises out of the power to regulate the frequency of the visit of a material to different work stations. Operations flexibility is the availability of the next operation to be performed on each material on the basis of the up-to-date knowledge of the work stations' availabilities for different operations.

Kusiak [Kusi 85a, 86a] considered four types of flexibility in order to derive the system measure; *flexible manufacturing module* flexibility, *MHS* flexibility, *computer system* flexibility and *organizational system* flexibility encompassing *job* flexibility, *scheduling* flexibility, *short-term* flexibility and *long-term* flexibility. Browne and Rathmill [Brow 83] offered six types of flexibility in terms of the major characteristics of non-mass jobbing production; *mix* flexibility, *materials* flexibility, *routing* flexibility, *design change* flexibility, *volume* flexibility and *customizing* flexibility. Primrose and Leonard [Prim 86] identified 25 different aspects of flexibility which are required at each four different types of FMS in such a way that it can be related to both technical specification and financial benefits.

Here, we mainly review the eight types of flexibility proposed by Browne *et al* [Brow 84].

#### *Machine flexibility*

Machine flexibility represents the capability of a machine to perform a variety of processing operations for a given set of material types. This flexibility can be measured by set-up time, therefore it is called as machine set-up flexibility in [Bara 88]. Machine flexibility fall into the category of short term and actual flexibility.

#### *Process flexibility*

Process flexibility can be defined as the ability to vary the steps necessary to process a given set of material types. That is, this type of flexibility can be represented as a combination of process variety, interchangeability and redundancy of unit operations, coupled with transfer possibilities, allowing utilization of the process variety for a given product mix. This flexibility can be attained by having machine flexibility, and measured by the number of material types that can be simultaneously processed without using batches. Process flexibility is putted into the class of both potential and actual flexibility.

#### *Product flexibility*



Product flexibility is the capability to change a product mix inexpensively and quickly. This flexibility can also be attained by having machine flexibility, and measured by the time required to switch from one material mix to another, not necessarily of the same material type. This type of flexibility fall into the category of long term flexibility .

#### *Routing flexibility*

Routing flexibility represents the ability to process a given set of materials on alternative machines, in alternate sequences. It is considered that this flexibility is the core of the flexibility concept [Bara 88]. Routing flexibility is measured by the robustness of FMS when breakdowns occur. This flexibility can be potential for fixed material routing or actual for dynamic routing.

#### *Volume flexibility*

Volume flexibility can be described as the ability to operate an FMS profitably at different production volumes. This flexibility can be attained by having routing flexibility and increased by a higher level of automation. Volume flexibility is measured by the availability of system running profitably in terms of the smallest unit of volumes for each material type.

#### *Expansion flexibility*

Expansion flexibility is the ability to easily and modularly add the capacity to FMS as needed. This flexibility can be attained by having routing flexibility, and measured by how large the FMS can become. Expansion flexibility has to be included in the original design of FMS. It is noted that this flexibility is called as machine flexibility by Gustavsson [Gust 84].

#### *Operation flexibility*

Operation flexibility represents the ability to interchange ordering of several operations on each material while complying with the design restrictions. This flexibility is measured by the level of existing technology.

#### *Production flexibility*

Production flexibility is the capability to produce various products without adding major capital equipments, even though new tooling or other resources may be required. This flexibility requires the capabilities of all flexibilities. Although this flexibility becomes a

potential, it provides an overall flexibility index [Stec 85b].

These flexibilities clearly make the FMS efficient and offer a high level of productivity. The definition of various flexibility type indicates that the flexibility is not a self-contained concept [Gupt 89]. Furthermore, all of these flexibility types are not independent. The relationships among these flexibility types are described in [Stec 85b]. The basic flexibility types among flexibilities which are required for any FMS are machine and routing flexibility.

## 2) Types of FMS

So far, many different kinds of FMS were developed depending upon the geometrical form, size and production amount of materials to be processed and also upon the philosophy of production [Sata 88]. Typically, FMS has two types of machining alternatives. One typical type involves general purpose machines, which are most economical for very small batch production of many different products. General purpose equipment is designed to accommodate a wide array of tools and processing functions in return for limiting its rate of production as well as other capabilities in respect to any particular operations [Gold 82]. The second type involves single purpose or dedicated machinery such as transfer lines, suited for very large batch or mass production of a specific product [Gerw 81]. However, a dedicated type system permits only minor adjustments in product designs or processing methods. In general, FMS can be divided into various types at the base of different criterion such as the differences in the flexible machines and MHS used. For example, according to the flow of materials by pattern of MHS, the FMS can be largely classified into (1) *linear flow* type; material flow is one direction through a product-oriented layout (e.g. transfer line system), (2) *loop flow* type; material flow is flexible relative to the processing sequence, but has a fixed transfer loop (e.g. loop conveyor system), (3) *random access* type; material flow is flexible both relative to the processing sequence and to the transfer of materials through a process-oriented layout (e.g. job shop type system).

There are many papers which discussed various types of FMS. Among them, Kusiak [Kusi 85] divided types of FMS into five classes based on the number of NC machines and their layout; *flexible manufacturing module*, *flexible manufacturing cell*, *flexible manufacturing group*, *flexible production system* and *flexible manufacturing line*.

Browne *et al.* [Brow 84] classified the FMS according to the types of flexibilities; (1) *flexible manufacturing cell*, (2) *flexible machining system*, (3) *flexible transfer line*, (4)



*flexible transfer multi-line*. Stecke and Browne [Stec 85b] discussed types of FMS which proposed by Browne *et al.* [Brow 84] regarding types of MHS. They categorized many existing FMS installations with their corresponding MHS types in order to provide real examples indicating the wide variety of different FMS.

Primrose and Leonard [Prim 88] divided the FMS into four categories using the nature of the financial benefits; *single machine flexible manufacturing modules*, *multi-machine flexible manufacturing system*, *flexible transfer lines* and *fully integrated flexible manufacturing factory*.

Here, we arrange the classification of FMS appearing in some literatures, according to the number and functions of work stations.

1. *Flexible Manufacturing Cell (FMC)* represents a basic cellular work station for FMS. For FMC, we will discuss in Section 1.1.3. The FMC further can be subdivided into two categories according to NC machine function as follows.
  - (a) *Flexible Machining Cell (FMaC)* is a cellular unit in which machining operations are only performed. The FMaC is suitable for processing a group of similar material types which require little variation in the sequence of machining operations [Sata 88].
  - (b) *Flexible Assembly Cell (FAC)* is the basic unit for automated assembly systems and mainly a single loading/unloading robot-based cellular unit for assembling two or more parts to be joined or fastened together. In FAC, a machining operation of parts is permitted only for assembling purpose.
2. *Flexible Machining System (FMaS)* consists of a set of FMaCs where various types of material are machined through the system in small to medium lot sizes. The FMaS makes possible the automated multi-step and multi-product manufacturing [Warn 83]. It is noted that in many literature the FMaS is generally considered as what is called the FMS [Stec 85b].
3. *Flexible Assembly System (FAS)* is a special type where subassemblies are assembled periodically in small to medium batch sizes, which consists of two or more FACs. In particular, the FAS is viewed as a collection of flexible work stations for assembly connected by automated materials handling devices, where each work station may perform any subset or all of the required operations on a variety of parts [Dona 88]. From this viewpoint, the FAS is the combination of the benefits of a highly

productive assembly line and a flexible bench assembly process where each part has a pre-determined number of operations which are performed at any assembly work station [Rank 83]. Typically, the FAS can sequence material randomly and respond quickly to various assembly orders. It is noted that some researchers does not consider the FAS as the FMS.

4. *Flexible Transfer Line (FTL)* contains a sequence of several work stations which are connected to each other by an automated material flow system according to the line principle. The FTL is a type of FMS which introduces the flexibility of batch production into mass production while still retaining the advantages of conventional transfer lines. Therefore, the FTL system is capable of simultaneously or sequentially manufacturing different type of materials in large volume, which run through the system along the same path. Generally, the potential benefits of FTL come from two broad areas [Prim 88]; (1) improvements in operating performance, (2) the ability to introduce new product designs without the constraints of high capital cost and long downtime between product models. Still the concept of FTL is new, and actual application is quite difficult to evaluate because the borders between FTL and a conventional transfer line overlap smoothly in many cases [Warn 83]. However, with the development of FTL, the distinction between mass and batch production will be disappeared. The FTL can be subdivided into the following two classes.
  - (a) *Flexible Transfer Single-Line (FTSL)* consists of several dedicated NC machines which are arranged in series, and requires that each material has a fixed route in a down stream direction through the system. The FTSL can be used as the building block for FTL, and produce more than one product on the single line.
  - (b) *Flexible Transfer Multi-Line (FTML)* consists of a set of sequence of work stations that are interconnected by automated MHS [Stec 85b]. The FTML is a type of multi-stage and multi-work station system.
5. *Flexible Production System (FPS)* contains several different types of FMCs representing different work areas interlinked by several kinds of automated MHS in a way which enables simultaneously the machining or assembling of different type of materials or parts which pass through the system along different routes. The FPS is considered as the interface between manufacturing and assembly system, and achieved by integration of any or all types of automated system.



Undoubtedly, new types of FMS will be developed with the technological advances in processing, mechanization, computerization, related communication and instrumentation capabilities, programmable controls, and robotics. However, the essential aspects of each new design will not differ from those of existing systems.

### 1.1.3 Flexible Manufacturing Cell

Most FMSs that are currently operated in many countries, especially Japan, U. S. A. and Germany, are large scale integrated automatic production systems. But, unfortunately large scale FMS requires the large initial capital investments to install. For the most small and medium-sized factories, the capital investment is the very important decision factor which determines whether to install the FMS or not. Therefore, the research on the development of FMC which can be installed and operated with the small capital investment is required. In this respect, the great interest of FMC as a small scale FMS is expanding much more rapidly than the bigger FMS in the industrialized countries.

FMC is a computer controlled cellular unit capable of performing various manufacturing operations. Generally, the FMC is considered as the fundamental building blocks for FMS. In reality, the most typical FMS is recognized as a variety of cellular manufacturing system which is realistic and effective unit of production system based on the concept of group technology [Oba 88]. A typical FMC consists of three basic components. Three components are a versatile work station, an automatic material handling equipment, and storage buffers for work-in-process. A versatile work station is composed of a stand-alone machine centre (MC) or several flexible machines with peripheral equipments such as automatic tool changer (ATC), automatic pallet changer (APC), and automatic control and supervision subsystems. Flexible machines at work station are made up of any one of NC machines, direct numerical control (DNC) machines or computerized numerical control (CNC) machines. A stand-alone MC or flexible machine at work station is performing a variety of operations such as drilling, milling, cutting, turning, boring, grinding, etc. In an FMC system, an automatic material handling equipment is mainly a group of industrial robots because they are highly flexible, easily reprogrammable and low volume oriented. Moreover, industrial robots can be easily incorporated into the FMC system for performing a number of different functions such as loading/unloading, tool changing, inspection, etc. They certainly become a major factor in FMC system with the constant increasing of their roles.

There are many types of FMC system. The typical types, FMaC and FAC system,

were explained in Section 1.1.2. Among other types, multiple FMCs system with common input storage buffers was proposed by [Gree 86] in order to find out a proper loading and scheduling policies. The FMC system with serial stages, each of which has one or more identical parallel work stations with a finite buffer storage was considered by Han and McGinnis [Han 88] to study the maximization of the FMC throughput. The FMC system with automatic set-up equipment was presented to analyze the minimization of the total production time in [Hito 89]. Their FMC consists of an MC, an industrial robot, a loading/unloading station and an index-pallet changer with clamping devices.

There are some papers concerned with the FMC system. An overview of cellular manufacturing technology are presented in [Purd 82], [Scho 83], [Zisk 83], and various problems arising in FMCs are treated by [Chak 87, 90], [Noh 88], [Seid 85a, 85b, 87], [Shaw 87]. Characteristics of cellular manufacturing systems are presented in [Blac 83].

## 1.2 Problems and Analytical Techniques of FMS

Although the FMS was recognized as the key to resolve the inherent limitation of conventional manufacturing systems, a lot of new problems concerned was resulted in. It would be very difficult to effectively carry out the FMS without solving these problems.

To date, many researches were performed in order to investigate and study various problems arising in the wide scope of FMS [Hild 80], [Kalk 86], [Kusi 86b], [Sol 77], [Suri 81], [Yao 85a]. These researches demonstrated that analytical techniques for solving FMS problems have to be developed.

### 1.2.1 Overview of FMS's Problems

For the installation and operation of FMS, there are many kinds of problems to be resolved, such as strategic, technological, economical and labor problems. Strategic problems are arising due to the change of markets, competition, and technology. The problem of cost concerning FMS must be considered in order to obtain the optimal FMS design [Adam 88], [Tom 88]. Furthermore, the FMS may well engender serious labour problems. A use of computerized automation (e.g. robots which are introduced as direct replacements for labour) reduces substantial employment in manufacturing factories. Technical problems in FMS are mainly resulted from the following areas [Lern 81]; (1) to increase the versatility of the systems, (2) to automate the heavy burden of maintenance, (3) to automate the difficult area. As a general rule, the technical complexity causes various



problems such as frequent breakdowns.

Some technical problems frequently occur due to various factors in the machines, materials being processed, MHSs, computers, or some combination. Especially, among technical problems, two groups arising in FMS design, planning and scheduling are of particular importance. The first group is concerned with the optimal selection of FMS's components, and the second with their optimal utilization [Kusi 85a]. In general, the optimal selection is very difficult because it is faced with numerous highly technical decisions that have to be made under little reliable information and considerable pressure for correct choices.

In this section, we review these problems discussed in [Buza 86a], [Kusi 85a, 86b], [Stec 83, 85a, 86b] and divide into three categories; FMS design problem, FMS operational problem and FMS control problem.

### 1) FMS Design Problems

The FMS design begins with a survey of the manufacturing requirements of products in the factory [Klah 81]. That is, it begins by defining goals and developing a conceptual design for processes, equipments, control systems and information systems, and considering the integration of FMS with the factory-wide production. In FMS design, the function, capability and number of work stations, type of MHS, type of storage buffers and others should be determined at the initial stage. At the detailed stage it is necessary to determine tool change systems, method of feeding and locating materials at machines, capacity of storage, etc. The difficulty of designing FMS is mainly caused by the numerous and complex design factors to be considered. Therefore, the following issues must be considered in FMS design.

- (1) The selection of the type of FMS.
- (2) The range of material (or part) types to be processed.
- (3) Issues on FMS components.
  - The capability and number of each type of machines required at each work station.
  - The function and type of MHS.
  - The type and capacity of storage buffers for work-in-process.
- (4) The type and amount of flexibility needed.

- (5) The layout of FMS components considering their connection via MHSs.
- (6) The hierarchy of control structure, interconnection of computers and data handling devices.
- (7) The strategies for running FMS.

Doubtlessly, these issues interact so extensively that an optimal design can be devised only through a system approach.

### 2) FMS Operational Problems

Once the FMS is installed, then the assignment of associated tools to machines, the routing of materials, the sequencing of materials flow into FMS, and so on must be considered because the appropriate production planning and scheduling can sufficiently assure the utilization of FMS. Furthermore, since the FMS has to be integrated into the overall manufacturing operations of the factory, there are a variety of problems relating to the operational control issues. FMS operational problems are decomposable into the production planning and scheduling problem.

#### Production Planning Problems

Decisions on FMS production planning problems have to be made and implemented in order to utilize capabilities of FMS sufficiently before FMS begin to produce products. In particular, FMS production planning must be carried out effectively at many different levels in order to maximize the system throughput over a certain time period. FMS planning problems are divided into the following subproblems, which can be solved sequentially or iteratively.

##### *Material (or part) selection problem*

From a set of material types that have production requirements, a subset of material types must be determined to be processed immediately and simultaneously in specific time intervals.

##### *Machine grouping problem*

NC machines must be partitioned into machine groups in such a way that each machine in the same group can perform the same operations. This would be achieved in practice by providing the same tools to each NC machine in the group.

##### *Production ratio problem*



This problem is concerned with material type mix determination. The relative ratios in which the selected material types will be produced to attain a good utilization must be determined.

#### *Allocation problem*

In this problem, the assignment of various appropriate tools to NC machines must be determined. Namely, the optimal number and type of pallets and fixtures to be reserved for each material type must be determined.

#### *Machine loading problem*

In order to balance workload among NC machines, appropriate tools must be determined to load into appropriate tool magazines of machines. This is the allocation problem of the operations and associated tools of the selected material types among the machine groups subject to the technological and capacity constraints on FMS.

### **Scheduling Problems**

One of the most difficult problems generated by FMS is the scheduling problem. FMS scheduling problems are concerned with running FMS, which involve determining the input sequence of materials into the system and the processing sequence at each work station. In general, complexity of scheduling depends on the shop configuration, material arrival process, processing time distribution, scheduling objective [Nels 86]. Especially, scheduling problems in FMS are very complex due to the following factors; (1) multifunctionalization of machines, (2) alternative machines in parallel work stations, (3) finite storage buffers at machines, (4) transportation system among machines.

FMS scheduling problem is decomposed into several subproblems such as sequencing, dispatching, routing problems. The following issues must be considered.

- (1) The appropriate sequence policy to determine which materials to be taken into the system.
- (2) The optimal dispatching rule to determine the priorities among materials which are waited to be processed at a work station.
- (3) The development of applicable algorithms to schedule the operations of all materials through the system.

### **3) FMS Control Problems**

The actual operations of the system are dealt with in FMS control. In FMS control problems, the effect of different control structures on information handling requirements, and the range of easily implementable control rules should be studied. Generally, FMS control problems are associated with the continuous monitoring of the system status, breakdowns, performance, and the tracking of production operations through the system in order that production requirements and due dates are being met as scheduled. The following policies must be determined to efficiently operate the FMS.

- (1) Breakdown handling policies to automatically handle any breakdown situations.
- (2) Maintenance policies to prevent breakdowns of machines.
- (3) Inspection policies for work-in-process and finished products.
- (4) Procedure policies for tool life, process and machine condition monitoring.

### **1.2.2 FMS's Analytical Techniques**

In general, the methodology available for solving FMS problems includes direct experiment method, analytical procedure and simulation method. Among them, the direct experiment method is very costly and not feasible in many situations of FMS.

After researches on the development of analytical techniques for FMS began in the 1970's, various techniques were developed to solve FMS problems. For example, simulation techniques were developed to investigate and compare between routing policies, and mathematical programming was applied to develop scheduling and operating procedures [Stec 83]. Also the queueing theory has been applied as an application tools to evaluate the performance measures such as the work-in-process and machine utilization resulting from particular routing policies [Buza 80].

In this section, we briefly review major analytical tools that are widely used to resolve many problems associated with FMS. Comprehensive surveys on analytical techniques for solving FMS problems are found in [Buza 86a], [Kalk 86], [Stec 86b], [Suri 85a].

#### **Queueing Theory**

Among various analytical tools, the queueing theory is a well-used technology to model FMS and applied to analyze various performance measures of FMS. The first direct application of the queueing theory to FMS was due to Solberg [Sol 77]. He pointed out that



most FMS can be represented as a closed queueing network. In solving the FMS problems, the use of closed queueing networks became widespread by the virtue of researches of Jackson [Jack 63], Gordon and Newell [Gord 67].

Some researchers approximately analyzed and evaluated performance measures of FMS using queueing network theory. Suri [Suri 81] examined analytical results in queueing network theory employing the concepts of operational analysis. Yao and Buzacott [Yao 87a] demonstrated that some known results and tools in queueing networks with finite buffers can be applied to a wide class of FMS with finite local buffers at stations. Koenigsberg and Mamer [Koen 82] modelled an FAS as a set of cyclic queues and analyzed through decomposition.

Numerous applications of the queueing theory in FMS were made much more efficient in [Basa 88], [Buza 85, 86b], [Coff 88], [Ko 90a, 90b, 91], [Yao 85b, 85c, 86a, 86b, 87b]. We provide in detail the application of the queueing theory for FMS in Section 1.3.

### Simulation method

For the FMS design and analysis, simulation was recognized as one of the most practical and effective tools as it allowed detailed investigation of the effects of parameter variations and dynamic interactions in the system. In reality, most researches related to dynamic operations planning of FMS mainly applied simulation approach. Furthermore, the complexity and amount of expense involved in FMS require the need for simulation technique. Fortunately, by the advent of general purpose simulation languages such as GASP, GPSS, Q-GERT, SLAM, SIMAN and SIMSCRIPT, the FMS modelling process is simplified.

To date, there are largely three main areas within the field of application of simulation for FMS [Mill 83]; (1) general guidelines for the design and control of FMS, (2) the development of low-cost fast-response simulation methods, (3) the development of the present simulation languages. Stecke and Solberg [Stec 81] investigated alternative loading and scheduling strategies using simulation language GASP IV for the system built by Sunstrand for Caterpillar Tractor Company. Jain and Foley [Jain 86] introduced an overview of requirements of a generic FMS simulator, and developed general design criteria for the simulator. Adamov and Dukarskii [Adam 88] pointed out that it must rely upon the simulation method in order to exactly obtain the more expected values for performance evaluation of FMS.

Many applications of simulation to FMS design, planning and scheduling were reported by [Brow 83], [Brun 86], [Bull 87], [Carr 84, 86], [Clay 82], [Denz 87], [Fuji 88], [Had 88],

[Inou 86], [Kay 82], [Klei 88], [Noh 88], [Ozed 88], [Rath 83, 86].

### Mathematical programming

Generally, it is considered that mathematical programming is a valuable technique for formulating and resolving the design and operational problems of FMS. In reality, many researchers applied mathematical programming such as linear or nonlinear integer programming, dynamic programming, branch and bound method, and heuristic approach to solve the problems concerned with the design, planning, and scheduling of FMS.

Afentakis [Afen 86] used the graph theory in order to investigate the various aspects of FMS design, and developed heuristic solution algorithms based on decomposition approach for FMS layout design problem. Kusiak and Heragu [Kusi 87] surveyed various formulations and twelve heuristic algorithms for modelling and solving the FMS layout problem. They compared these algorithms based on the solution quality and computation time. Stecke [Stec 83] formulated the FMS grouping and loading problems as a nonlinear 0-1 mixed integer program, and used linearization procedures for comparing the loading methods and the work flow control policies in FMS. Berrad and Stecke [Berr 86] solved a similar problem, focusing on the loading objective of balancing the workload on all machines, while assigning each operation to only one machine. Kimemia and Gershwin [Kime 83] have formulated the higher level scheduling problem as a continuous time dynamic program considering both the demand rate and the system state with respect to machine failures.

Other research works using this technique for the FMS design and operational problems are provided in [Avon 88a, 88b], [Baso 88], [Co 90], [Han 89a, 89b], [Hera 88], [Hwan 89], [Kuma 86a], [Kusi 85b], [Lash 87], [Maim 88], [Ogra 87], [Sari 87], [Shan 86], [Shak 89], [Soml 88], [Stec 85a, 86a], [Wilh 85], [Wils 89].

### Perturbation Analysis

As a nice tool which pulled out much more informations normally provided by a simulation's output, perturbation analysis was originally developed by Ho *et al.* [Ho 79] for studying the sensitivity of system performance to its parameters. This new technique has potential applications to both simulation and real-time operation of FMS with the following two important advantages. One advantage is that it is not necessary to re-run the system or simulation because all the predictions are obtained from one observation. The other is that the modelling assumptions required are minimal because it can work directly off real data. But, this analysis can not predict accurately the effects of large



change in decisions.

Using perturbation analysis as the basic tools, Suri and Dille [Suri 85b] studied some design problems in FMS through analyzing the sensitivity of relevant system parameters. An overview of perturbation analysis is provided in [Ho 85] and applications to the FMS can be found in [Ho 84], [Suri 85a].

### Artificial Intelligence

An expert system is defined as an intelligent problem-solving computer program that uses knowledge and inference procedures to achieve a high level of performance in some specialized problem domain [Shar 87]. Recently, Artificial Intelligence (AI) technique emerges as a major tool to systematically represent FMS scheduling and control problems. Especially, AI technique is useful to take into account unexpected situations in FMS, such as tool breakage [Wall 83].

Among researchers who applied AI expert system to FMS, Novak [Nova 83] discussed over the concept of AI in order to improve and simplify machine control with the application of adaptive control in FMS. Subramanyam and Askin [Subr 86] provided a general introduction to the important concepts of expert system, and described a prototype expert scheduling system for selecting a part for processing in FMS. Young and Rossi [Youn 88] examined the application of AI technique to create a knowledge based system for solving FMS control problems. Shaw and Whinston addressed the application of AI technique to FMS planning and scheduling [Shaw 86], and employed knowledge based system to handle the flexibility of machines and dynamic changes in FMS scheduling [Shaw 88, 89].

### Petri Net

Petri net theory provides a compact model and general graphical tools to capture the intricacies of system interactions. Therefore, Petri net theory is applied to investigate real-time, steady-state, and transient scheduling and control problems of FMS such as the determination of relevant control rules to order operations waiting for each machine, the determination of the minimum number of pallets to maintain the required production ratios. That is, FMS can be systematically modelled and analyzed to gain insights into the qualitative and quantitative behaviour by using Petri net. However, Petri net technique is not useful for FMS having many machines with finite buffer storage and real time routing policies.

A comprehensive review over Petri net technique can be found in [Nara 85] and applications to FMS appear in [Alla 86], [Bara 88], [Nof 80], [Vala 90].

Consequently, the choice of analytical techniques on FMS problems must be carefully considered according to awareness and experience with FMS issues. The indicative choices of analytical techniques provided by [Suri 85a] are as follows.

- (1) Mathematical programming techniques for problems on appropriate FMS mechanism.
- (2) Queueing theory, simulation and perturbation analysis for problems on a number of machines which will be required to meet the specified production mix.
- (3) Queueing theory, simulation, perturbation analysis and Petri nets techniques for problems on performance measures and routing policies.
- (4) Simulation techniques for problems on FMS layout.
- (5) Simulation and perturbation analysis for problems on adequate buffer sizes.
- (6) Simulation and Petri nets techniques for problems on material selection to meet a level production mix.

## 1.3 Queueing Models of FMS

Evaluative models for FMS were generally established by using the techniques mentioned in Section 1.2.2 and provided insight into a number of key design issues. Also they were used to determine how the overall production capacity of a system was affected by the mix of material types, the number and capability of machines, the size of storage buffers, machine breakdowns and blocking. In this section, we briefly review queueing models of FMS appeared in the literature.

Generally, when FMS is modelled in the queueing theory, most issues concerned are network, routing and loading, and storage buffer problems. It is clearly shown by many papers that queueing models of FMS are very useful in the evaluation and prediction of stochastic behaviour of material flow in a system, as well as in the preliminary design of FMS. However, these models are currently limited by their inability to handle finite buffer capacity at a station and their dependence on simplifying assumptions related to processing times.

### 1.3.1 Network Models

After a queueing network model was identified as a tool for studying job-shop operations by Jackson [Jack 63], queueing network theory was frequently used to model and



analyze the FMS. In FMS, queueing network models were employed to explain dynamics, interactions and uncertainties for materials competing for the same work stations. But, these network models contained unrealistic factors, and did not represent certain features of FMS such as an interaction among transporters. Moreover, most network models of FMS provided average values of performance measures which have an assumption of a steady-state operation. Also it was proved that queueing network theory was not useful for studying breakdown situations in the scheduling and control problems of FMS. However, these queueing network models efficiently gave reasonable estimates of FMS's performance measures. There were advantages that the standard analytical techniques of queueing network were directly or indirectly used to evaluate the performance measure of FMS.

In general, a queueing network model of FMS consists of a set of  $M$  work stations which are interconnected by materials flow. Station  $i$  may have one or more identical NC machines. The total number of materials in the system is a fixed constant, which can be viewed as the total number of pallets available in the system. A material after being processed at station  $i$  proceeds to some station  $j$  with routing probability  $p_{ij}$ . The routing probabilities are independent of the system state. The system state  $\mathbf{n}$  is an  $M$ -tuple  $(n_1, n_2, \dots, n_M)$  where  $n_i$  denotes the number of materials at station  $i$ .

Detailed discussion on the application of queueing network theory to FMS and several queueing network models of FMS can be found in [Buza 86a], [Suri 81, 85a], [Yao 85a, 85c, 86a, 86b, 87a]. An FMS can be modelled as either open queueing network (OQN) or closed queueing network (CQN).

In OQN models, the number of materials within the system is a random variable, and materials arrive externally according to a Poisson process and leave the system when they are processed. Shanthikumar and Buzacott [Shan 81] proposed an OQN model of dynamic job shops with general service times and FCFS or shortest processing time service discipline. They developed an approximate decomposition approach to analyze the OQN model, and compared its accuracy with simulation results. Yao and Buzacott [Yao 85a] developed an OQN model with general service times to evaluate the performance of FMS, considering the finite storage capacity at each work station. They developed a flow equivalent decomposition approach to derive approximate solutions to the model. Other applications can be found in [Buza 80, 82a, 85], [Shan 86], [Yao 85b, 87b].

In CQN models, the number of materials within the system is a constant. That is to say, materials which have been processed are instantaneously replaced by the corresponding

new materials, or the system always has a fixed number of pallets which carry materials among work stations. In general, it is proved that the CQN model is useful in providing insights into how system components interact as materials compete for the same limited resources. CQN models are primarily used to obtain the production capacity of FMS under different machine loading conditions, and can also be used to study the effects of material flow control on the production capacity. Solberg [Sol 77] observed that the CQN approach can be effectively used as FMS's performance models. He developed the FMS model by applying the CQN theory, and analyzed the CQN model of FMS by using Buzen's algorithm. Since the application of the CQN technique in modelling FMS was first suggested by Solberg, many researchers applied the CQN technique to represent FMS and evaluate their performance measures. There are many papers which clearly established the use of CQN models [Cava 82], [Hild 80], [Secc 79], [Seid 86], [Sol 77].

Maione *et al.* [Maio 86] derived closed form analytical formulae to evaluate the performance measures of FMS by using the Z-transform method and adopting the symbols, definitions and assumptions provided in [Denn 78]. They considered two types of systems, that is, balanced system and completely unbalanced system, and introduced a decomposition algorithm for computing the performance measures. Yao and Buzacott [Yao 86b] developed an exponentialization approach to model FMS with general processing times in the CQN. They formulated the exponentialization approach as a fixed point problem which can be solved through a path-following algorithm proposed by Zangwill and Garcia in 1981. Kimemia and Gershwin [Kime 85] used the CQN technique to model FMS, and studied the optimal routing/loading problem in order to maximize the production rate.

CQN models of FMS are based on several assumptions that are not expected to be satisfied in actual FMS. The following two basic assumptions are made in the CQN model of FMS [Yao 86a].

- (1) At each station with FCFS queue discipline, the processing times are exponentially distributed and all material types have the same service rate.
- (2) Each station has a local storage buffer large enough to accommodate all materials if necessary, so that no material is blocked at any station.

The application of queueing network theory to the performance evaluation of FMS usually requires the assumptions of exponentially distributed processing times and FCFS discipline. In reality, processing times in FMS are more likely to be deterministic rather



than exponential. The assumption of exponentially distributed processing times was generally believed to be the most restrictive among assumptions. However, some researchers demonstrated that results based upon the exponential distribution were robust. They found that some performance measures obtained under the assumption of exponential processing time distribution were rather insensitive to errors involved in the estimation of system parameters such as a service distribution. Suri [Suri 82, 83] gave more rigorous theoretical justification by investigating the 'robustness' of a CQN model from an operational analysis viewpoint. In general, the CQN model of FMS has not yielded satisfactory performance evaluations for the system with FCFS queue discipline, in which the processing time distribution is not exponential [Buza 86a].

The CQN model with infinite local buffers was called the classical model, which is a cornerstone of CQN models of FMS. Although most CQN models of FMS developed so far assumed the exponential processing time distribution at all stations, it was proved that in case of infinite local buffers, the exponential assumption did not yield adequate results if the processing time distribution had a coefficient of variation greater than one [Yao 86a]. In the case of the CQN model with finite local buffers, it was appeared that the exponential approach was a good approximation and was valid in the case of dynamic routing. Yao and Buzacott [Yao 86a] considered the CQN model of FMS to study the relation between materials routing and loading at any work station in FMS. They developed three models, which are (1) the fixed-routing model, (2) the fixed-loading model, (3) the dynamic-routing model, according to different assumptions on the operation of the system.

There are several categories of CQN models connected with FMS [Seid 86]. Here, we illustrate two models which are well-used as the technique in analyzing the performance measures of FMS. These models are becoming popular in FMS design and operation as a good compromise between the efficiency of the model and the accuracy of predictions.

### CAN-Q Model

The CAN-Q technique offers realistic modelling capability and provides several performance measures, such as work station utilization, average queue length, throughput rate, etc. that can be used for FMS design.

The CAN-Q model is the most widely known analytical model related to FMS for studying the effect of various strategies on system throughput. The CAN-Q model was developed as a single material type CQN model by Solberg [Sol 77] in order to study some issues in the preliminary FMS design and production planning. It was enriched in [Stec 81] and extended by [Dubo 83]. The CAN-Q model was found to be robust by Co

and Wysk [Co 86]. They demonstrated the robustness of the CAN-Q model by providing statistical evaluation for a variety of restrictive assumptions in modelling FMS based on an existing Caterpillar system.

### MVA-Q Model

Mean Value Analysis (MVA) is a computational technique to analyze networks with many closed chains by applying a heuristic algorithm. The MVA-Q technique is not originally developed for FMS modelling. But the advantages of MVA technique enabled us to model an FMS in order to obtain quite reasonable predictions of performance in FMS planning and control problems. One remarkable advantage is that the MVA technique can model more detailed features than CAN-Q without loss of efficiency.

The MVA-Q model is a CQN model using simple but sophisticated mathematical techniques, and is solved computationally by using the exact MVA algorithm. The MVA-Q model yields exact solutions for FMS network where service demand at each FCFS station is independent of the material type. Especially, the MVA-Q model of FMS can explicitly define the transporters and the load/unload stations, and measure the operations in terms of average queue size, average waiting time and throughput. Generally, the MVA-Q and CAN-Q models provide similar performance measures, but the effect of limiting the supply of pallets can be studied by using MVA-Q model.

In addition to the basic functions performed by CAN-Q approach, the MVA-Q technique provides additional features such as modelling of multiple part types, low computer memory required, and accurate large system performance. Particularly, the MVA-Q technique is suited to the multiple part type capability. The CQN model of multiple material types with exponential processing times at each machine was presented by Hildebrandt [Hild 80], which was based on the MVA method proposed by Reiser and Lavenberg in 1980. However this model required a large computer memory. More efficient implementation was carried out on the CQN model with deterministically distributed processing times by Cavaille and Dubois [Cava 82], but their approach could not be applied to FMS with several identical machines. Suri [Suri 83] justified the use of MVA techniques for FMS, and Suri and Hildebrandt [Suri 84] employed MVA technique for modelling FMS with several identical machines.

Among other CQN models, the priority mean value analysis (PMVA) model gave insights into various priority assignment policies in FMS. The PMVA model was developed by Shalev-Oren *et al.* [Shal 85] in order to model the FMS with parallel stations, each



station having several identical machines and following priority scheduling scheme which can be either FCFS, AS (ample-server) or HOL (head of the line) non-preemptive policy. It was shown that throughputs and flow times for various material types could be quite sensitive to the choice of priorities at heavily loaded stations. The variety of performance measures computed by the PMVA model was similar to that of the MVA-Q model.

The complex interaction in FMS makes it almost impossible to make a detailed analytical model in a realistic way. It is almost impossible to exactly analyze the network model of FMS which covers interactions of operating factors at all production stations.

### 1.3.2 FMC Models

FMC models are largely divided into the two categories, which are FMC models in the conveyor-serviced production system and robot-based FMC models. An FMC model connected with a material handling conveyor system is provided by [Beig 68], [Coff 88], [Mats 75, 78], [Reis 67]. A robot-based FMC model consisting of several work stations positioning around a robot is presented by [Inab 82], [Seid 85a, 85b, 87], [Noh 88]. Other types of FMC model are found in [Davi 79], [Gree 86], [Han 88], [Hito 89], [Seid 84], [Yao 85a, 85b], [Tang 88a, 88b]. Here, we review FMC models where the queueing theory is fruitfully applied to evaluate various average performance measures. In general, the important performance criterion in FMC models are

- (1) meeting due date
- (2) maximizing system and machine utilization
- (3) minimizing work-in-process inventory
- (4) maximizing system and machine throughput
- (5) minimizing set up time and tool changes
- (6) minimizing material flow time
- (7) balancing machine utilization.

There were many conveyor-serviced FMC models for analyzing the problems arising in the design and operation of FMC. For example, Reis *et al.* [Reis 67] modeled the individual work station with fixed range policy as a Markov process in order to analyze delays associated with a material handling conveyor at work stations. Beightler and Crisp [Beig

68] extended the Reis's model for analyzing the work-in-process storage requirements of the individual work station that is operated in conjunction with a conveyor on the assumption that each work station has a stationary Bernoulli arrival process. They employed the discrete time queueing analysis and proposed an operating policy, the *Sequential Range Policy (SRP)*, as a method for efficiently operating the work stations in the conveyor-serviced production system. Their model was simulated in [Cris 69] using the GPSS III language for testing the validation of Bernoulli assumption in the operating policy.

Matsui and Fukuta [Mats 75] proposed a conveyor-serviced FMC model with the general arrival, which was the extension of Beig's model provided in [Beig 68]. They applied the imbedded Markov process, and analyzed the stochastic behaviour of reserve state in the conveyor-serviced work station with the SRP. Their model was extended in [Mats 78] for analyzing the operating policy, the *State-dependent Sequential Range Policy (SdSRP)*, which proposed by Crisp in 1967, with the application of the queueing theory.

In latest studies, Coffman, Jr. *et al.* [Coff 88] presented two FMC models to study the interaction between a conveyor and a single station with infinite storage buffers. One model had one robot that cannot return a processed material to the conveyor while unloading a new material for processing. The other model had two robots to allow simultaneous loading and unloading of the conveyor. They analyzed with the M/G/1 queueing technique to obtain the proper distance between the two points at which material leaves and rejoins the conveyor, with respect to the system performance, such as average queue length.

Among the robot-based FMC models, Seidmann and Nof [Seid 85a] proposed the model with stochastic material feedback flow, in which one type material was produced at a time, for analyzing a material recirculation in the robot-based FMC system. The extension of their model could be found in [Seid 85b]. For other types of FMC model, Seidmann and Schweitzer [Seid 84] considered an FMC model producing different parts for several emanating lines, each of which had a finite input storage buffers. They applied an undiscounted semi-Markovian process to solve the FMC control problem. An FMC system having a set of parallel machines with finite storage buffers was proposed by Yao and Buzacott [Yao 85a, 85b] for studying the FMC design problem within FMS. In the case that both the interarrival and processing times have a small squared coefficient of variation (svc), the FMC system was modelled as G/G/c/N queue technique and analyzed with diffusion approximation in [Yao 85a]. For the case of large svc's, the FMC system was modelled as  $C_2/C_2/c/N$  queue and analyzed with Coxian laws of two phases in [Yao



85b].

## 1.4 Overview of the Dissertation

In this dissertation, the major purpose is to study the performance evaluation of an FMC system in view of a basic unit within FMS as well as stand-alone cellular manufacturing unit. This study is motivated by the fact that comparing FMS, FMC systems are particularly little studied with respect to analyzing the performance measures with the application of the queueing theory.

In general, it is very difficult to exactly analyze an FMC with finite storage buffers in the conveyor-serviced production system because of the effects resulted from the relation between the finite storage capacity and operations of conveyor. Especially, there are little studies on the analysis of outstanding characteristics of an FMC system such as changing necessary tools automatically when required. In an FMC system with an automatic tool changer, tool switching times become to be an important factor of system performances.

In Chapter 2, we consider the FMC system connected with the conveyor, which have two waiting storage buffers where the first storage can keep at most one material and the second storage is of infinite capacity. We assume that the FMC system can be modelled as a discrete time queueing system with a stationary Bernoulli arrival process and service process. We exactly analyze the FMC model using the queueing theory in order to investigate the influence of material handling conveyor system on the storage buffers. Consequently, we derive the stationary probability generating function to evaluate its average performance measures, and present numerical results of average queue length and waiting time.

In Chapter 3, we consider the more general FMC model wherein the first storage is of finite capacity and the second storage is of infinite capacity, which is the extension of the FMC model provided in Chapter 2. It is noted that the FMC model considered in Chapter 2 is a special case of this model. The stationary probability generating functions of FMC model are derived by exact analysis to evaluate various performance measures. We provide numerical examples of the average waiting time and buffer occupancy.

In the above chapters, we provide the exact analysis of FMC with finite input storage buffers in the conveyor-serviced production system. In Chapter 4, we consider the FMC system having a single NC machine with a finite buffer storage and an automatic tool changer which has a fixed-finite tool magazine. In the FMC system, tool changing times are considered as two types of materials are processed at the single machine. In general,

tool changing times have been ignored, but the tool switching time is found to significantly affect the machine utilization. We model the FMC system as the discrete time M/G/1/K queueing system, and derive the moment generating function of waiting time. Numerical experiment is performed to present the average performance measure of FMC model.

Finally, in Chapter 5, conclusion and suggestions for future research topics are provided.

The results discussed in Chapter 2 is mainly taken from [Ko 90a], Chapter 3 from [Ko 90b], Chapter 4 from [Ko 91].



## Chapter 2

# Analysis of an FMC in a Conveyor-Serviced Production System

### 2.1 Introduction

In this chapter, we consider the exact analysis of an FMC with loading/unloading robots, which is connected to the conveyor-serviced production system. In FMS, *Industrial robots* and *conveyors* are the foremost material handling equipments.

Industrial robots have a wide range of capabilities and can be easily incorporated into an FMS. In these respects, they certainly become an essential component of FMS. In reality, industrial robots play a major role in FMS, and the different types of applications are increasingly proposed. Generally, typical operations performed by robots in FMS are largely classified into (1) material handling, (2) tool operation such as coating, welding, trimming, etc. and (3) assembly [Kusi 85]. Especially, robots offer many advantages over customized loading and unloading devices incorporated into work stations in FMS. However, to date, their coverage in FMS was limited to a few isolated units. Therefore, industrial robots must be more developed to match various circumstances required in FMS. Various functions and types of industrial robot in the automated factory were discussed in [Otti 81, 82].

Among several types of MHS used, conveyor is one of the most popular equipment in FMS, mainly due to the following three reasons.

1. The basic function of conveyor is handling materials for delivering from one production station to another.
2. Conveyor relatively has a well-understood mechanism and furthermore, it's operat-



ing cost per foot lower than any other automatic material handling equipments.

3. Conveyor can be used to transport any type of materials.

The main objectives of conveyor are to ensure the smooth flow of materials and the maximum utilization of available space with maximum flexibility. Advantages and disadvantages for various types of conveyor are provided in [Kama 82]. For the more detail discussions on conveyor see [Kwo 58], [Muth 79], [Reis 63b].

A number of studies were performed in which the queueing theory was fruitfully applied to the analysis of production station problems related to the conveyor system (see, Section 1.3.2). Up to the present, problems arising in a production station connected with conveyor systems was extensively studied by several researchers [Naka 87], [Proc 77], [Reis 63a]. Disney [Disn 63] studied a power and free conveyor system having two unloading stations respectively with finite storage buffers as a multichannel queueing model problem. Gupta [Gupt 66] analyzed the two-channel queueing problem posed by [Disn 63] in order to obtain the queue size distribution for each of the two service stations with finite queue length. Pritsker [Prit 66] studied the conveyor system which include  $m$  unloading stations with infinite storage buffers allowed only at the last unloading station, as a specialied queueing problem. That is, he generalized the analysis provided by [Disn 63]. Additionally, he used simulation method to get the performance measures associated with the loop conveyor system. Gregory and Litton [Greg 75] presented the Markovian approach to analyze the discrete loop conveyor model with  $m$  work stations. In their model, storage buffers and recirculation of materials were not allowed. Muth [Muth 77] analyzed a closed loop conveyor with a loading station and an unloading station, in which random material flow and recirculation of materials were taken into account. El Sayed *et al.* [El 76, 77] investigated the steady state behaviour of two and three-channel ordered entry conveyor-serviced system with multiple arrivals having independent poisson distribution. In their model, either homogenous or heterogeneous servers were allowed at the service channels. They demonstrated that the solution of multi-item conveyor system is possible through the application of the queueing theory.

We consider the performance of a production station with two waiting storage buffers to store materials before and after processing in the conveyor-serviced production system. As for such an FMC system, we analyze an important class of problems arising in the interaction between conveyor operations and waiting storage buffers, in order to consider the optimal layout of FMS and the optimal production scheduling and to make the op-

eration of FMC efficient. We analyze the system to evaluate the average performance measures, such as the average length at queue 2.

This chapter is made up of as follows. In Section 2.2, we describe the FMC system, and mathematical model is presented in Section 2.3. In Section 2.4, the exact analysis is performed on the case in which the first storage has a one buffer size, and the second storage has an infinite buffer size, and also the average performance measures on the second storage buffers are derived. The numerical results and the concluding remarks of this chapter are provided in Section 2.5.

## 2.2 Description of FMC System

Our FMC system is shown in Fig.2.1 and consists of three basic components as follows.

1. a single production station to process materials.
2. two conveyors to transport materials and loading/unloading robots.
3. input and output storage buffers to temporarily hold materials to be processed and to have been processed.

A single production station, that we encounter in real manufacturing system, consists of MC or a set of NC machine tools with several pallets. In this chapter, we do not take into account these factors in the analysis. In the production station, arriving materials are served. There are two conveyors in this system. Two conveyors move at a constant speed and are divided into fixed-size cells, each able to hold at most one material. The conveyor which is called the loading conveyor carries materials to the input spot by shifting down to the right-hand side and the conveyor called the unloading conveyor takes materials away. Input and output buffer storages are located in front and rear of the production station. These are called queue 1 and queue 2, respectively.

There are two spots where the production station has access to the conveyors. The input spot is the place the station has access to the loading conveyor, at which new arriving materials enter queue 1. The output spot is the connection point between the station and the unloading conveyor, at which materials having been processed are put on the unloading conveyor. Materials to be processed at the production station leave the loading conveyor at the input spot, wait in queue 1, receive a service, wait in queue 2, and are put on the unloading conveyor at the output spot. Materials to be processed elsewhere



stay on the loading conveyor. To be put on the unloading conveyor, the materials having been processed must wait for an empty cell arriving at the output spot.

### 2.3 Mathematical Model

First of all, we make an assumption on the time to make the analysis tractable. We define the time required for each cell to pass the input spot as unit-time slot. Therefore, at each slot time, a new cell arrives at the input spot and the output spot respectively. The arriving materials at the input spot in different time slots are assumed to be statistically independent. Thus, the conveyor cell arriving at the input spot is assumed to be independently in either of the following states with fixed probability.

1. empty, with probability  $(1 - \lambda)$ .
2. occupied. Each cell
  - (a) contains a material which requires service from the production station, with probability  $\alpha\lambda$ .
  - (b) contains a material which is headed for other production stations, with probability  $(1 - \alpha)\lambda$ .

Each coming material waits in queue 1 and then occupies the production station for a random service time. Service times are independent and identically distributed. They are geometrically distributed with mean  $1/\mu$ . In order to join the unloading conveyor, the material at queue 2 must wait the empty cell at the output spot. In our system, there are three computer-controlled specialized robots. The first robot which is beside the input spot picks up arriving materials transported by the loading conveyor, the second robot loads and/or unloads on and off the production station, the third robot at the output spot puts materials having been processed by the station on the unloading conveyor.

The FMC model is described by a Markov chain with the stationary joint probabilities denoted by  $\pi(i, j)$  for two queues in tandem ( $i = 0, 1, \dots, j = 0, 1, \dots$ ), where  $i$  and  $j$  indicate the number of materials in queue 1 and queue 2, respectively. In this system, we consider the model where queue 1 has a finite buffer of size  $K$  and queue 2 has an infinite buffer.

### 2.4 Analysis

In this section, we present the exact analysis of the above-mentioned FMC model, which has a Bernoulli input process, service process and output process with parameters  $\beta$ ,  $\mu$  and  $\gamma$ , respectively.  $\beta$  is the probability that a new arriving material at the input spot enters queue 1 during the one slot time i.e.,  $\beta = \alpha\lambda$  and  $\gamma$  denote the probability that materials waiting at the queue 2 can be put on the unloading conveyor during the one slot time.

We analyze the particular case where queue 1 size is one (namely,  $K=1$ ) in order to examine the stochastic behavior of the model. In this case, it is easy to find the balance equation as follows.

$$\pi(0, 0) = (1 - \beta)\pi(0, 0) + (1 - \beta)\gamma\pi(0, 1). \quad (2.1)$$

$$\begin{aligned} \pi(0, 1) &= (1 - \beta)(1 - \gamma)\pi(0, 1) + (1 - \beta)\gamma\pi(0, 2) \\ &\quad + (1 - \beta)\mu\pi(1, 0) + (1 - \beta)\mu\gamma\pi(1, 1). \end{aligned} \quad (2.2)$$

$$\begin{aligned} \pi(1, 0) &= \beta\pi(0, 0) + \beta(1 - \mu)\gamma\pi(0, 1) \\ &\quad + (1 - \beta)(1 - \mu)\pi(1, 0) + (1 - \beta)(1 - \mu)\gamma\pi(1, 1). \end{aligned} \quad (2.3)$$

$$\begin{aligned} \pi(0, j) &= (1 - \beta)(1 - \gamma)\pi(0, j) + (1 - \beta)\gamma\pi(0, j + 1) \\ &\quad + (1 - \beta)\mu(1 - \gamma)\pi(1, j - 1) + (1 - \beta)\mu\gamma\pi(1, j), \quad j \geq 2. \end{aligned} \quad (2.4)$$

$$\begin{aligned} \pi(1, j) &= \beta(1 - \mu)(1 - \gamma)\pi(0, j) + \beta(1 - \mu)\gamma\pi(0, j + 1) \\ &\quad + \beta\mu(1 - \gamma)\pi(1, j - 1) + [\beta\mu\gamma + (1 - \beta)(1 - \mu)(1 - \gamma)]\pi(1, j) \\ &\quad + (1 - \beta)(1 - \mu)\gamma\pi(1, j + 1) + (1 - \beta)\mu(1 - \gamma)\pi(2, j - 1) \\ &\quad + (1 - \beta)\mu\gamma\pi(2, j), \quad j \geq 1. \end{aligned} \quad (2.5)$$

Now we introduce probability generating functions for the preceding steady state probabilities.

$$P_0(\omega) = \sum_{j=0}^{\infty} \pi(0, j)\omega^j, \quad (2.6)$$

$$P_1(\omega) = \sum_{j=0}^{\infty} \pi(1, j)\omega^j. \quad (2.7)$$

$P_0(\omega)$  and  $P_1(\omega)$  are analytic in the unit circle  $|\omega| \leq 1$ . The equations on probability generating functions can be obtained in principle by multiplying  $\omega$  and performing appropriate summations of the basic set of the steady state equations (2.1), (2.2), (2.3), (2.4),



(2.5). A straightforward calculation yields,

$$\begin{aligned} & [1 - (1 - \beta)C(\omega)]P_0(\omega) - (1 - \beta)\mu\omega C(\omega)P_1(\omega) \\ & = (1 - \beta)[1 - C(\omega)]P_0(0) + (1 - \beta)\mu\omega[1 - C(\omega)]P_1(0), \end{aligned} \quad (2.8)$$

$$\begin{aligned} & - \beta C(\omega)P_0(\omega) + [1 - (1 - \mu + \beta\mu\omega)C(\omega)]P_1(\omega) \\ & = \beta[1 - C(\omega)]P_0(0) + (1 - \mu + \beta\mu\omega)[1 - C(\omega)]P_1(0). \end{aligned} \quad (2.9)$$

where

$$C(\omega) = (1 - \gamma + \gamma/\omega).$$

From the equations (2.8) and (2.9), we can obtain

$$P_0(\omega) = \frac{\gamma}{f(\omega)} \cdot \{G(\omega)\omega P_0(0) + (1 - \beta)\mu\omega^2 P_1(0)\} \quad (2.10)$$

$$P_1(\omega) = \frac{\gamma}{f(\omega)} \cdot \{\beta\omega P_0(0) + [H(\omega) + \beta\mu\omega]\omega P_1(0)\} \quad (2.11)$$

where

$$G(\omega) = (1 - \beta)[1 - (1 - \mu)C(\omega)]$$

$$H(\omega) = (1 - \mu)[1 - (1 - \beta)C(\omega)]$$

and

$$f(\omega) = \beta\mu(1 - \gamma)\omega^2 + \gamma[\beta\mu - 1 + (1 - \beta)(1 - \mu)(1 - \gamma)]\omega + (1 - \beta)(1 - \mu)\gamma^2.$$

Since  $P_0(\omega)$  and  $P_1(\omega)$  are analytic in the region  $|\omega| \leq 1$ , whenever the denominator  $f(\omega)$  of the right-hand side of equations (2.10) and (2.11) has zeros in that region, so must the numerator. This fact is henceforth used to evaluate  $P_0(0)$  and  $P_1(0)$ . Applying Rouché's theorem to the  $f(\omega)$ , we can prove that the equation precisely has one unique root  $\omega^*$  in the  $|\omega| \leq 1$ , if the stability condition of the system is satisfied, that is,

$$\frac{\beta\mu}{1 - (1 - \beta)(1 - \mu)} < \gamma.$$

By substituting 1 and  $\omega^*$  for  $\omega$  we can get two related equations required to determine the state of  $P_0(0)$  and  $P_1(0)$ . When  $\omega = 1$ , we have

$$P_0(1) = \frac{\gamma}{f(1)} \cdot \{(1 - \beta)\mu P_0(0) + (1 - \beta)\mu P_1(0)\}. \quad (2.12)$$

Therefore, we can easily get the values of  $P_0(1)$  and  $P_1(1)$  from the equations which obtain by substituting  $\omega = 1$  in the equations (2.11) and (2.12) together with the boundary equation  $P_0(1) + P_1(1) = 1$ , which are given below.

$$P_0(1) = \frac{(1 - \beta)\mu}{1 - (1 - \beta)(1 - \mu)},$$

$$P_1(1) = \frac{\beta}{1 - (1 - \beta)(1 - \mu)}.$$

Hence, we obtain the necessary equation as follows

$$P_0(0) + P_1(0) = \frac{f(1)}{\{1 - (1 - \beta)(1 - \mu)\}\gamma}. \quad (2.13)$$

In the same way as before calculated, we proceed to derive the another equation from the equations (2.10) and (2.11) by subsequently using  $\omega = \omega^*$ . Considering  $f(\omega^*) = 0$ , it is not difficult to obtain an additional equations.

$$\beta\omega^* P_0(0) + \{[1 - (1 - \beta)C(\omega^*)](1 - \mu) + \beta\mu\omega^*\}\omega^* P_1(0) = 0. \quad (2.14)$$

Therefore, it is clear that we can obtain the values of  $P_0(0)$  and  $P_1(0)$  using the equations (2.13) and (2.14) such as

$$P_0(0) = \frac{f(1)}{I\gamma} \left\{ 1 - \frac{\beta}{\beta K(\omega^*) - H(\omega^*)} \right\}. \quad (2.15)$$

$$P_1(0) = \frac{\beta f(1)}{I\gamma\{\beta K(\omega^*) - H(\omega^*)\}}. \quad (2.16)$$

where

$$K(\omega^*) = (1 - \mu\omega^*),$$

$$H(\omega^*) = (1 - \mu)[1 - (1 - \beta)C(\omega^*)],$$

and

$$I = [1 - (1 - \beta)(1 - \mu)].$$

Then, we have

$$P_0(\omega) = \frac{f(1)\{(1 - \beta)\beta\mu\omega - [\beta\mu\omega^* + H(\omega^*)]G(\omega)\}\omega}{I\{\beta K(\omega^*) - H(\omega^*)\}f(\omega)}. \quad (2.17)$$

$$P_1(\omega) = \frac{f(1)\{-\beta[\beta\mu\omega^* + H(\omega^*)] + \beta[\beta\mu\omega + H(\omega)]\}\omega}{I\{\beta K(\omega^*) - H(\omega^*)\}f(\omega)}. \quad (2.18)$$



Equations above provide the steady state probabilities in the FMC model where queue 1 has one buffer. The cases in which queue 1 buffer size is two and three (i.e.,  $K = 2$ ,  $K = 3$ ) can be treated in the same way, although it is rather complex. We will treat these cases in next chapter.

From the equations (2.17) and (2.18), we can readily obtain expected values of several performance measures of this FMC system. The generating function of the number of materials at queue 2 can be obtained by  $P_0(\omega)$  plus  $P_1(\omega)$ , which is given as follows

$$L(\omega) = \frac{f(1)\{\beta[\mu\omega + H(\omega)] - [\beta\mu\omega^* + H(\omega^*)][\beta + G(\omega)]\}\omega}{I\{\beta K(\omega^*) - H(\omega^*)\}f(\omega)} \quad (2.19)$$

The average queue 2 length is given by

$$L_{q2} = \frac{d}{d\omega} L(\omega) \Big|_{\omega=1} \quad (2.20)$$

By use of Little's formula, we can get the average waiting time in the queue 2 which is written as

$$W_{q2} = \left\{ \frac{1 - (1 - \beta)(1 - \mu)}{\beta\mu} \right\} \cdot L_{q2} \quad (2.21)$$

where  $\beta(1 - \mu)/[1 - (1 - \beta)(1 - \mu)]$  is the blocking probability in the queue 1 and  $\beta\mu/[1 - (1 - \beta)(1 - \mu)]$  is the average number of materials to be actually entered in the queue 2 during one slot time.

## 2.5 Numerical Results and Concluding Remarks

Numerical calculation is carried out to obtain the values of  $L_{q2}$  and  $W_{q2}$  and to get some useful observations. In the numerical experiments, we change the arrival rate of materials and the service rate of the station while fixing other parameters. In other words, both parameters of  $\alpha$  and  $\gamma$  remain constant at 0.2, and the service rate of the station  $\mu$  takes on the values 0.3, 0.4, 0.5, while the arrival rate of materials  $\lambda$  varies from 0.1 to 1.0. For the FMC model with  $K = 1$ , the values of the average number of materials in queue 2 is calculated. Fig.2.2 shows such an example for  $\alpha = 0.2, \gamma = 0.2$ . The results of numerical analysis of average waiting time in queue 2 with the foregoing parameters are shown in Fig.2.3. In Figs.2.2 and 2.3, we can observe the relationship between the average performance measures and the parameter of  $\lambda$  and  $\mu$ .

In this chapter, we analyzed the FMC model in the conveyor-serviced production station, in which the first queue has one buffer size and the second queue has an infinite

buffer size. We have obtained the steady state solution described in Section 2.4, to evaluate average performance measures of the FMC model with loading and unloading robots in the conveyor-serviced production system. In general, the exact solution of FMC system conjuncted with conveyor is not ease to get.



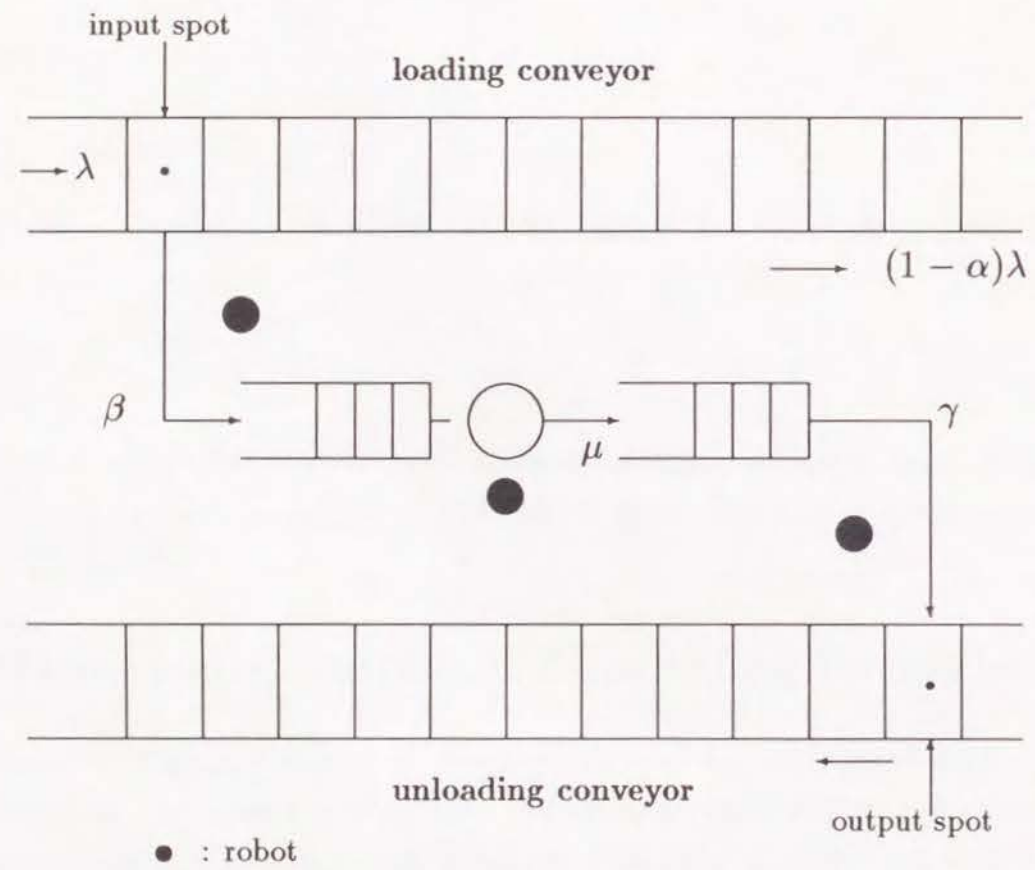
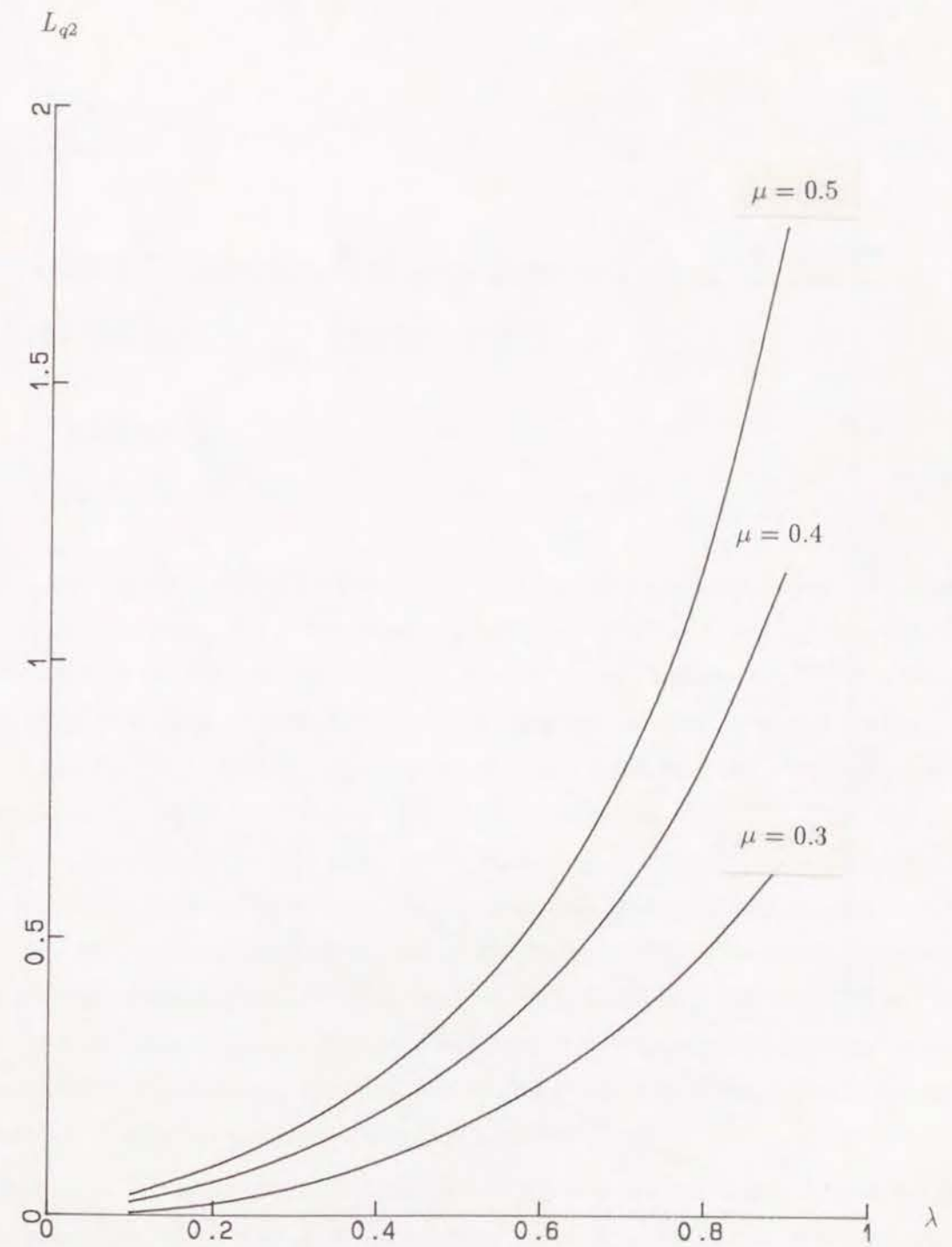


Figure 2.1: Configuration of an FMC model.

Figure 2.2: Average queue 2 length for  $\alpha = 0.2$ ,  $\gamma = 0.2$ .



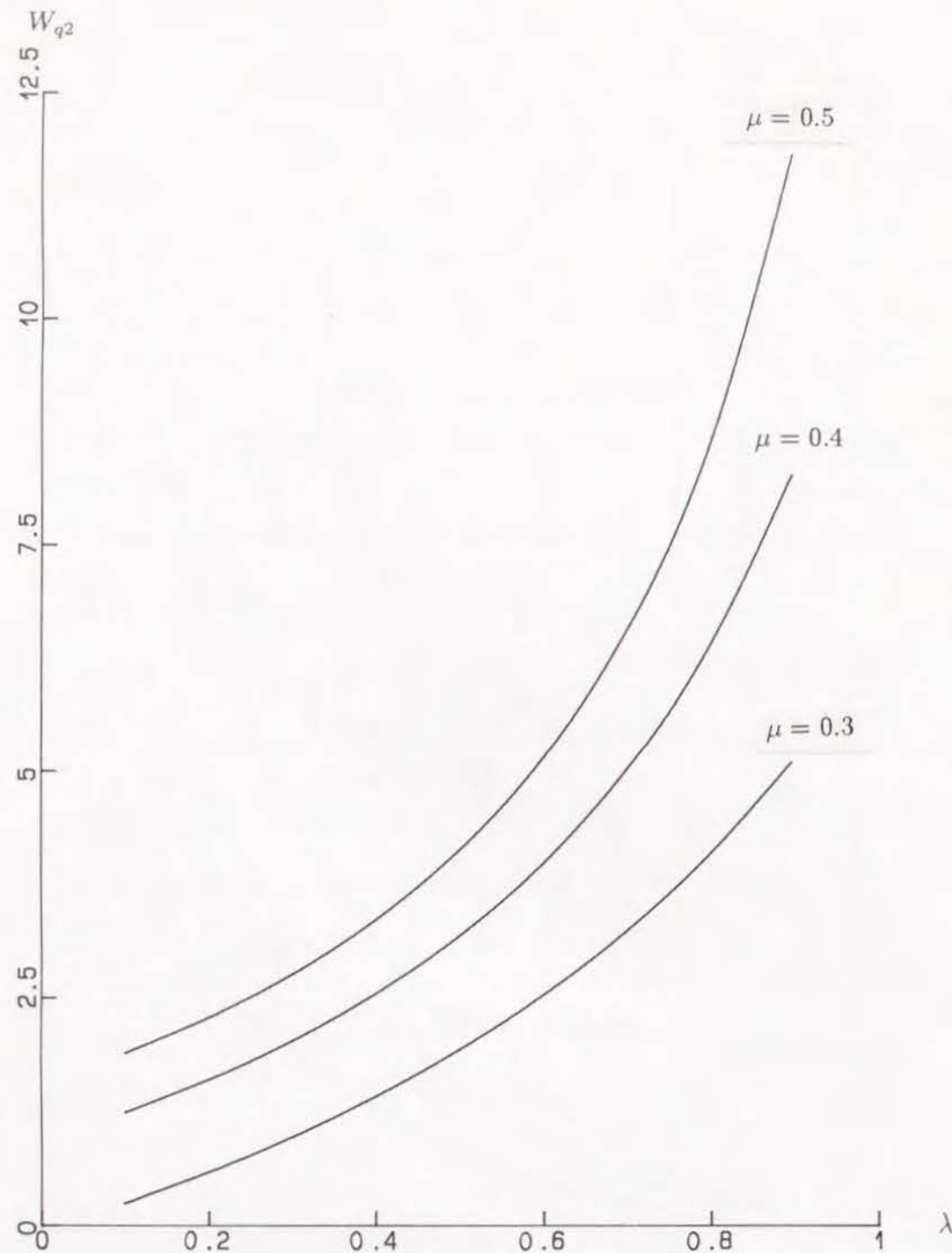


Figure 2.3: Average waiting time in queue 2 for  $\alpha = 0.2$ ,  $\gamma = 0.2$ .

## Chapter 3

# Performance Analysis of an FMC with Waiting Storage

### 3.1 Introduction

In this chapter, we consider the extension of FMC model provided in Chapter 2 in order to investigate the effect of a buffer storage capacity on FMC efficiency. In general, one way of improving the efficiency of FMS is to provide storage buffers. If an FMS is linked by an MHS that permits no storage, and a NC machine breaks down at a work station, all other stations that process materials either going to or coming from the failed station may be forced down. This disruption eventually result in affecting all work stations in FMS. Therefore, it is desire to provide some buffer storage space either in front of individual work station or the system as a whole. As a general rule, storage buffers for materials in FMS reduce the blocking delay effects caused by the variability of processing operation time, work station breakdown and the diversity of material routing. Furthermore, in FMS design, the location and capacity of storage buffers are very important parameters because they contribute to maximizing the work station utilization, and make control easier as a large number of different materials are always available.

There are two basic alternatives which can be used in providing storage buffers for the system. One alternative is local storage buffers for each station. The other is central storage buffers for the system, which is accessible by all work stations. The purpose of local buffer storage is to reduce delays while the MHS transports materials between work stations or between stations and the central storage place. In general, the local buffer storage requires the space for only one or two materials. One method of achieving central storage buffers is to consider a loop conveyor which link each station. As compared with the local buffer storage, the central buffer storage has an advantage that automatically



achieves control over the number of materials in the system. Generally, it is considered that the central buffer storage is superior to the local buffer storage.

The justification for providing storage buffers in a particular FMS requires a complete economic and technical study which takes into account the cost of providing storage buffers and the benefit of increased FMS efficiency [Buza 67]. Up to the present time, considerable research efforts were spent in investigating the behaviour of storage buffers in FMS by [Buza 78], [Conw 88], [Smit 88]. A two-stage storage buffers in tandem which was of finite capacity was analyzed by [Konh 76], and the optimal buffer storage capacity for the multi-product production line system was considered by [Ande 69]. Other researches can be found in Chapter 1 (see, Section 1.3.1).

In this chapter, we analyze an FMC system with tandem storage buffers, where the first storage is of finite capacity and the second storage is of infinite capacity, as the discrete time queueing model. The discrete time system with storage buffers in tandem was not treated extensively in the literature. The behaviour of tandem storage buffers having infinite capacity respectively in the discrete time system was studied with considering the geometric arrival and Markovian output in [Hsu 76], and the internal and external arrival sequence at the second storage buffers in [Morr 79].

In Section 3.2, we describe a mathematical model of FMC's. In Section 3.3, the procedure to analyze the FMC model is presented, and also the average performance measures are derived. It is noted that the analytical approach is based on that in Chapter 2. Numerical results are shown in Section 3.4, and finally, we conclude this work in Section 3.5.

### 3.2 Description of FMC Model

A mathematical model of FMC's is proposed and analyzed in Chapter 2. In Chapter 2, the FMC model consists of three basic components (see, Fig.2.1).

1. *A single production station* where arriving materials are processed.
2. *Two conveyors* which load and unload materials independently.
3. *Input and output buffer storages* which are located in front and rear of the production station and temporarily hold materials before and after processing. These are called queue 1 and queue 2, respectively.

In the model two conveyors are divided into unit spaces of constant size which are called cells, each cell able to hold at most one material. The loading conveyor transports materials to the station and the unloading conveyor carries away materials processed at the station. Arriving materials leave the loading conveyor, wait in queue 1, receive a production service, wait in queue 2, and are put on the unloading conveyor. Materials to be processed elsewhere stay on the loading conveyor. After being processed, the materials to be get back into the unloading conveyor must wait for an empty cell arriving at the output spot.

In Chapter 2, the mathematical model was presented on the assumption that the one cell for passing through the input spot requires one unit-time called a slot. The number of arriving materials at the input spot in different time slots are assumed to be statistically independent. The length of processing service time for each material is drawn from a geometric distribution with mean  $1/\mu$ . Service times are independent of all other random variables in the FMC model. In order to get back into the unloading conveyor, each material at queue 2 must wait for an empty cell at the output spot. The following notations can be defined.

$\lambda$  : the probability that a cell carries a material.

$\alpha$  : the probability that a material arriving at the input spot enters queue 1.

$\beta$  : the probability that a new material arrives at the input spot and enters queue 1 (namely,  $\beta = \alpha\lambda$ ).

$\gamma$  : the probability that the material waiting at the queue 2 is placed on the unloading conveyor.

In the FMC model described in Chapter 2, the first storage can keep at most one material and the second storage is of infinite capacity. In this chapter, we consider a more general FMC model. That is, queue 1 has a buffer of finite capacity and queue 2 has a buffer of infinite capacity. The stochastic behaviour of the FMC model is also described by a Markov chain with the stationary state probabilities denoted by  $\pi(i, j)$  for two queues in tandem ( $i = 0, 1, \dots, K, j = 0, 1, \dots$ ), where  $i$  and  $j$  indicate the numbers of materials present at queue 1 and queue 2, respectively. We apply the queueing theory to analyze the system and evaluate its performance measures such as the average queue length and the average waiting time in the FMC system.



### 3.3 Analysis

In this section, we present the exact analysis of the FMC model in the conveyor production system, where queue 1 is of finite capacity  $K$ . We derive a set of balance equations describing the stochastic behaviour of this FMC model under the steady state condition as follows.

$$\pi(0,0) = (1-\beta)\pi(0,0) + (1-\beta)\gamma\pi(0,1) \quad (3.1)$$

$$\begin{aligned} \pi(0,1) = & (1-\beta)(1-\gamma)\pi(0,1) + (1-\beta)\gamma\pi(0,2) \\ & + (1-\beta)\mu\pi(1,0) + (1-\beta)\mu\gamma\pi(1,1) \end{aligned} \quad (3.2)$$

$$\begin{aligned} \pi(1,0) = & \beta\pi(0,0) + \beta(1-\mu)\gamma\pi(0,1) \\ & + (1-\beta)(1-\mu)\pi(1,0) + (1-\beta)(1-\mu)\gamma\pi(1,1) \end{aligned} \quad (3.3)$$

$$\begin{aligned} \pi(i,0) = & \beta(1-\mu)\pi(i-1,0) + \beta(1-\mu)\gamma\pi(i-1,1) \\ & + (1-\beta)(1-\mu)\pi(i,0) + (1-\beta)(1-\mu)\gamma\pi(i,1) \quad 2 \leq i \leq K-1 \end{aligned} \quad (3.4)$$

$$\begin{aligned} \pi(K,0) = & \beta(1-\mu)\gamma\pi(K-1,1) + \beta(1-\mu)\pi(K-1,0) \\ & + (1-\beta)(1-\mu)\gamma\pi(K,1) + (1-\beta)(1-\mu)\pi(K,0) \end{aligned} \quad (3.5)$$

$$\begin{aligned} \pi(0,j) = & (1-\beta)(1-\gamma)\pi(0,j) + (1-\beta)\gamma\pi(0,j+1) \\ & + (1-\beta)\mu(1-\gamma)\pi(1,j-1) + (1-\beta)\mu\gamma\pi(1,j) \quad j \geq 2 \end{aligned} \quad (3.6)$$

$$\begin{aligned} \pi(i,j) = & \beta(1-\mu)(1-\gamma)\pi(i-1,j) + \beta(1-\mu)\gamma\pi(i-1,j+1) \\ & + \beta\mu(1-\gamma)\pi(i,j-1) + [\beta\mu\gamma + (1-\beta)(1-\mu)(1-\gamma)]\pi(i,j) \\ & + (1-\beta)(1-\mu)\gamma\pi(i,j+1) + (1-\beta)\mu(1-\gamma)\pi(i+1,j-1) \\ & + (1-\beta)\mu\gamma\pi(i+1,j) \quad 1 \leq i \leq K-1, \quad j \geq 1 \end{aligned} \quad (3.7)$$

$$\begin{aligned} \pi(K,j) = & \beta(1-\mu)\gamma\pi(K-1,j+1) + \beta(1-\mu)(1-\gamma)\pi(K-1,0) \\ & + (1-\beta)(1-\mu)\gamma\pi(K,j+1) + \beta\mu(1-\gamma)\pi(K,j-1) \\ & + \{\beta\mu\gamma + (1-\beta)(1-\mu)(1-\gamma)\}\pi(K,j). \quad j \geq 1 \end{aligned} \quad (3.8)$$

Then, we can obtain the explicit expressions for the probability generating functions of the aforementioned equilibrium probabilities.

$$P_i(\omega) = \sum_{j=0}^{\infty} \pi(i,j)\omega^j, \quad 0 \leq i \leq K. \quad (3.9)$$

$P_i(\omega)$ 's ( $0 \leq i \leq K$ ) are analytic functions in the interior of the unit disk  $|\omega| \leq 1$ . The set of equations of probability generating functions can be straightforwardly calculated from

equations (3.1) to (3.8).

$$\begin{aligned} [1 - (1-\beta)C(\omega)]P_0(\omega) - (1-\beta)\mu\omega C(\omega)P_1(\omega) \\ = (1-\beta)[1 - C(\omega)]P_0(0) + (1-\beta)\mu\omega[1 - C(\omega)]P_1(0) \end{aligned} \quad (3.10)$$

$$\begin{aligned} \beta C(\omega)P_0(\omega) - \{1 - [(1-\beta)(1-\mu) + \beta\mu\omega]C(\omega)\}P_1(\omega) \\ + (1-\beta)\mu\omega C(\omega)P_2(\omega) \\ = -\beta[1 - C(\omega)]P_0(0) - [(1-\beta)(1-\mu) + \beta\mu\omega][1 - C(\omega)]P_1(0) \\ - (1-\beta)\mu\omega[1 - C(\omega)]P_2(0) \end{aligned} \quad (3.11)$$

$$\begin{aligned} \beta(1-\mu)C(\omega)P_{i-1}(\omega) - \{1 - [(1-\beta)(1-\mu) + \beta\mu\omega]C(\omega)\}P_i(\omega) \\ + (1-\beta)\mu\omega C(\omega)P_{i+1}(\omega) \\ = -\beta(1-\mu)[1 - C(\omega)]P_{i-1}(0) - [(1-\beta)(1-\mu) + \beta\mu\omega][1 - C(\omega)]P_i(0) \\ - (1-\beta)\mu\omega[1 - C(\omega)]P_{i+1}(0) \quad 2 \leq i \leq K-1 \end{aligned} \quad (3.12)$$

$$\begin{aligned} \beta(1-\mu)C(\omega)P_{K-1}(\omega) - \{1 - [1 - \mu + \beta\mu\omega]C(\omega)\}P_K(\omega) \\ = -\beta(1-\mu)[1 - C(\omega)]P_{K-1}(0) - (1-\mu + \beta\mu\omega)[1 - C(\omega)]P_K(0) \end{aligned} \quad (3.13)$$

where  $C(\omega) = 1 - \gamma + \gamma/\omega$ .

It is readily seen that the equations (3.10) to (3.13) can be simply rewritten by using matrix representation.

$$\mathbf{G}(\omega)\vec{P}(\omega) = \gamma(\omega-1)\mathbf{B}(\omega)\vec{P}(0) \quad (3.14)$$

where

$$\vec{P}(\omega) = (P_0(\omega), \dots, P_K(\omega))$$

$$\mathbf{G}(\omega) = \begin{bmatrix} \delta(\omega) & \zeta(\omega) & & & 0 \\ \eta(\omega) & \theta(\omega) & \zeta(\omega) & & \\ & \kappa(\omega) & \theta(\omega) & \zeta(\omega) & \\ & & \ddots & \ddots & \ddots \\ & & & \ddots & \ddots & \zeta(\omega) \\ 0 & & & & \kappa(\omega) & \epsilon(\omega) \end{bmatrix}$$



$$\begin{aligned}
\delta(\omega) &= [1 - (1 - \beta)(1 - \gamma)]\omega - (1 - \beta)\gamma \\
\zeta(\omega) &= -(1 - \beta)\mu(1 - \gamma)\omega^2 - (1 - \beta)\mu\gamma\omega \\
\eta(\omega) &= -\beta(1 - \gamma)\omega - \beta\gamma \\
\theta(\omega) &= -\beta\mu(1 - \gamma)\omega^2 + [1 - (1 - \beta)(1 - \mu)(1 - \gamma) - \beta\mu\gamma]\omega - (1 - \beta)(1 - \mu)\gamma \\
\kappa(\omega) &= -\beta(1 - \mu)(1 - \gamma)\omega - \beta(1 - \mu)\gamma \\
\epsilon(\omega) &= -\beta\mu(1 - \gamma)\omega^2 + [1 - (1 - \mu)(1 - \gamma) - \beta\mu\gamma]\omega - (1 - \mu)\gamma
\end{aligned}$$

$$\mathbf{B}(\omega) = \begin{bmatrix} (1 - \beta) & \chi(\omega) & & & & & & & & 0 \\ \beta & \phi(\omega) & \chi(\omega) & & & & & & & \\ & \beta(1 - \mu) & \phi(\omega) & \chi(\omega) & & & & & & \\ & & \ddots & \ddots & \ddots & & & & & \\ & & & \ddots & \ddots & \ddots & & & & \\ 0 & & & & & & \beta(1 - \mu) & \psi(\omega) & \chi(\omega) & \end{bmatrix}$$

$$\begin{aligned}
\chi(\omega) &= (1 - \beta)\mu\omega \\
\phi(\omega) &= (1 - \beta)(1 - \mu) + \beta\mu\omega \\
\psi(\omega) &= 1 - \mu + \beta\mu\omega
\end{aligned}$$

The determinant and the adjoint of the matrix  $\mathbf{G}(\omega)$  are denoted by  $|\mathbf{G}(\omega)|$ ,  $\hat{\mathbf{G}}(\omega)$  respectively. We make use of the following iterative equations to compactly represent the expression of  $|\mathbf{G}(\omega)|$  and of entries in  $\hat{\mathbf{G}}(\omega)$ .  $H_i(\omega)$ ,  $F_i(\omega)$  and  $D_i(\omega)$  ( $i = -2, \dots, K - 2$ ) are the functions of satisfying the recursive equations.

1.  $H_i(\omega)$ 's satisfy

$$\begin{aligned}
H_i(\omega) &= 1 & i < -1 \\
H_i(\omega) &= \epsilon(\omega) & i = -1 \\
H_i(\omega) &= H_{i-1}(\omega) \cdot \theta(\omega) - H_{i-2}(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega) & 2 \leq i \leq K - 2
\end{aligned}$$

2.  $F_i(\omega)$ 's satisfy

$$\begin{aligned}
F_i(\omega) &= 1 & i = -2 \\
F_i(\omega) &= \delta(\omega) & i = -1 \\
F_i(\omega) &= F_{i-1}(\omega) \cdot \theta(\omega) - F_{i-2}(\omega) \cdot \zeta(\omega) \cdot \eta(\omega) & i = 0 \\
F_i(\omega) &= F_{i-1}(\omega) \cdot \theta(\omega) - F_{i-2}(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega) & 1 \leq i \leq K - 2
\end{aligned}$$

3.  $D_i(\omega)$ 's satisfy

$$\begin{aligned}
D_i(\omega) &= 1 & i = -2 \\
D_i(\omega) &= \theta(\omega) & i = -1 \\
D_i(\omega) &= D_{i-1}(\omega) \cdot \theta(\omega) - D_{i-2}(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega) & 0 \leq i \leq K - 3.
\end{aligned}$$

Therefore, the equation (3.14) can be expressed as,

$$\vec{P}(\omega) = \frac{\hat{\mathbf{G}}(\omega)}{|\mathbf{G}(\omega)|} \gamma(\omega - 1) \mathbf{B}(\omega) \vec{P}(0) \quad (3.15)$$

where

$$|\mathbf{G}(\omega)| = H_K(\omega) \cdot \delta(\omega) - H_{K-2}(\omega) \cdot \zeta(\omega) \cdot \eta(\omega), \quad K \geq 1. \quad (3.16)$$

$$\hat{\mathbf{G}}(\omega) = \begin{bmatrix} g_{00} & g_{10} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & g_{K0} \\ g_{01} & g_{11} & g_{21} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & g_{12} & g_{22} & g_{32} & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & g_{i-1j} & g_{ij} & g_{i+1j} & \cdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & g_{K-2K-1} & g_{K-1K-1} & g_{KK-1} \\ g_{0K} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & g_{K-1K} & g_{KK} \end{bmatrix}.$$

The entries of  $\hat{\mathbf{G}}(\omega)$  are given by

if  $i = j$

$$\begin{aligned}
g_{ii} &= (-1)^{i+j} F_{i-2}(\omega) \cdot \{D_{K-i-3}(\omega) \cdot \epsilon(\omega) - D_{K-i-4}(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega)\} \\
& 1 \leq i \leq K - 2, \quad K \geq 3
\end{aligned}$$

if  $i \neq j$

$$\begin{aligned}
g_{ij} &= (-1)^{i+j} F_{r-2}(\omega) \cdot \zeta^m(\omega) \cdot \kappa^p(\omega) \cdot \{D_{K-q-3}(\omega) \cdot \epsilon(\omega) - D_{K-q-4}(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega)\} \\
& 1 \leq i \leq K - 2, \quad 1 \leq j \leq K - 2, \quad K \geq 4
\end{aligned}$$

where

$$m = \frac{1}{2} \{(i - j) + |i - j|\}, \quad p = \frac{1}{2} \{(j - i) + |j - i|\}$$

$$q = \max\{i, j\}, \quad r = \min\{i, j\}.$$



We obtain the other entries of the matrix  $\hat{\mathbf{G}}(\omega)$  as follows

$$\begin{aligned}
g_{i0} &= (-1)^i \zeta^i(\omega) \{D_{K-i-3}(\omega) \cdot \epsilon(\omega) - D_{K-i-4}(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega)\} \\
&\quad 0 \leq i \leq K-2 \quad K \geq 2 \\
g_{0j} &= (-1)^j \eta(\omega) \cdot \kappa^{j-1}(\omega) \{D_{K-j-3}(\omega) \cdot \epsilon(\omega) - D_{K-j-4}(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega)\} \\
&\quad 1 \leq j \leq K-2 \quad K \geq 3 \\
g_{iK-1} &= (-1)^{i+K-1} F_{i-2}(\omega) \cdot \kappa^{K-i-1}(\omega) \cdot \epsilon(\omega) \\
&\quad 1 \leq i \leq K-1 \quad K \geq 2 \\
g_{iK} &= (-1)^{i+K} F_{i-2}(\omega) \cdot \kappa^{K-i}(\omega) \\
&\quad 1 \leq i \leq K \quad K \geq 1 \\
g_{K-1j} &= (-1)^{K-1+j} F_{j-2}(\omega) \cdot \zeta^{K-j-1}(\omega) \cdot \epsilon(\omega) \\
&\quad 1 \leq j \leq K-2 \quad K \geq 3 \\
g_{Kj} &= (-1)^{K+j} F_{j-2}(\omega) \cdot \zeta^{K-j}(\omega) \\
&\quad 1 \leq j \leq K-1 \quad K \geq 2 \\
g_{K-10} &= (-1)^{K-1} \zeta^{K-1}(\omega) \cdot \epsilon(\omega) \quad K \geq 1 \\
g_{0K-1} &= (-1)^{K-1} \eta(\omega) \cdot \kappa^{K-2}(\omega) \cdot \epsilon(\omega) \quad K \geq 2 \\
g_{K0} &= (-1)^K \zeta^K(\omega) \quad K \geq 1 \\
g_{0K} &= (-1)^K \eta(\omega) \cdot \kappa^{K-1}(\omega) \quad K \geq 1.
\end{aligned}$$

These are proved in Appendix A.

We denote  $f(\omega)$  as the equation by dividing  $|\mathbf{G}(\omega)|$  with  $(\omega - 1)$ . Then, the equation (3.14) is rewritten as

$$\vec{P}(\omega) = \frac{\gamma}{f(\omega)} \hat{\mathbf{G}}(\omega) \mathbf{B}(\omega) \vec{P}(0). \quad (3.17)$$

Since  $P_i(\omega)$ 's are analytic in the unit disk  $|\omega| \leq 1$ , the numerator must equal zero at the roots of denominator  $f(\omega)$ . Accordingly, the unknown  $\vec{P}(0)$  is determined. We apply Rouché's theorem to the equation (3.17) in order to find the roots. By substituting 1 and  $\omega_i^*$ 's which are the roots for the denominator in the equation (3.17), we get a set of equations to determine the state probability vector  $\vec{P}(0)$ . When  $\omega = 1$ , we have one linear equation by using the equation  $\sum_{i=0}^K P_i(1) = 1$  as

$$\frac{\gamma}{f(1)} \hat{\mathbf{G}}(1) \mathbf{B}(1) \vec{P}(0) \cdot \mathbf{e} = 1 \quad (3.18)$$

where  $\mathbf{e} = (1, \dots, 1)^t$ .

Another  $K$  equations can be derived from the equation (3.17) by substituting  $\omega = \omega_i^*$ . We note that additional linear equations can be in principle written as

$$\hat{\mathbf{L}}(\omega_i^*) \vec{P}(0) = (0, \dots, 0)^t \quad (3.19)$$

where  $\hat{\mathbf{L}}(\omega_i^*)$  is the matrix obtained by substituting  $\omega = \omega_i^*$  into  $\gamma \hat{\mathbf{G}}(\omega) \mathbf{B}(\omega) \vec{P}(0)$ . Therefore, solving these linear equations obtained from the equations (3.18) and (3.19), it is not difficult to obtain the values of probability vector  $\vec{P}(0)$ . Then, substituting these values to the equation (3.17), we have the equations to provide the steady state probability generating functions in the FMC model where queue 1 is of finite capacity and queue 2 is of infinite capacity.

From the steady state probability generating functions, the average performance measures for this FMC model are obtained. The average number of materials at queue 1 and at queue 2 are given by

$$L_{q1} = \sum_{i=1}^K (i-1) P_i(1) \quad (3.20)$$

$$L_{q2} = \left. \frac{d}{d\omega} Q(\omega) \right|_{\omega=1} \quad (3.21)$$

where  $Q(\omega) = \sum_{i=0}^K P_i(\omega)$ .

From Little's formula, the average waiting time in queue 1 and queue 2 are presented as

$$W_{q1} = \left\{ \frac{L_{q1}}{\beta[1 - (1-\mu)P_K(1)]} \right\} \quad (3.22)$$

$$W_{q2} = \left\{ \frac{L_{q2}}{\beta[1 - (1-\mu)P_K(1)]} \right\} \quad (3.23)$$

where  $(1-\mu)P_K(1)$  is the blocking probability in queue 1 and  $\beta[1 - (1-\mu)P_K(1)]$  is the probability of materials actually entering at queue 1.

### 3.4 Numerical Results

We carry out the numerical calculation and get some useful observations. We consider that in most existing systems, the FMC system probably has small buffer to keep one or two waiting materials. Hence, we focus on the average performance measures for the value of a small buffer size of queue 1. Especially, we concentrate our attention on the



case which the buffer size of queue 1 is two and three (i.e.  $K = 2$ ,  $K = 3$ ). The numerical results of  $K = 1$  have been provided in Chapter 2.

In the numerical experiments, we employed the parameters which were setted in Chapter 2. That is, both parameters of  $\alpha$  and  $\gamma$  remain constant at 0.2, and  $\mu$  takes on the values 0.3, 0.4, 0.5, while  $\lambda$  varies from 0.1 to 1.0. For  $K = 2$ , measures of effectiveness for this FMC model, the values of the average number of materials in queue 1 and queue 2, has been respectively calculated. Figs.3.1 and 3.2 show such an example for  $K = 2$ ,  $\alpha = 0.2$ ,  $\gamma = 0.2$ . In Figs.3.1 and 3.2, the relationship between the average queue length and the parameter of  $\lambda$  and  $\mu$  can be observed. The results of numerical analysis of average spent time in queue 1 and in queue 2 for the case with the foregoing parameters are respectively shown in Figs.3.3 and 3.4. We can get some useful observations from Figs.3.3 and 3.4.

In Figs.3.5 and 3.6,  $L_{q1}$  and  $L_{q2}$  are drawn versus  $\lambda$  and  $\mu$  for the case  $K = 3$ ,  $\alpha = 0.2$ ,  $\gamma = 0.2$ . In Fig.3.5, we can easily observe that  $L_{q1}$  for  $\mu = 0.3$  is very sensitive to the arrival rate of materials in the input spot. The numerical results for the average waiting time are presented in Figs.3.7 and 3.8. As can be seen in Fig.3.7, it is evident that the average waiting time in queue 1 is changed of a great deal for each of  $\mu$ . When  $\mu = 0.3$ , we can observe that the average waiting time in queue 1 becomes very large as the value of  $\lambda$  becomes large. In Fig.3.8, observing intuitively, when  $\lambda = 1.0$ , the average waiting time in queue 1 for  $\mu = 0.5$  is very longer than the others.

A brief discussion in the case of  $K = 2$  and  $K = 3$  presents the difference of the performance measures. That is, comparing the average performance measures, the performance of  $K = 2$  is intuitively inferior to the performance of  $K = 3$ . However, when the parameter  $\lambda$  approaches lower values, the performance for the two case largely behave in the same way. These results are supported by the weight of considerable computer calculations, and provide a clear picture of important effects.

### 3.5 Concluding Remarks

In this chapter, we analyzed the FMC model with two waiting storage buffers, wherein the first buffer is of finite capacity and the second buffer is of infinite capacity. We derived stationary probability generating functions for the FMC model. Numerical calculation was performed in order to evaluate the average performance measures of the FMC model such as the average queue length and the average waiting time. The numerical results showed the relation between the average performance measures and buffer size of queue

1.

In order to construct the optimal FMC system, the performance of various FMC models must be exactly analyzed and evaluated. Undoubtedly, the cost concerned must be considered. It was shown that operating costs of an FTL system are influenced by the capacity of storage buffers for work-in-process [Ande 69]. In general, the rigorous solution for the discrete time FMC system with tandem storage buffers is not easily obtained. Although the FMC model we studied here is the simplest one, it can be still improved.



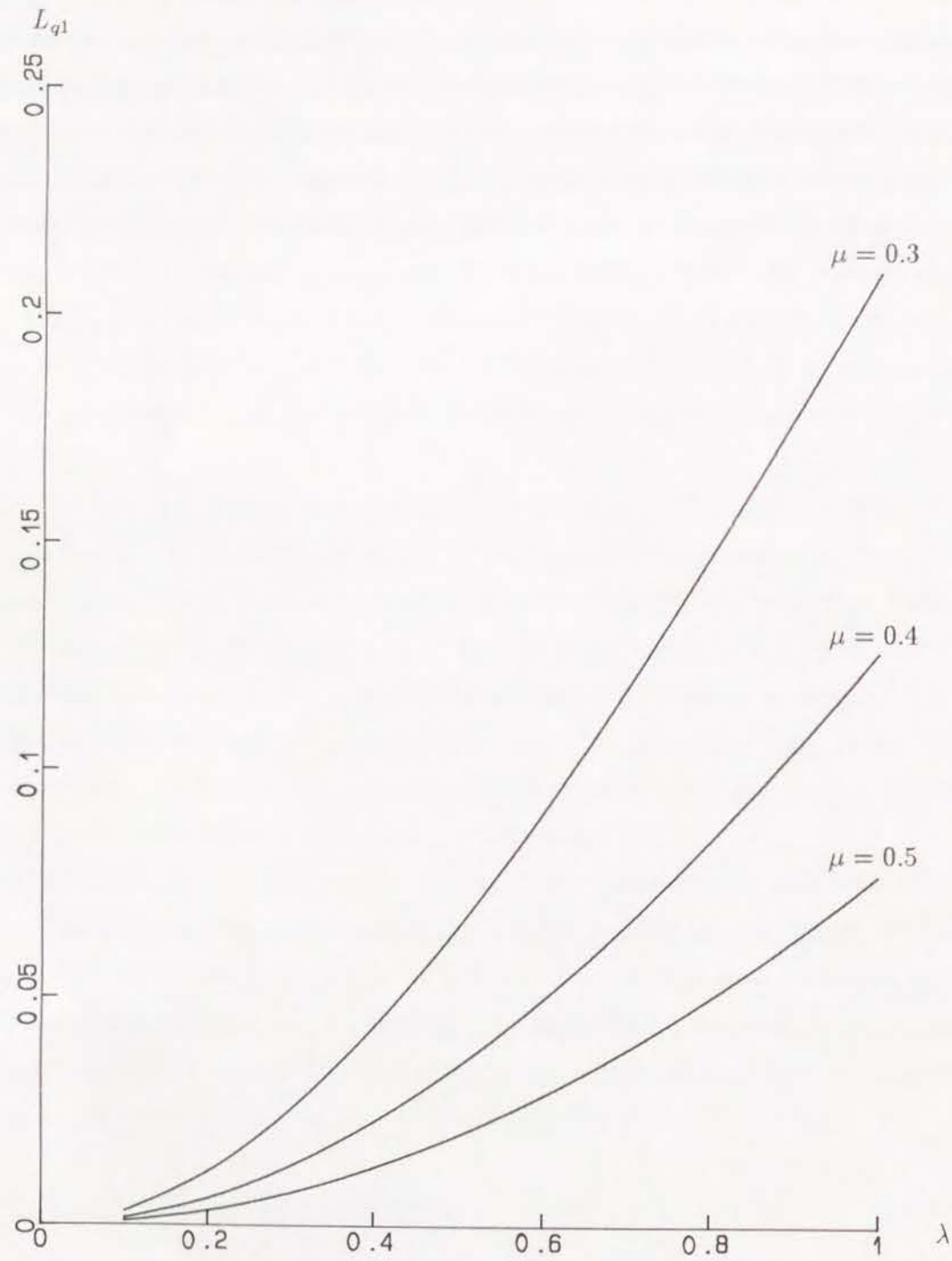


Figure 3.1: Average queue 1 length for  $K = 2$ ,  $\alpha = 0.2$ ,  $\gamma = 0.2$ .

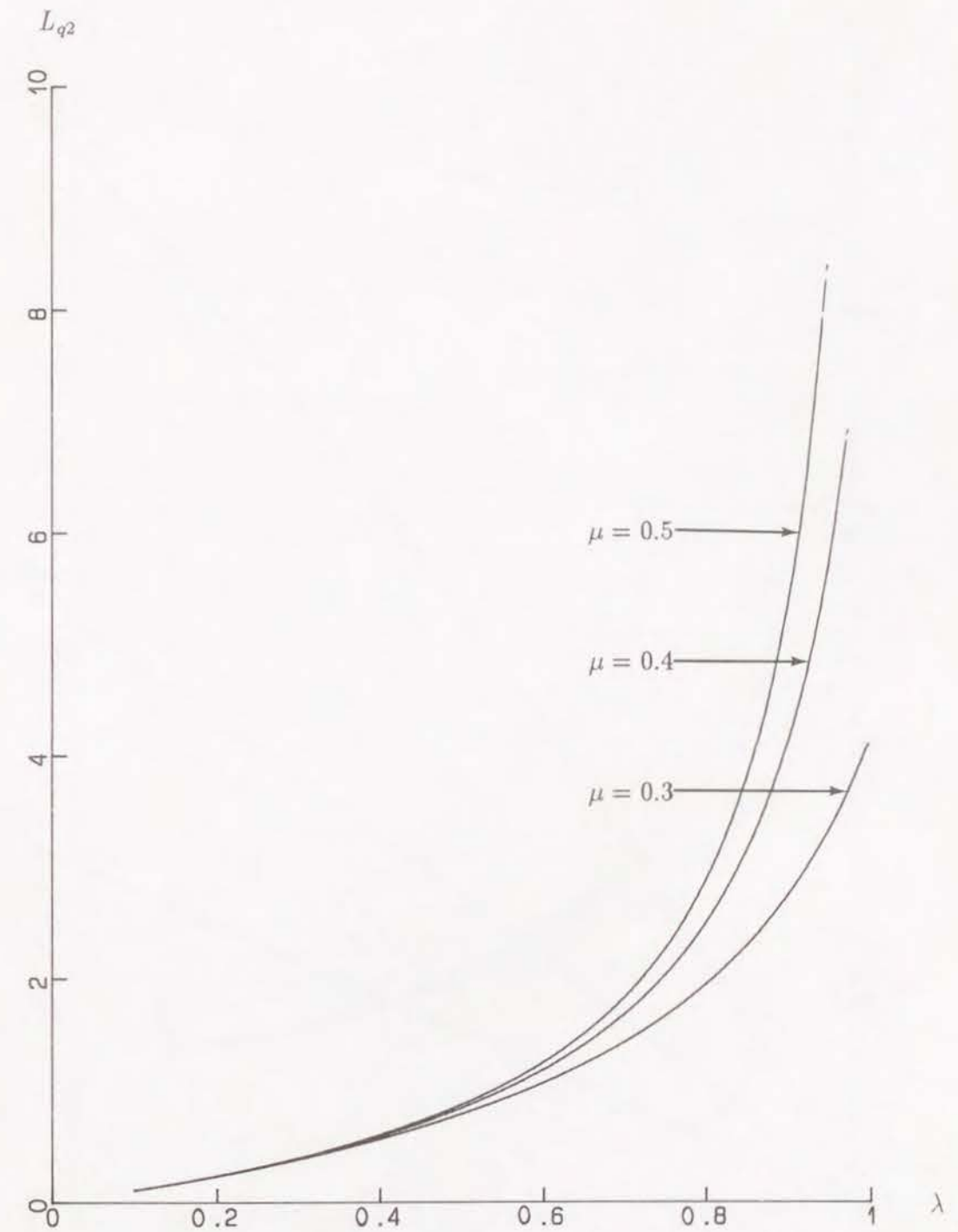


Figure 3.2: Average queue 2 length for  $K = 2$ ,  $\alpha = 0.2$ ,  $\gamma = 0.2$ .



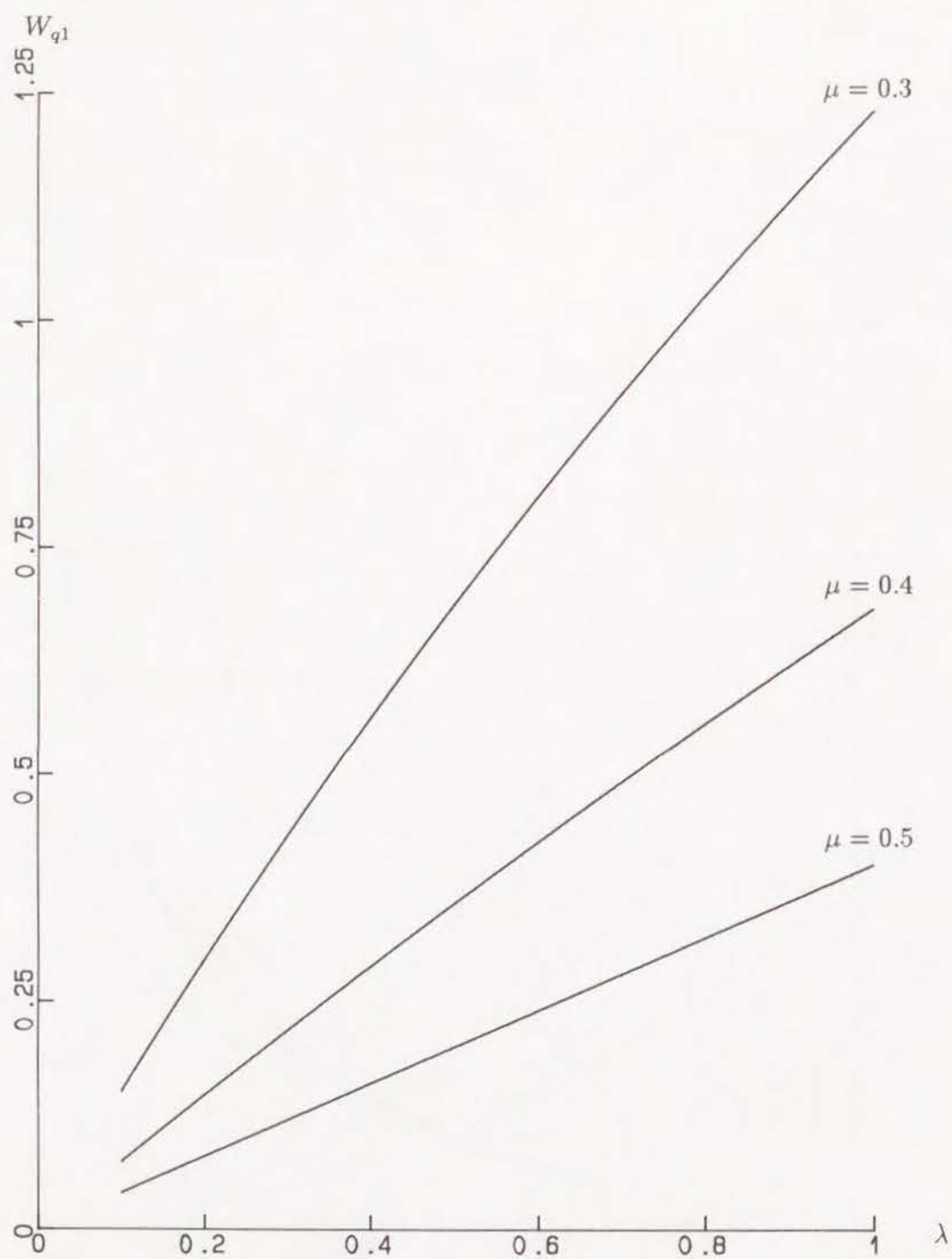


Figure 3.3: Average waiting time in queue 1 for  $K = 2$ ,  $\alpha = 0.2$ ,  $\gamma = 0.2$ .

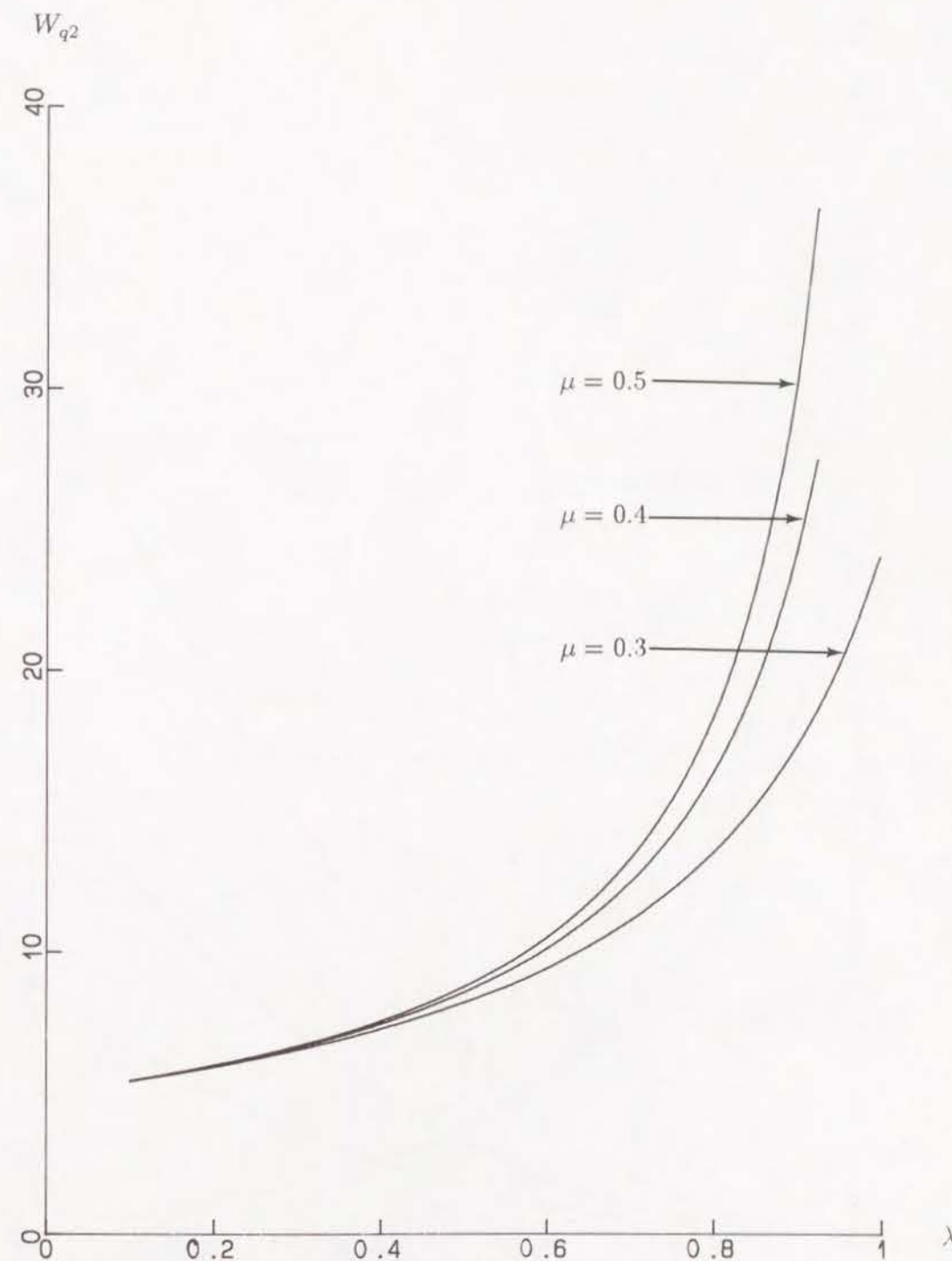


Figure 3.4: Average waiting time in queue 2 for  $K = 2$ ,  $\alpha = 0.2$ ,  $\gamma = 0.2$ .



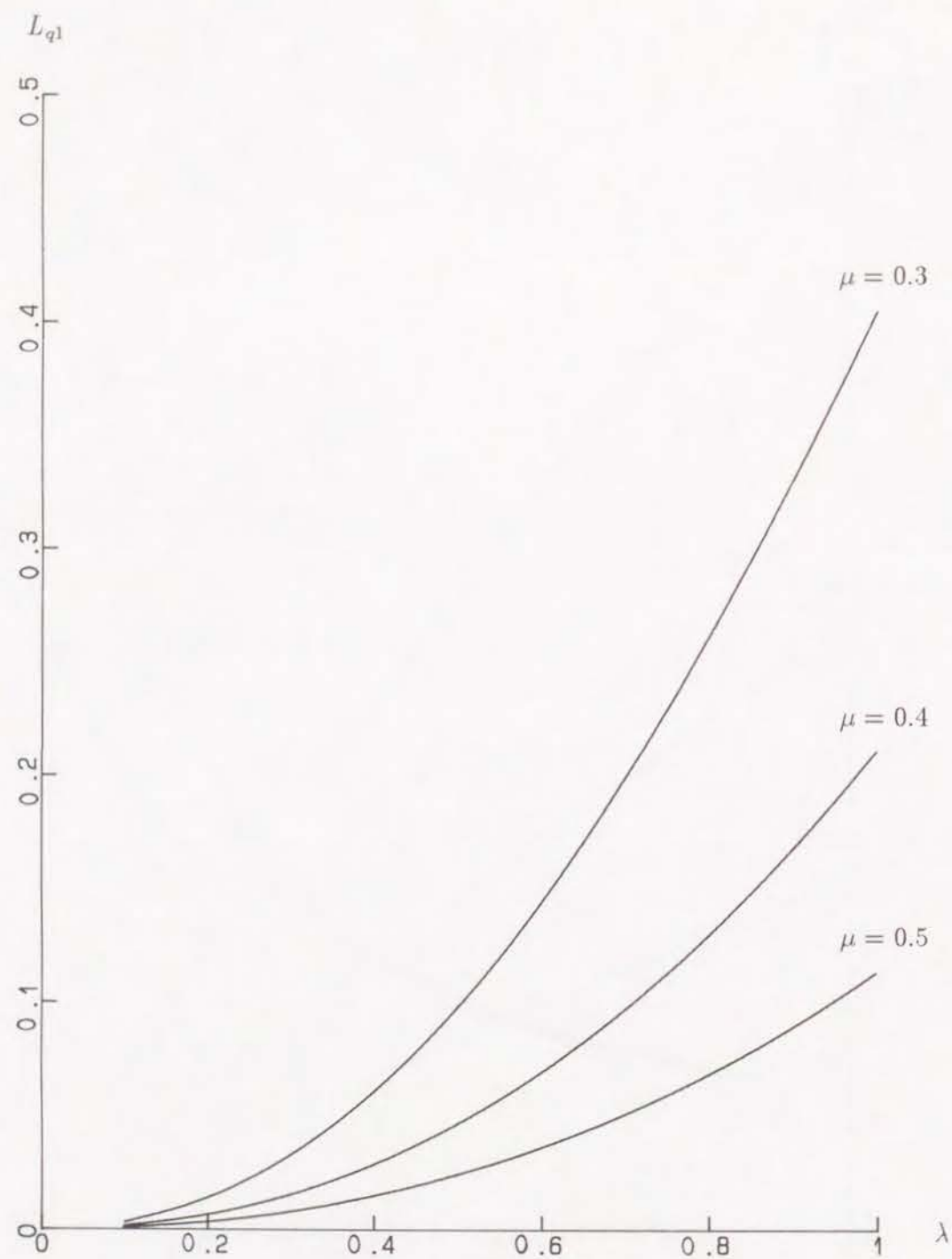


Figure 3.5: Average queue 1 length for  $K=3$ ,  $\alpha=0.2$ ,  $\gamma=0.2$ .

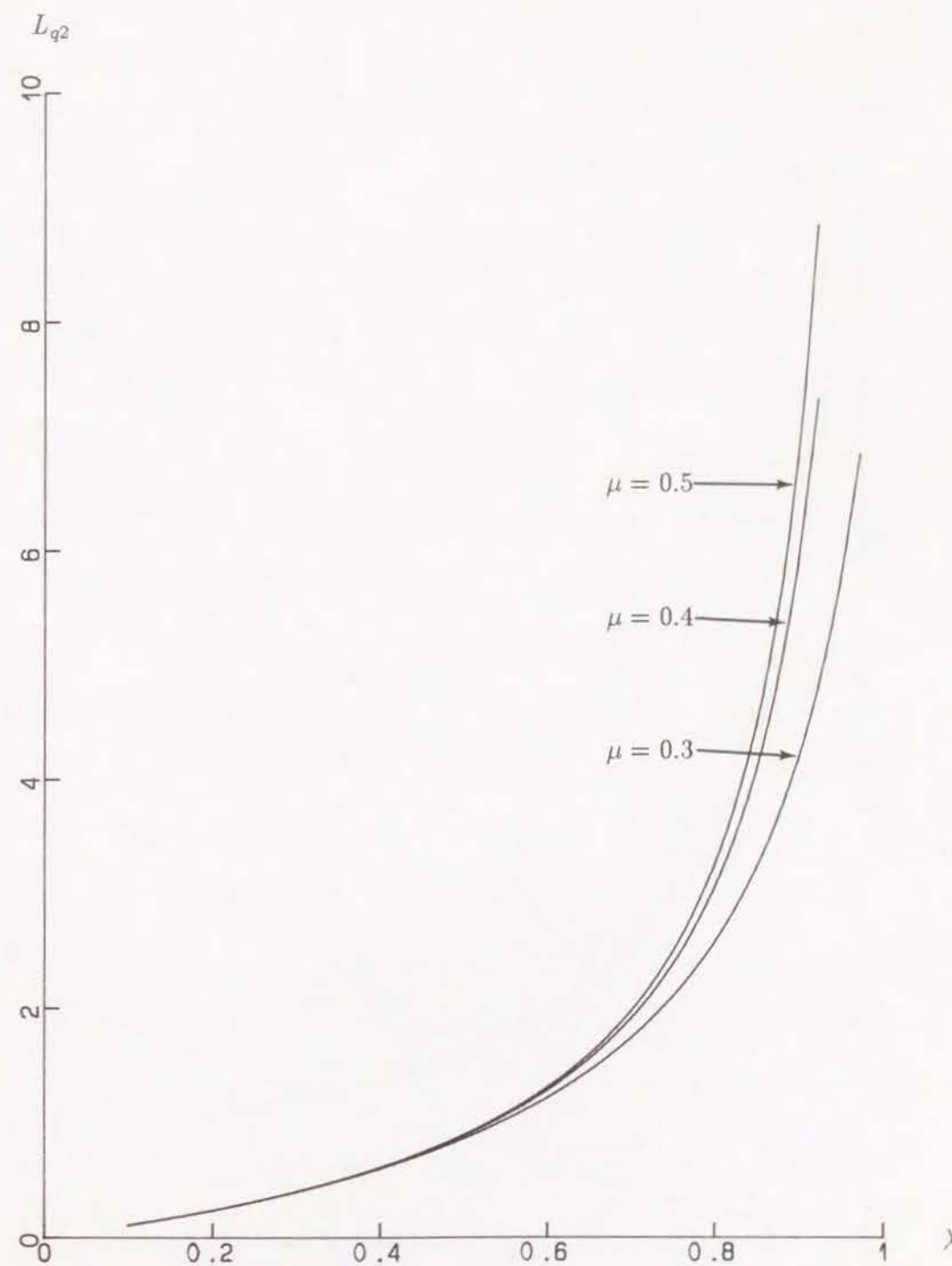


Figure 3.6: Average queue 2 length for  $K=3$ ,  $\alpha=0.2$ ,  $\gamma=0.2$ .



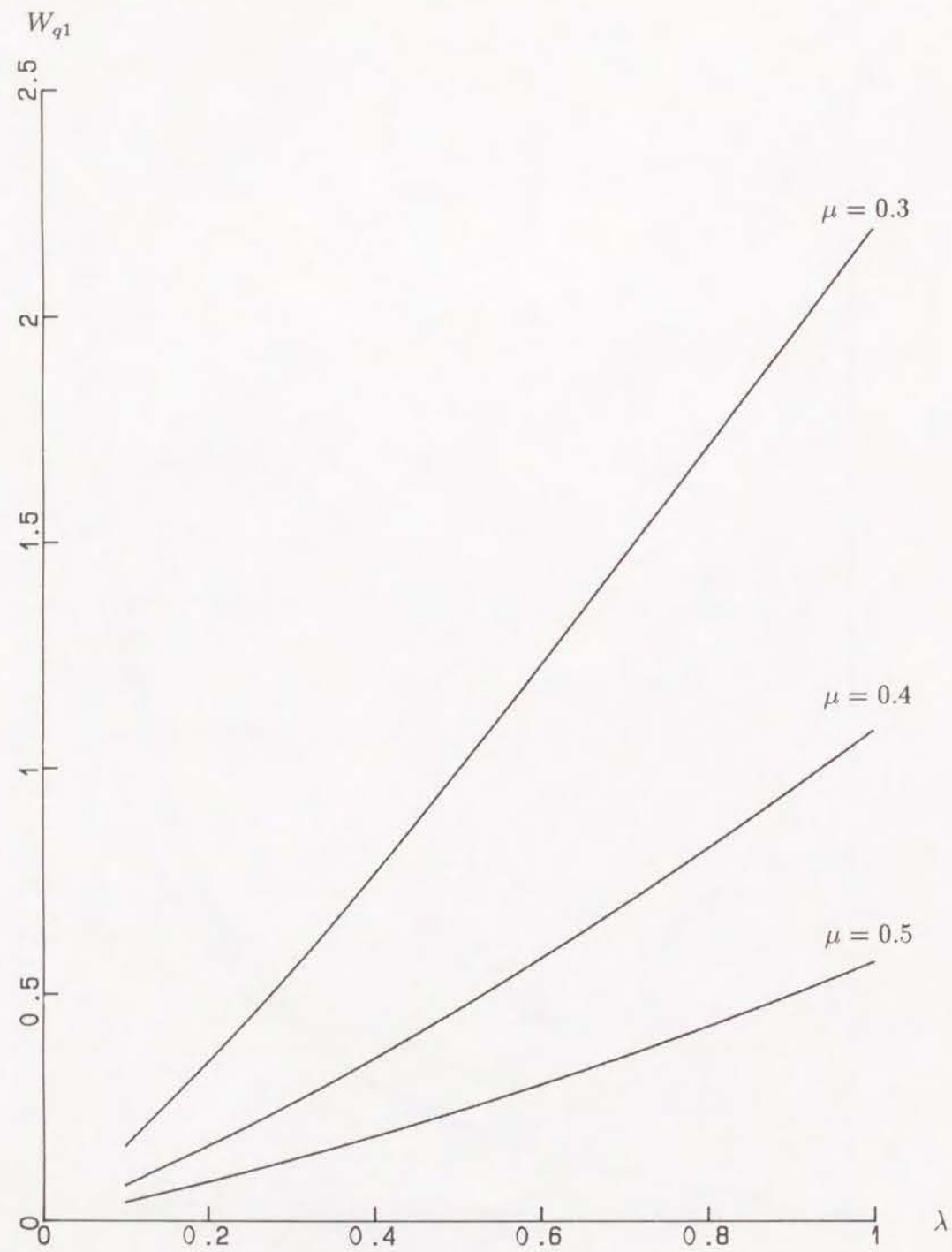


Figure 3.7: Average waiting time in queue 1 for  $K = 3$ ,  $\alpha = 0.2$ ,  $\gamma = 0.2$ .

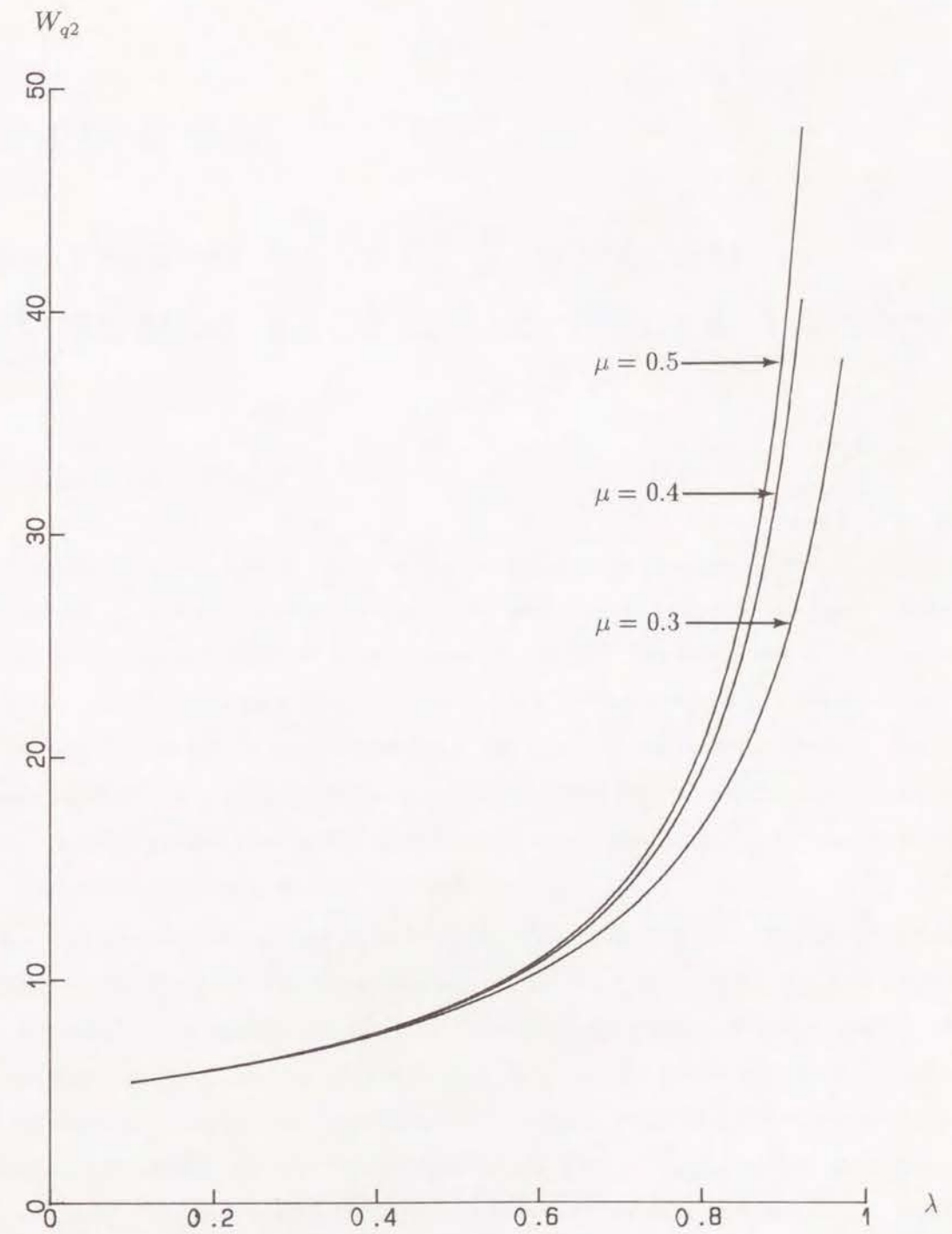


Figure 3.8: Average waiting time in queue 2 for  $K = 3$ ,  $\alpha = 0.2$ ,  $\gamma = 0.2$ .



## Chapter 4

# Analysis of an FMC with an Automatic Tool Interchange Device

### 4.1 Introduction

As reliability, rigidity and accuracy of the tooling system guaranteed excellent results in NC machines, it was proved that automatic tool changing in NC machines showed its worth in numerous industrial applications [Thom 83]. An automatic tool changing equipment can be easily integrated not only in FMC but also in many existing machine tools. In some literatures, it was assumed that the time needed to switch tools at flexible machines with ATC is negligibly small. However, it seems that in reality a tool switching time significantly affects the system performance and becomes a major aspect of the material scheduling problem of FMC's as well as FMS's.

There are some papers treating tool switching time. Sahney [Sahn 72] studied a static sequencing problem considering the switching time in two parallel machines with a single server. He developed a branch-and-bound procedure to solve the problem for minimizing the mean flow time of materials. Buzacott and Gupta [Buza 86b] considered a network model where each material visits a machine no more than once, and investigated the impact of different scheduling rules at the flexible machines taking into account constant switching times. They compared the approximation results with the simulation results, and discussed the implication of different scheduling policies in order to describe advantages of providing flexible machines in automated manufacturing systems.

Tang and Denardo considered an FMC model with an NC machine requiring fine tuning during the tool changing to study a job scheduling problem. They treated the minimization of the number of tool switches with integer programming, and solved by a heuristic procedure in [Tang 88a]. In [Tang 88b], they formulated the problem as a job



grouping problem and developed a branch-and-bound procedure to solve the minimization of the total number of switching instants. Some papers give analytical results for the tool switching problem in the single machine [Gave 62, 63], [Syke 70], [Eise 71, 72, 79].

In this chapter, we consider that the work station is equipped with an automatic device which can interchange a set of tools simultaneously between a tool magazine and a tool storage. All tools needed to process two types of material are kept in the tool storage. An NC machine of the work station performs a processing service by using the tools placed in its tool magazine. If requisite tools are not placed on the tool magazine, then appropriate tools must be switched from the tool storage before processing.

The main purpose of this chapter is to provide an FMC model and to analyze with the application of the queueing theory. We derive the probability generating function of waiting time under the steady state condition, and evaluate the average performance measures such as the average waiting time and average response time.

In the next section, we describe the FMC model and in Section 4.3, an analytical solution is provided. In Section 4.4, we present numerical results of the average performance measures for the FMC model. Concluding comments are addressed in Section 4.5.

## 4.2 Description of FMC model

The FMC system comprises a stand-alone machining centre with an ATC having a tool magazine, pick-up robots and a finite input storage. Following assumptions are made.

1. Materials are transported by a loading conveyor which consists of fixed-size cells, each cell is assumed to have the capacity of carrying at most one material.
2. Arriving materials wait to be processed at an input buffer storage, which is located in front of the station.
3. Arrival materials are processed one by one at a work station.
4. Materials having been processed are immediately unloaded to an automatic transportation device.

We define time required for each conveyor cell to pass an entrance spot as the unit slot time. Hence, in every slot, a new conveyor cell which can contain at most one material arrives at the entrance spot. Materials are assumed to arrive independently. We make the following assumptions for the FMC model.

1. All coming materials are classified into two classes, type 1 and type 2.
2. A type  $l$  material arrives in each slot with probability  $\lambda_l$  ( $\lambda^* = \lambda_1 + \lambda_2$ ).
3. The service time  $B^{(l)}$  ( $l = 1, 2$ ) for a type  $l$  material is independent and identically distributed according to a general distribution. Let  $\bar{b}^{(l)}$  and  $B^{(l)}(z)$  denote the mean service time and the probability generating function of  $B^{(l)}$  respectively.
4. It requires the tool switching time  $S^{(kl)}$  to switch tools for a type  $l$  material after completing service of a type  $k$  ( $l \neq k$ ) material.  $S^{(kl)}$  is also independent and identically distributed according to a general distribution. Let  $\bar{s}^{(kl)}$  and  $S^{(kl)}(z)$  denote the mean tool switching time and the probability generating function of  $S^{(kl)}$  respectively.
5. The input buffer storage is of finite capacity. The maximum number of materials that can be present in the system is  $K$ .
6. Materials are processed according to the order of their arrivals (FCFS).

Then, this FMC model is described by an imbedded Markov chain. We denote by  $\pi_j$  the stationary state probabilities of queue length ( $0 \leq j \leq K - 1$ ), where  $j$  indicates the number of materials in queue. We analyze the FMC model in application of the  $M/G/1/K$  queueing system and derive the moment generating function of waiting time. We consider two kinds of waiting time for a type  $l$  material. The first is an ordinary waiting time from the arrival epoch of a material to showing up the head of buffer storage. The second is a set-up waiting time from the arrival epoch of a material to the beginning of processing. The difference between these two kinds of waiting time is the possible tool switching time.

## 4.3 Analysis

We define the following notations to analyze the number of materials in the FMC system by using imbedded Markov chain.

$A_n$  : number of arriving materials during processing time of the  $n$ th material.

$C_n$  : number of arriving materials during tool switching time plus processing time of the  $n$ th material.

$M_n$  : number of materials to be left behind in the system just after the  $n$ th departure.



$d_n$  : type of the  $n$ th material.

A set of imbedded Markov points are chosen at slot boundaries when materials leave the system after service completion. Then, the system state can be represented by  $M_n$ , which satisfies the following relation

$$M_{n+1} = \begin{cases} [M_n - 1]^+ + A_{n+1} & \text{for } d_{n+1} = d_n \\ [M_n - 1]^+ + C_{n+1} & \text{for } d_{n+1} \neq d_n \end{cases}$$

where  $[\dots]^+ = \max[0, \dots]$ .

We define  $\alpha^*$  as the probability that an arriving material is of type 1 and  $\beta^* (= 1 - \alpha^*)$  for type 2. Therefore,  $\lambda_1 = \lambda^* \alpha^*$  and  $\lambda_2 = \lambda^* \beta^*$ . Let us define other probabilities to get transition probabilities for the Markov chain.

$a_i^{(l)}$  : probability that  $i$  materials arrive during processing time of type  $l$  material, ( $l = 1, 2, i = 0, 1, 2, \dots$ ).

$\gamma_i^{(l)}$  : probability that  $i$  materials arrive during tool switching time plus processing time for type  $l$  material, ( $l = 1, 2, i = 0, 1, 2, \dots$ ).

Let  $P_{ij}^{(kl)}$  ( $k, l = 1, 2, i, j = 0, 1, \dots$ ) denote the state transition probabilities which are defined as

$$P_{ij}^{(kl)} = \text{Prob}[d_{n+1} = l, M_{n+1} = j \mid d_n = k, M_n = i].$$

They are given by

(1) for  $0 \leq j \leq K - 2, k, l = 1, 2$

$$P_{0j}^{(kl)} = \begin{cases} \gamma_j^{(l)} & \text{for } k \neq l \\ a_j^{(l)} & \text{for } k = l \end{cases} \quad (4.1)$$

(2) for  $i - 1 \leq j \leq K - 2, 1 \leq i \leq K - 1$

$$P_{ij}^{(kl)} = \begin{cases} \gamma_{j-i+1}^{(l)} & \text{for } k \neq l \\ a_{j-i+1}^{(l)} & \text{for } k = l \end{cases} \quad (4.2)$$

(3) for  $j = K - 1$

$$P_{0j}^{(kl)} = \begin{cases} 1 - \sum_{m=0}^{K-2} \gamma_m^{(l)} & \text{for } k \neq l \\ 1 - \sum_{m=0}^{K-2} a_m^{(l)} & \text{for } k = l \end{cases} \quad (4.3)$$

$$P_{ij}^{(kl)} = \begin{cases} 1 - \sum_{m=0}^{K-i-1} \gamma_m^{(l)} & \text{for } k \neq l, 1 \leq i \leq K - 1 \\ 1 - \sum_{m=0}^{K-i-1} a_m^{(l)} & \text{for } k = l, 1 \leq i \leq K - 1. \end{cases} \quad (4.4)$$

Let us define

$$\pi_j^{(l)} = \lim_{n \rightarrow \infty} \text{Prob}[M_n = j, d_n = l] \quad 0 \leq j \leq K - 1, \quad l = 1, 2. \quad (4.5)$$

Then, these limiting probabilities satisfy the following equations

$$\pi_j^{(1)} = \alpha^* \pi_0^{(1)} a_j^{(1)} + \alpha^* \pi_0^{(2)} \gamma_j^{(1)} + \alpha^* \sum_{i=1}^{j+1} \pi_i^{(1)} a_{j-i+1}^{(1)} + \alpha^* \sum_{i=1}^{j+1} \pi_i^{(2)} \gamma_{j-i+1}^{(1)} \quad 0 \leq j \leq K - 2 \quad (4.6)$$

$$\pi_{K-1}^{(1)} = \alpha^* \pi_0^{(1)} \left(1 - \sum_{m=0}^{K-2} a_m^{(1)}\right) + \alpha^* \pi_0^{(2)} \left(1 - \sum_{m=0}^{K-2} \gamma_m^{(1)}\right) + \alpha^* \sum_{i=1}^{K-1} \pi_i^{(1)} \left(1 - \sum_{m=0}^{K-i-1} a_m^{(1)}\right) + \alpha^* \sum_{i=1}^{K-1} \pi_i^{(2)} \left(1 - \sum_{m=0}^{K-i-1} \gamma_m^{(1)}\right) \quad (4.7)$$

$$\pi_j^{(2)} = \beta^* \pi_0^{(1)} \gamma_j^{(2)} + \beta^* \pi_0^{(2)} a_j^{(2)} + \beta^* \sum_{i=1}^{j+1} \pi_i^{(1)} \gamma_{j-i+1}^{(2)} + \beta^* \sum_{i=1}^{j+1} \pi_i^{(2)} a_{j-i+1}^{(2)} \quad 0 \leq j \leq K - 2 \quad (4.8)$$

$$\pi_{K-1}^{(2)} = \beta^* \pi_0^{(1)} \left(1 - \sum_{m=0}^{K-2} \gamma_m^{(2)}\right) + \beta^* \pi_0^{(2)} \left(1 - \sum_{m=0}^{K-2} a_m^{(2)}\right) + \beta^* \sum_{i=1}^{K-1} \pi_i^{(1)} \left(1 - \sum_{m=0}^{K-i-1} \gamma_m^{(2)}\right) + \beta^* \sum_{i=1}^{K-1} \pi_i^{(2)} \left(1 - \sum_{m=0}^{K-i-1} a_m^{(2)}\right) \quad (4.9)$$

and

$$\sum_{j=0}^{K-1} (\pi_j^{(1)} + \pi_j^{(2)}) = 1. \quad (4.10)$$

Thus,  $\{\pi_j^{(l)} : 0 \leq j \leq K - 1\}$  are determined by solving equations (4.6) to (4.10).

We consider the remaining processing time and the remaining tool switching time plus processing time in order to derive the joint probability distribution of the number of materials in the system and the remaining time for a type  $l$  material under processing at an arbitrary time. First, we analyze the remaining processing time. We denote by  $L^{(l)}$  the processing time of a type  $l$  material which is observed by an arriving material. The elapsed processing time and the remaining processing time for a type  $l$  material in processing at an arbitrary time are denoted by  $X$  and  $Y$  which satisfy

$$L^{(l)} = X + Y.$$

By setting  $L^{(l)} = N$ , we derive the conditional generating function of the elapsed time and the remaining time

$$\begin{aligned} E[z^X \omega^Y \mid L^{(l)} = N] &= \sum_{i=1}^N \frac{1}{N} z^{i-1} \omega^{N-i} \\ &= \frac{\omega^N - z^N}{N(\omega - z)}. \end{aligned} \quad (4.11)$$



The probability distribution of type  $l$  processing time observed by an arbitrary material becomes

$$\text{Prob}[L^{(l)} = N] = \frac{N \cdot \text{Prob}[B^{(l)} = N]}{b^{(l)}}. \quad (4.12)$$

We note that

$$B^{(l)}(1 - \lambda^* + \lambda^* z) = \sum_{i=0}^{\infty} a_i^{(l)} z^i. \quad (4.13)$$

Therefore, by removing the condition on the length of processing time, we have

$$\begin{aligned} E[z^X \omega^Y] &= \sum_{N=1}^{\infty} E[z^X \omega^Y | L^{(l)} = N] \cdot \text{Prob}[L^{(l)} = N] \\ &= \frac{B^{(l)}(\omega) - B^{(l)}(z)}{b^{(l)} \cdot (\omega - z)}. \end{aligned} \quad (4.14)$$

Let  $N^{(l)}(X)$  denote the number of materials that arrive at the FMC system during the elapsed processing time  $X$  of type  $l$  material, and define

$$g_i^{(l)}(\omega) = E[\omega^Y | N^{(l)}(X) = i] \text{Prob}[N^{(l)}(X) = i]. \quad (4.15)$$

With equations (4.13) and (4.15), we get the generating function of  $g_i^{(l)}(\omega)$

$$\begin{aligned} \sum_{i=0}^{\infty} g_i^{(l)}(\omega) z^i &= E[z^{N^{(l)}(X)} \omega^Y] \\ &= \frac{B^{(l)}(\omega) - B^{(l)}(1 - \lambda^* + \lambda^* z)}{b^{(l)} \cdot (\omega - 1 + \lambda^* - \lambda^* z)} \\ &= \frac{B^{(l)}(\omega) - \sum_{i=0}^{\infty} a_i^{(l)} z^i}{b^{(l)} \cdot (\omega - 1 + \lambda^* - \lambda^* z)} \\ &= \frac{1}{b^{(l)} \cdot (\omega + \lambda^* - 1)} \\ &\quad \cdot \left\{ B^{(l)}(\omega) \sum_{i=0}^{\infty} \left( \frac{\lambda^*}{\omega + \lambda^* - 1} \right)^i z^i - \sum_{i=0}^{\infty} a_i^{(l)} z^i \sum_{j=0}^{\infty} \left( \frac{\lambda^*}{\omega + \lambda^* - 1} \right)^j z^j \right\}. \end{aligned} \quad (4.16)$$

Since  $g_i^{(l)}(\omega)$  is the coefficient of  $z^i$ , we can obtain the generating function of the remaining processing time as

$$g_i^{(l)}(\omega) = \frac{1}{b^{(l)} \cdot (\omega + \lambda^* - 1)} \cdot \left\{ B^{(l)}(\omega) \left( \frac{\lambda^*}{\omega + \lambda^* - 1} \right)^i - \sum_{n=0}^i a_n^{(l)} \left( \frac{\lambda^*}{\omega + \lambda^* - 1} \right)^{i-n} \right\}. \quad (4.17)$$

Similarly to  $g_i^{(l)}(\omega)$ , we can derive the generating function  $\phi_i^{(l)}(\omega)$  for the remaining time of the tool switching time plus processing time. Then,  $\phi_i^{(l)}(\omega)$  is given by

$$\begin{aligned} \phi_i^{(l)}(\omega) &= \frac{1}{(s^{(kl)} + b^{(l)}) \cdot (\omega + \lambda^* - 1)} \\ &\quad \cdot \left\{ S^{(kl)}(\omega) B^{(l)}(\omega) \left( \frac{\lambda^*}{\omega + \lambda^* - 1} \right)^i - \sum_{n=0}^i \gamma_n^{(l)} \left( \frac{\lambda^*}{\omega + \lambda^* - 1} \right)^{i-n} \right\}. \end{aligned} \quad (4.18)$$

The average interdeparture time, which is denoted by  $H$ , can be obtained as follows

$$\begin{aligned} H &= \sum_{j=0}^{K-1} \pi_j^{(1)} \{ \alpha^* b^{(1)} + \beta^* (s^{(12)} + b^{(2)}) \} + \pi_0^{(1)} \cdot \frac{1}{\lambda^*} \\ &\quad + \sum_{j=0}^{K-1} \pi_j^{(2)} \{ \alpha^* (s^{(21)} + b^{(1)}) + \beta^* b^{(2)} \} + \pi_0^{(2)} \cdot \frac{1}{\lambda^*} \\ &= \alpha^* b^{(1)} + \beta^* b^{(2)} + \alpha^* \beta^* (s^{(12)} + s^{(21)}) + (\pi_0^{(1)} + \pi_0^{(2)}) \cdot \frac{1}{\lambda^*}. \end{aligned} \quad (4.19)$$

We define the generating function  $W_n^{(l)}(z)$  ( $l = 1, 2$ ) for the time required to complete processing of  $n$  materials left behind in the system just after the departure of a type  $l$  material. Then, we derive a recurrence relation among  $W_n^{(l)}(z)$

$$W_n^{(1)}(z) = \alpha^* B^{(1)}(z) W_{n-1}^{(1)}(z) + \beta^* S^{(12)}(z) B^{(2)}(z) W_{n-1}^{(2)}(z) \quad (4.20)$$

$$W_n^{(2)}(z) = \alpha^* S^{(21)}(z) B^{(1)}(z) W_{n-1}^{(1)}(z) + \beta^* B^{(2)}(z) W_{n-1}^{(2)}(z). \quad (4.21)$$

It is readily seen that equations (4.20) and (4.21) can be simply rewritten by using matrix representation.

$$\begin{aligned} \begin{pmatrix} W_n^{(1)}(z) \\ W_n^{(2)}(z) \end{pmatrix} &= \begin{pmatrix} \alpha^* B^{(1)}(z) & \beta^* S^{(12)}(z) B^{(2)}(z) \\ \alpha^* S^{(21)}(z) B^{(1)}(z) & \beta^* B^{(2)}(z) \end{pmatrix} \begin{pmatrix} W_{n-1}^{(1)}(z) \\ W_{n-1}^{(2)}(z) \end{pmatrix} \\ &= \begin{pmatrix} \alpha^* B^{(1)}(z) & \beta^* S^{(12)}(z) B^{(2)}(z) \\ \alpha^* S^{(21)}(z) B^{(1)}(z) & \beta^* B^{(2)}(z) \end{pmatrix}^n \begin{pmatrix} W_0^{(1)}(z) \\ W_0^{(2)}(z) \end{pmatrix} \\ &= \frac{1}{\delta(z)} \begin{pmatrix} \epsilon(z) & \zeta(z) \\ \eta(z) & \kappa(z) \end{pmatrix} \begin{pmatrix} W_0^{(1)}(z) \\ W_0^{(2)}(z) \end{pmatrix}. \end{aligned} \quad (4.22)$$

$\delta(z)$ ,  $\epsilon(z)$ ,  $\zeta(z)$ ,  $\eta(z)$  and  $\kappa(z)$  are shown in Appendix B. Therefore, for a type  $l$  material arriving at an arbitrary time in the FMC model we have the generating function  $G_1(z)$  of waiting time which is made up of the remaining processing time, switching time if the material under service is of type  $2$ , and processing time of materials left in the system.

$$\begin{aligned} G_1(z) &= \frac{1}{1 - P_{Loss}} \left\{ \left( \sum_{k=0}^{K-2} \frac{1}{H} \pi_0^{(1)} \alpha^* b^{(1)} g_k^{(1)}(z) W_k^{(1)}(z) \right. \right. \\ &\quad \left. \left. + \sum_{j=1}^{K-1} \sum_{k=0}^{K-j-1} \frac{1}{H} \pi_j^{(1)} \alpha^* b^{(1)} g_k^{(1)}(z) W_{j+k-1}^{(1)}(z) \right) \right. \\ &\quad \left. + \left( \sum_{k=0}^{K-2} \frac{1}{H} \pi_0^{(1)} \beta^* (s^{(12)} + b^{(2)}) \phi_k^{(2)}(z) W_k^{(2)}(z) \right. \right. \\ &\quad \left. \left. + \sum_{j=1}^{K-1} \sum_{k=0}^{K-j-1} \frac{1}{H} \pi_j^{(1)} \beta^* (s^{(12)} + b^{(2)}) \phi_k^{(2)}(z) W_{j+k-1}^{(2)}(z) \right) \right. \\ &\quad \left. + \frac{1}{H} \pi_0^{(1)} \frac{1}{\lambda^*} W_0^{(1)}(z) \right\} \end{aligned}$$



$$\begin{aligned}
& + \left( \sum_{k=0}^{K-2} \frac{1}{H} \pi_0^{(2)} \alpha^* (s^{(21)} + b^{(1)}) \phi_k^{(1)}(z) W_k^{(1)}(z) \right. \\
& \quad \left. + \sum_{j=1}^{K-1} \sum_{k=0}^{K-j-1} \frac{1}{H} \pi_j^{(2)} \alpha^* (s^{(21)} + b^{(1)}) \phi_k^{(1)}(z) W_{j+k-1}^{(1)}(z) \right) \\
& + \left( \sum_{k=0}^{K-2} \frac{1}{H} \pi_0^{(2)} \beta^* b^{(2)} g_k^{(2)}(z) W_k^{(2)}(z) \right. \\
& \quad \left. + \sum_{j=1}^{K-1} \sum_{k=0}^{K-j-1} \frac{1}{H} \pi_j^{(2)} \beta^* b^{(2)} g_k^{(2)}(z) W_{j+k-1}^{(2)}(z) \right) \\
& + \frac{1}{H} \pi_0^{(2)} \frac{1}{\lambda^*} W_0^{(2)}(z) \Bigg\}. \tag{4.23}
\end{aligned}$$

where  $P_{Loss}$  is the probability that an arriving material is lost. Using the equation

$$P_{Loss} = 1 - \frac{1}{\lambda^* H},$$

equation (4.23) is rewritten as

$$\begin{aligned}
G_1(z) = & \sum_{k=0}^{K-2} \lambda^* \cdot \left\{ \alpha^* \left[ \pi_0^{(1)} b^{(1)} g_k^{(1)}(z) + \pi_0^{(2)} (s^{(21)} + b^{(1)}) \phi_k^{(1)}(z) \right] \cdot W_k^{(1)}(z) \right. \\
& \left. + \beta^* \left[ \pi_0^{(1)} (s^{(12)} + b^{(2)}) \phi_k^{(2)}(z) + \pi_0^{(2)} b^{(2)} g_k^{(2)}(z) \right] \cdot W_k^{(2)}(z) \right\} \\
& + \sum_{j=1}^{K-1} \sum_{k=0}^{K-j-1} \lambda^* \cdot \left\{ \alpha^* \left[ \pi_j^{(1)} b^{(1)} g_k^{(1)}(z) + \pi_j^{(2)} (s^{(21)} + b^{(1)}) \phi_k^{(1)}(z) \right] \cdot W_{j+k-1}^{(1)}(z) \right. \\
& \left. + \beta^* \left[ \pi_j^{(1)} (s^{(12)} + b^{(2)}) \phi_k^{(2)}(z) + \pi_j^{(2)} b^{(2)} g_k^{(2)}(z) \right] \cdot W_{j+k-1}^{(2)}(z) \right\} \\
& + \pi_0^{(1)} W_0^{(1)}(z) + \pi_0^{(2)} W_0^{(2)}(z). \tag{4.24}
\end{aligned}$$

The generating function of waiting time for a type 2 material can be derived in the same way as that of a type 1 material.

From equation (4.24), we can get the generating functions of the ordinary, and set-up waiting times for a type 1 material by setting  $W_0^{(l)}(z)$  appropriately. Note that the definitions of the ordinary, and set-up waiting times are presented in section 2. They are given as follows

1. for the ordinary waiting time

$$W_0^{(l)}(z) = 1 \quad \text{for } l = 1, 2 \tag{4.25}$$

2. for the set-up waiting time

$$W_0^{(l)}(z) = \begin{cases} 1 & \text{for } l = 1 \\ S^{(21)}(z) & \text{for } l = 2. \end{cases} \tag{4.26}$$

Furthermore, we can obtain the response time which is defined as the time including its set-up waiting time and processing time by inserting the following initial values into (4.24).

$$W_0^{(l)}(z) = \begin{cases} B^{(1)}(z) & \text{for } l = 1 \\ S^{(21)}(z) B^{(1)}(z) & \text{for } l = 2. \end{cases} \tag{4.27}$$

We define  $Q_0^{(l)}(z)$ ,  $V_0^{(l)}(z)$  and  $R_0^{(l)}(z)$  ( $l = 1, 2$ ) as the initial values for the ordinary, set-up waiting time and the response time.  $Q_1(z)$ ,  $V_1(z)$  and  $R_1(z)$  represent the generating functions of ordinary, set-up waiting time and response time, which can be obtained respectively by substituting these initial values into equations (4.22) and (4.24). Accordingly, the average performance measures for this FMC model can be obtained. We give an explicit expression for the average ordinary waiting time  $W_1$  of a type 1 material.

$$\begin{aligned}
W_1 = & \frac{d}{dz} Q_1(z) \Big|_{z=1} \\
= & \sum_{k=0}^{K-2} \left\{ \lambda^* \alpha^* \left[ \pi_0^{(1)} b^{(1)} \bar{g}_k^{(1)} + \pi_0^{(2)} (s^{(21)} + b^{(1)}) \bar{\phi}_k^{(1)} \right] \right. \\
& \left. + \alpha^* \left[ \pi_0^{(1)} \left( 1 - \sum_{n=0}^k a_n^{(1)} \right) + \pi_0^{(2)} \left( 1 - \sum_{n=0}^k \gamma_n^{(1)} \right) \right] \cdot \bar{Q}_k^{(1)} \right. \\
& \left. + \lambda^* \beta^* \left[ \pi_0^{(1)} (s^{(12)} + b^{(2)}) \bar{\phi}_k^{(2)} + \pi_0^{(2)} b^{(2)} \bar{g}_k^{(2)} \right] \right. \\
& \left. + \beta^* \left[ \pi_0^{(1)} \left( 1 - \sum_{n=0}^k \gamma_n^{(2)} \right) + \pi_0^{(2)} \left( 1 - \sum_{n=0}^k a_n^{(2)} \right) \right] \cdot \bar{Q}_k^{(2)} \right\} \\
& + \sum_{j=1}^{K-1} \sum_{k=0}^{K-j-1} \left\{ \lambda^* \alpha^* \left[ \pi_j^{(1)} b^{(1)} \bar{g}_k^{(1)} + \pi_j^{(2)} (s^{(21)} + b^{(1)}) \bar{\phi}_k^{(1)} \right] \right. \\
& \left. + \alpha^* \left[ \pi_j^{(1)} \left( 1 - \sum_{n=0}^k a_n^{(1)} \right) + \pi_j^{(2)} \left( 1 - \sum_{n=0}^k \gamma_n^{(1)} \right) \right] \cdot \bar{Q}_{j+k-1}^{(1)} \right. \\
& \left. + \lambda^* \beta^* \left[ \pi_j^{(1)} (s^{(12)} + b^{(2)}) \bar{\phi}_k^{(2)} + \pi_j^{(2)} b^{(2)} \bar{g}_k^{(2)} \right] \right. \\
& \left. + \beta^* \left[ \pi_j^{(1)} \left( 1 - \sum_{n=0}^k \gamma_n^{(2)} \right) + \pi_j^{(2)} \left( 1 - \sum_{n=0}^k a_n^{(2)} \right) \right] \cdot \bar{Q}_{j+k-1}^{(2)} \right\}. \tag{4.28}
\end{aligned}$$

In equation (4.28),  $\bar{g}_k^{(l)}$ ,  $\bar{\phi}_k^{(l)}$  and  $\bar{Q}_k^{(l)}$  are given as

$$\begin{aligned}
\bar{g}_k^{(l)} &= \frac{1}{\lambda^* b^{(l)}} \left\{ b^{(l)} - \frac{1}{\lambda^*} \left[ (k+1) \left( 1 - \sum_{n=0}^k a_n^{(l)} \right) + \sum_{n=1}^k n a_n^{(l)} \right] \right\} \\
\bar{\phi}_k^{(l)} &= \frac{1}{\lambda^* (s^{(kl)} + b^{(l)})} \left\{ (s^{(kl)} + b^{(l)}) - \frac{1}{\lambda^*} \left[ (k+1) \left( 1 - \sum_{n=0}^k \gamma_n^{(l)} \right) + \sum_{n=1}^k n \gamma_n^{(l)} \right] \right\} \\
\mathbf{Q}_k &\triangleq (\bar{Q}_k^{(1)}, \bar{Q}_k^{(2)})
\end{aligned}$$



$$= \begin{cases} [I + (k-1)\mathbf{B}]\mathbf{A} \cdot e & \text{for } k \geq 1 \\ (0, 0) & \text{for } k = 0 \end{cases}$$

where

$$e = (1, 1)^t$$

$$\mathbf{A} = \begin{pmatrix} \alpha^* b^{(1)} & \beta^* (s^{(12)} + b^{(2)}) \\ \alpha^* (s^{(21)} + b^{(1)}) & \beta^* b^{(2)} \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} \alpha^* & \beta^* \\ \alpha^* & \beta^* \end{pmatrix}$$

and  $t, I$  are the notation of transpose and an identity matrix, respectively.

We can also get the second moments of  $Q_1(z), V_1(z)$  and  $R_1(z)$  by successively differentiating. For instance, let  $\bar{W}_1$  denote the second moment of  $Q_1(z)$ , then  $\bar{W}_1$  can be given by

$$\begin{aligned} \bar{W}_1 = & \sum_{k=0}^{K-2} \left\{ \lambda^* \alpha^* \left[ \pi_0^{(1)} b^{(1)} h_k^{(1)} + \pi_0^{(2)} (s^{(21)} + b^{(1)}) \theta_k^{(1)} \right] \right. \\ & + 2\lambda^* \alpha^* \left[ \pi_0^{(1)} b^{(1)} \bar{g}_k^{(1)} + \pi_0^{(2)} (s^{(21)} + b^{(1)}) \bar{\phi}_k^{(1)} \right] \cdot \bar{Q}_k^{(1)} \\ & + \alpha^* \left[ \pi_0^{(1)} (1 - \sum_{n=0}^k a_n^{(1)}) + \pi_0^{(2)} (1 - \sum_{n=0}^k \gamma_n^{(1)}) \right] \cdot U_k^{(1)} \\ & + \lambda^* \beta^* \left[ \pi_0^{(1)} (s^{(12)} + b^{(2)}) \theta_k^{(2)} + \pi_0^{(2)} b^{(2)} h_k^{(2)} \right] \\ & + 2\lambda^* \beta^* \left[ \pi_0^{(1)} (s^{(12)} + b^{(2)}) \bar{\phi}_k^{(2)} + \pi_0^{(2)} b^{(2)} \bar{g}_k^{(2)} \right] \cdot \bar{Q}_k^{(2)} \\ & \left. + \beta^* \left[ \pi_0^{(1)} (1 - \sum_{n=0}^k \gamma_n^{(2)}) + \pi_0^{(2)} (1 - \sum_{n=0}^k a_n^{(2)}) \right] \cdot U_k^{(2)} \right\} \\ & + \sum_{j=1}^{K-1} \sum_{k=0}^{K-j-1} \left\{ \lambda^* \alpha^* \left[ \pi_j^{(1)} b^{(1)} h_k^{(1)} + \pi_j^{(2)} (s^{(21)} + b^{(1)}) \theta_k^{(1)} \right] \right. \\ & + 2\lambda^* \alpha^* \left[ \pi_j^{(1)} b^{(1)} \bar{g}_k^{(1)} + \pi_j^{(2)} (s^{(21)} + b^{(1)}) \bar{\phi}_k^{(1)} \right] \cdot \bar{Q}_{j+k-1}^{(1)} \\ & + \alpha^* \left[ \pi_j^{(1)} (1 - \sum_{n=0}^k a_n^{(1)}) + \pi_j^{(2)} (1 - \sum_{n=0}^k \gamma_n^{(1)}) \right] \cdot U_{j+k-1}^{(1)} \\ & + \lambda^* \beta^* \left[ \pi_j^{(1)} (s^{(12)} + b^{(2)}) \theta_k^{(2)} + \pi_j^{(2)} b^{(2)} h_k^{(2)} \right] \\ & + 2\lambda^* \beta^* \left[ \pi_j^{(1)} (s^{(12)} + b^{(2)}) \bar{\phi}_k^{(2)} + \pi_j^{(2)} b^{(2)} \bar{g}_k^{(2)} \right] \cdot \bar{Q}_{j+k-1}^{(2)} \\ & \left. + \beta^* \left[ \pi_j^{(1)} (1 - \sum_{n=0}^k \gamma_n^{(2)}) + \pi_j^{(2)} (1 - \sum_{n=0}^k a_n^{(2)}) \right] \cdot U_{j+k-1}^{(2)} \right\}. \quad (4.29) \end{aligned}$$

where  $h_k^{(l)}, \theta_k^{(l)}$  and  $U_k^{(l)}$  are given as

$$h_k^{(l)} = \frac{1}{\lambda^* b^{(l)}} \left\{ b_2^{(l)} - \frac{1}{\lambda^*} \left[ 2(k+1)b^{(l)} - \frac{1}{\lambda^*} \left( (k+1)(k+2) \left( 1 - \sum_{n=0}^k a_n^{(l)} \right) \right. \right. \right.$$

$$\left. \left. \left. + (2k+3) \sum_{n=1}^k n a_n^{(l)} - \sum_{n=1}^k n^2 a_n^{(l)} \right) \right] \right\}$$

$$\theta_i^{(l)} = \frac{1}{\lambda^* (s^{(kl)} + b^{(l)})} \left\{ (s_2^{(kl)} + 2s^{(kl)} b^{(l)} + b_2^{(l)}) - \frac{1}{\lambda^*} \left[ 2(k+1)(s^{(kl)} + b^{(l)}) \right. \right.$$

$$\left. \left. - \frac{1}{\lambda^*} \left( (k+1)(k+2) \left( 1 - \sum_{n=0}^i \gamma_n^{(l)} \right) + (2k+3) \sum_{n=1}^i n \gamma_n^{(l)} - \sum_{n=1}^i n^2 \gamma_n^{(l)} \right) \right] \right\}$$

$$\mathbf{U}_k \triangleq (U_k^{(1)}, U_k^{(2)})$$

$$= \begin{cases} (I + (k-1)\mathbf{B})\mathbf{D} \cdot e + 2\mathbf{A}\mathbf{Q}_{k-1} + 2\mathbf{B}\mathbf{A} \sum_{i=0}^{k-2} \mathbf{Q}_i & \text{for } k \geq 1 \\ (0, 0) & \text{for } k = 0 \end{cases}$$

$$\mathbf{D} = \begin{pmatrix} \alpha^* b_2^{(1)} & \beta^* (s_2^{(12)} + 2s^{(12)} b^{(2)} + b_2^{(2)}) \\ \alpha^* (s_2^{(21)} + 2s^{(21)} b^{(1)} + b_2^{(1)}) & \beta^* b_2^{(2)} \end{pmatrix}$$

and  $b_2^{(l)}, s_2^{(kl)}$  are the second moment of  $B^{(l)}(z)$  and  $S^{(kl)}(z)$ , respectively.

## 4.4 Numerical Results

The numerical calculation is performed for a small buffer size of queue. Especially, we concentrate our attention on the case that the buffer size is two (i.e.  $K = 3$ ). We assume that the processing and tool switching times are independent and geometrically distributed with mean  $1/\mu_l$  and  $1/\nu_{kl}$  ( $k, l = 1, 2, k \neq l$ ) respectively. We consider that the assumption of geometrical distribution shows the performance variation of the FMC system which processes multi-material type of arriving randomly.

In the numerical computations, we first calculate the performance measures for the case that parameters  $\alpha^*, \mu_l$  and  $\nu_{kl}$  are fixed whereas the arriving probability  $\lambda^*$  varies from 0.01 to 1. Next we change the probability  $\alpha^*$  from 0 to 0.5 while  $\lambda^*$  takes on the values 0.1, 0.5 and 1 keeping  $\mu_l$  and  $\nu_{kl}$  fixed. The values of the average ordinary waiting time  $W_1$ , set-up waiting time  $W_1^*$  and the average response time  $R_1$  of a type 1 material as well as their variances are calculated.

### 4.4.1 Average Waiting Time and Average Response Time

Fig.4.1 shows the relationship between the average performance of type 1 material and parameters  $\alpha^*, \mu_1 = \mu_2, \nu_{12} = \nu_{21}$ . In Fig.4.1,  $WR$  present the average waiting time and response time. We find that when  $\lambda^*$  is small,  $W_1, W_1^*$  and  $R_1$  increase because each type 1 material arrives to the FMC system with probability  $\lambda^* \alpha^*$ . But as  $\lambda^*$  becomes large the increases of  $W_1, W_1^*$  and  $R_1$  are small due to the dependence of  $W_1, W_1^*$  and  $R_1$  upon the value of probability that materials arrive to the buffer storage.



We denote by  $W$ ,  $R$  the average waiting time and the average response time, respectively. Fig.4.2 shows the average ordinary and set-up waiting times of type 1 materials in the case of  $\mu_1 = \mu_2 = 0.5$ ,  $\nu_{12} = \nu_{21} = 0.5$ . As can be seen in Fig.4.2, it is evident that when  $\lambda^* = 0.1$   $W_1^*$  decreases, and when  $\lambda^* = 0.5$ , the increase in  $W_1^*$  are small due to the influence of tool switching time, meanwhile  $W_1$  increases. Note that when  $\lambda^* = 0.5$ , 1 and  $\alpha^* = 0.5$   $W_1^*$  decreases. This result is intuitively understandable.

Let us examine the effect of tool switching time quantitatively on the performance measures of the FMC system by varying  $\nu_{kl}$ . Fig.4.3 exhibits the numerical results of  $W_1$  and  $W_1^*$  in the case of  $\nu_{12} = \nu_{21} = 0.1$  with the foregoing parameters. In this example, since the tool switching time is very short, the increase in  $W_1$  is very slow and the decrease in  $W_1^*$  is also very slow as  $\alpha^*$  becomes large. In addition there is little difference between  $W_1$  and  $W_1^*$  in consequence of the short tool switching time.

Comparison of the numerical results of  $R_1$  for the case of  $\nu_{12} = \nu_{21} = 0.5$  with  $\nu_{12} = \nu_{21} = 0.1$  is graphically presented in Fig.4.4, where  $R_1$  gets larger with  $\nu_{12}$  and  $\nu_{21}$ . It is noted that for  $0.3 \leq \alpha^* \leq 0.4$ , values of  $R_1$  under  $\lambda^* = 0.5$ ,  $\nu_{12} = \nu_{21} = 0.5$  closely approach those under  $\lambda^* = 1$ ,  $\nu_{12} = \nu_{21} = 0.1$ .

Fig.4.5 displays  $W_1$  and  $W_1^*$  for the case in which the FMC system has a different processing time for each type  $l$  material. When  $\alpha^* = 0.5$ , a close examination shows the interesting phenomenon that  $W_1$  becomes shorter than when  $\alpha^* = 0.4$  due to the shorter average processing time of type 1 material than type 2 material. Especially, on a nearer prospect, the effect of small buffer storage on the performance is dependent upon the value of  $\lambda^*$ .

In Fig.4.6,  $W_1$  and  $W_1^*$  are plotted for asymmetric switching times. It is very interesting that their behavior in Fig.4.6 is very similar to that in Fig.4.5. Particularly, for  $\lambda^* = 0.1$  we can observe these phenomena.

The numerical results of  $R_1$  under  $\mu_1 = 0.4$  and  $\nu_{12} = 0.1$  are shown in Fig.4.7. It also has the tendency similar to  $W_1$  and  $W_1^*$ . These behaviours can be explained in the same way as in Fig.4.6. Note that when  $\lambda^*$  is large,  $R_1$  in the case of  $\mu_1 = 0.4$  is sensitive to probability  $\alpha^*$  compared to  $\nu_{12} = 0.1$ .

Consequently, we observe that the tool switching time has a significant effect on the FMC performance.

#### 4.4.2 Variance of Waiting Time and Response Time

We denote by  $Var$ ,  $\hat{W}_1$ ,  $\hat{W}_1^*$  and  $\hat{R}_1$  variance, variances of ordinary, set-up waiting time and response time respectively. Fig.4.8 illustrates  $\hat{W}_1$  and  $\hat{W}_1^*$  in the case that the FMC

has different values of  $\mu_l$ . As  $\alpha^*$  increases, we see that  $\hat{R}_1$  of  $\lambda^* = 0.5$  is larger than that of  $\lambda^* = 1$ .  $\hat{W}_1$  and  $\hat{W}_1^*$  for the FMC system with asymmetric tool switching times are presented in Fig.4.9. We also observe that when  $\lambda^* = 0.5$ ,  $\hat{W}_1$  and  $\hat{W}_1^*$  become large as  $\alpha^*$  becomes large.

In Fig.4.10, we find that  $\hat{R}_1$  is very sensitive to  $\alpha^*$ . For  $\alpha^* \leq 0.3$ , the difference between  $\hat{R}_1$  of  $\mu_1 = 0.4$  and  $\nu_{12} = 0.1$  is large. But when  $\alpha^*$  is large, there is little difference between variances for the case of  $\mu_1 = 0.4$  and  $\nu_{12} = 0.1$ .

## 4.5 Concluding Remarks

In this chapter, we considered the FMC model with an automatic tool interchange device, and analyzed as a discrete time M/G/1 with a finite buffer size. We focused a great deal of interest on the problems arising in tool switching and finite buffer storage in order to investigate the optimal operations of the FMC system. Stationary probability generating functions for the FMC model were derived to analyze the performance measures such as the average waiting time and the average response time.

The numerical results showed the relation among the average performance measures, buffer size of queue and tool switching time. The results in this chapter give insight into FMC systems, and show the way to improve the performance of FMC systems. Furthermore, the methodology of the performance analysis is applicable to other FMC systems.



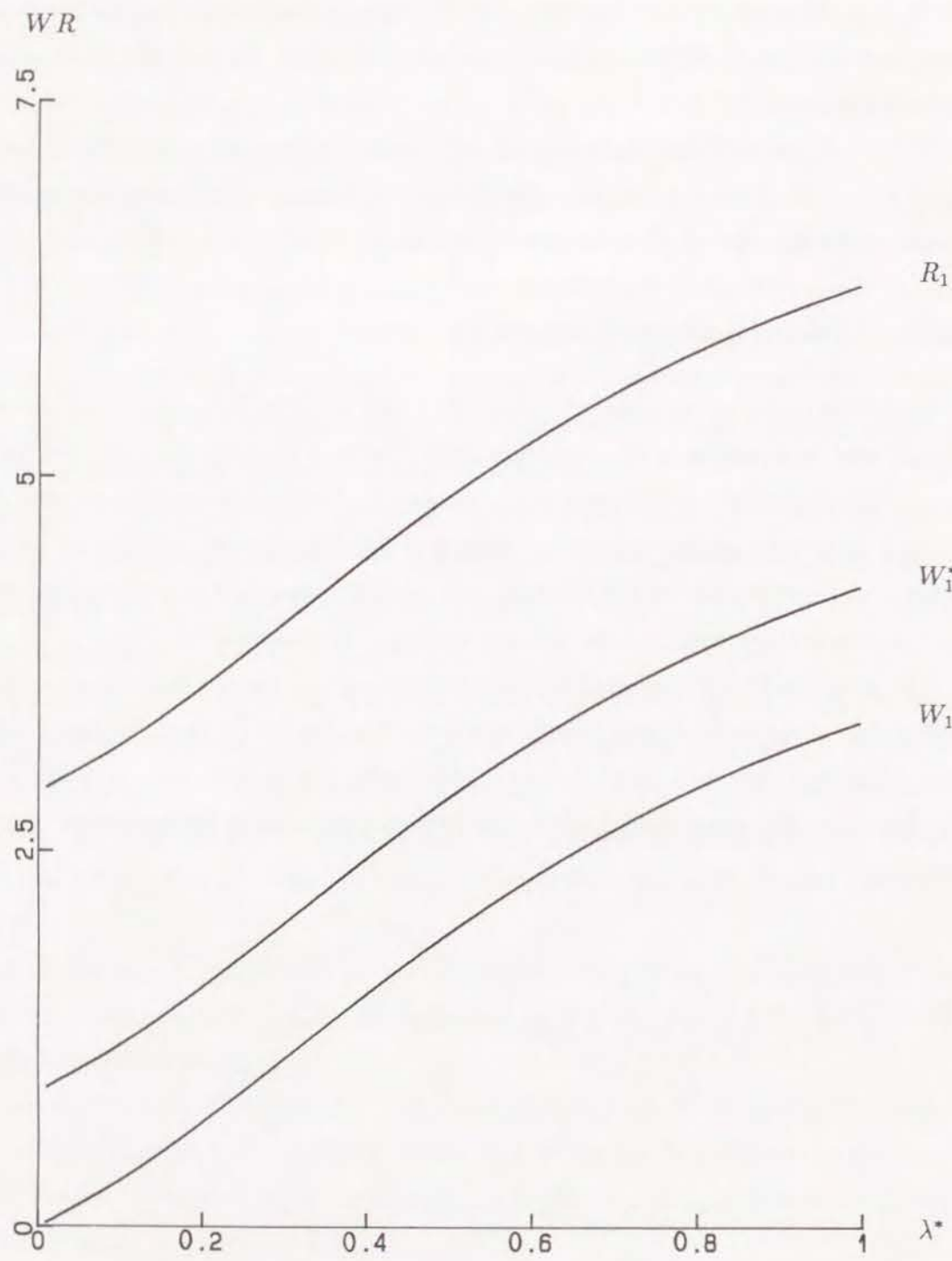


Figure 4.1: Average ordinary, set-up waiting time and response time for  $\alpha^* = 0.1$ ,  $\mu_1 = \mu_2 = 0.5$ ,  $\nu_{12} = \nu_{21} = 0.5$ .

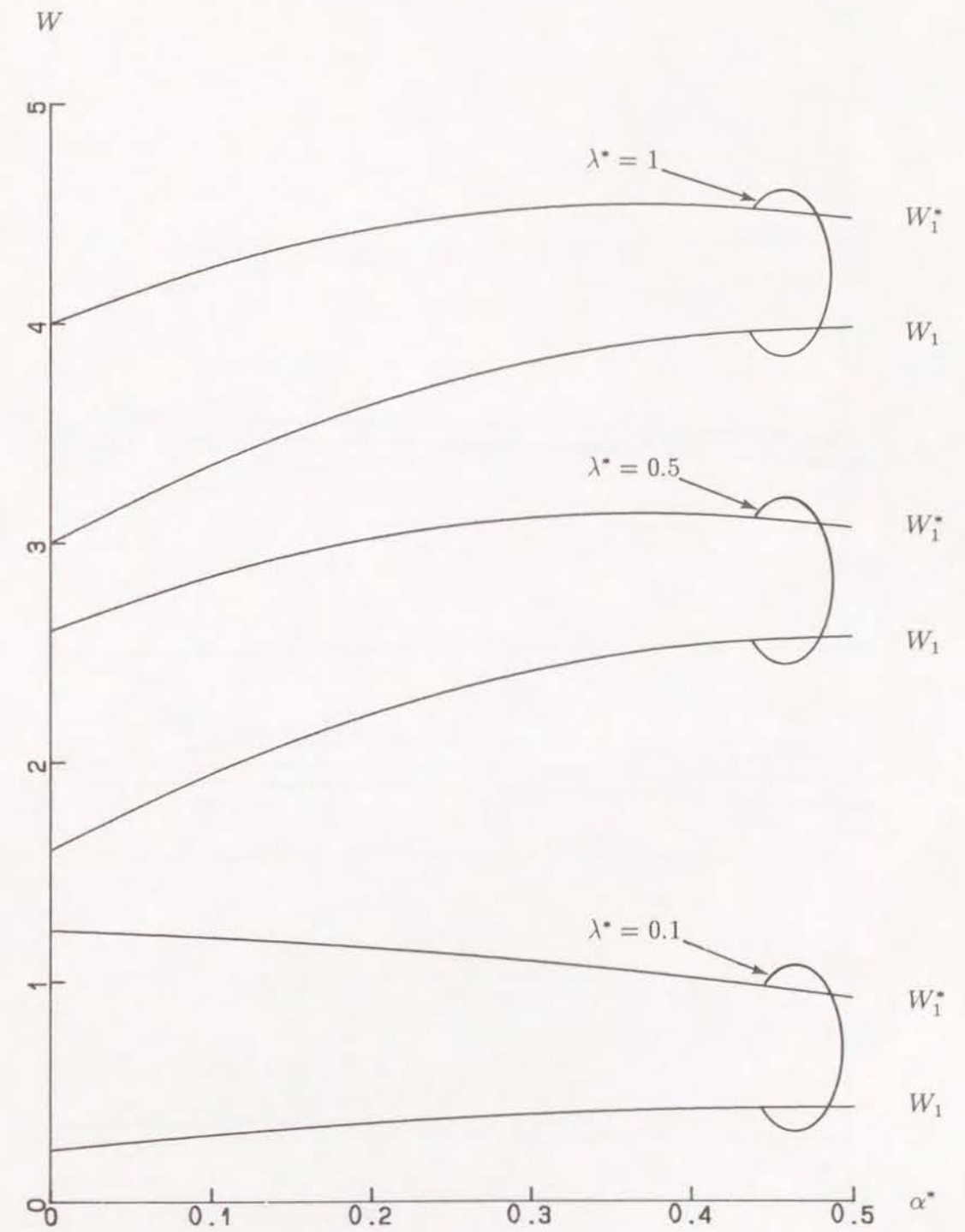


Figure 4.2: Average ordinary and set-up waiting time for  $\mu_1 = \mu_2 = 0.5$ ,  $\nu_{12} = \nu_{21} = 0.5$ .



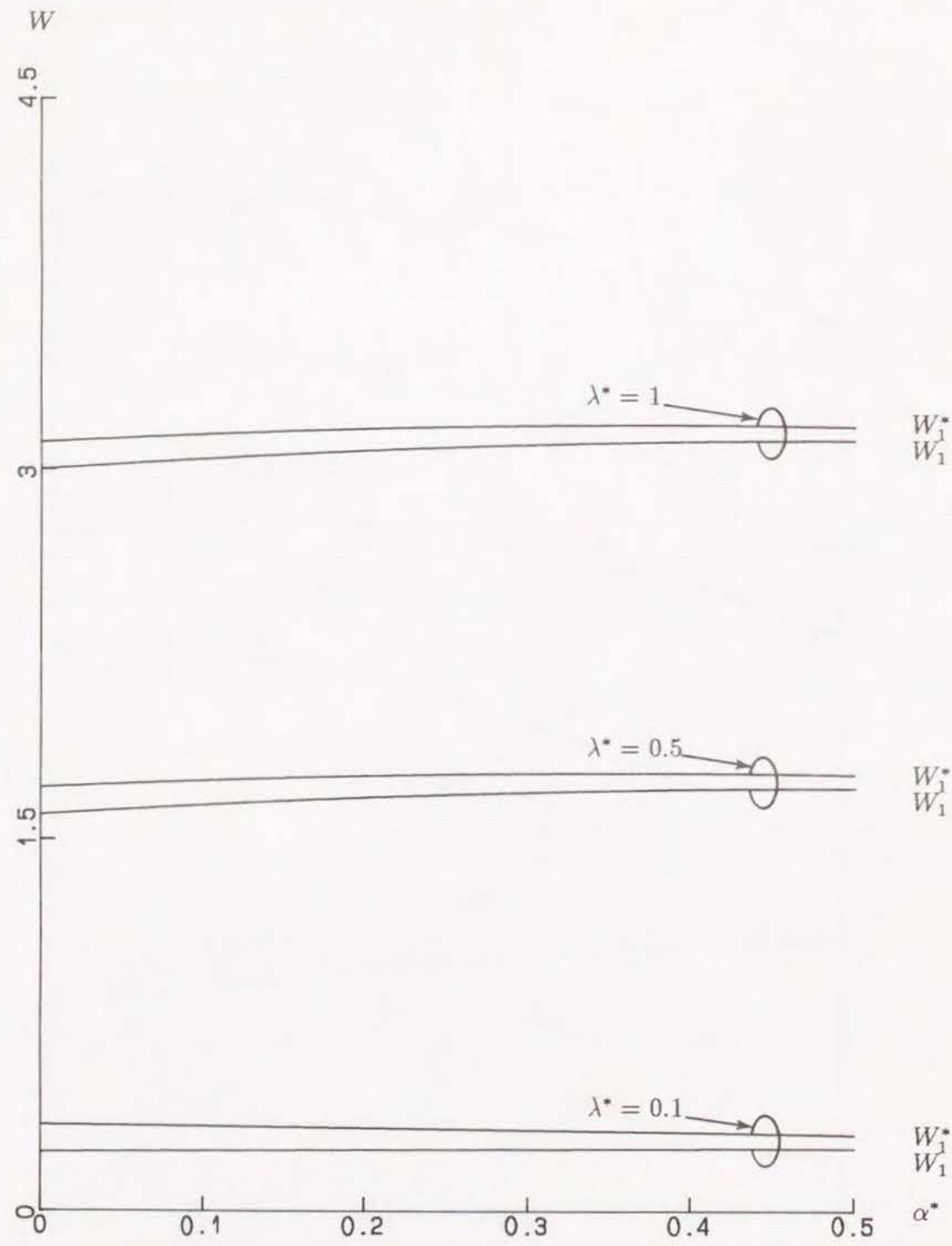


Figure 4.3: Average ordinary and set-up waiting time for  $\mu_1 = \mu_2 = 0.5$ ,  $\nu_{12} = \nu_{21} = 0.1$ .

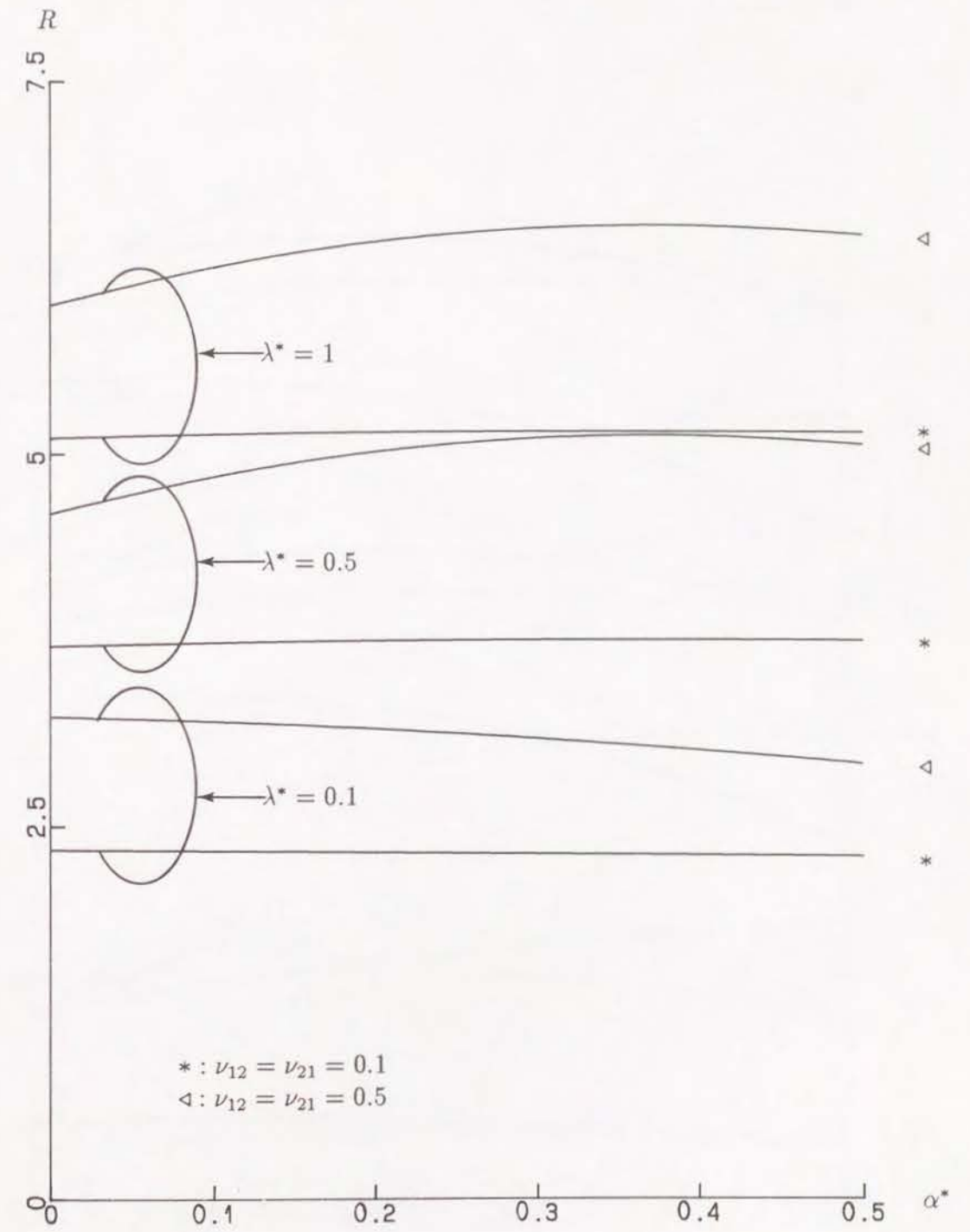


Figure 4.4: Average response time for  $\mu_1 = \mu_2 = 0.5$ .



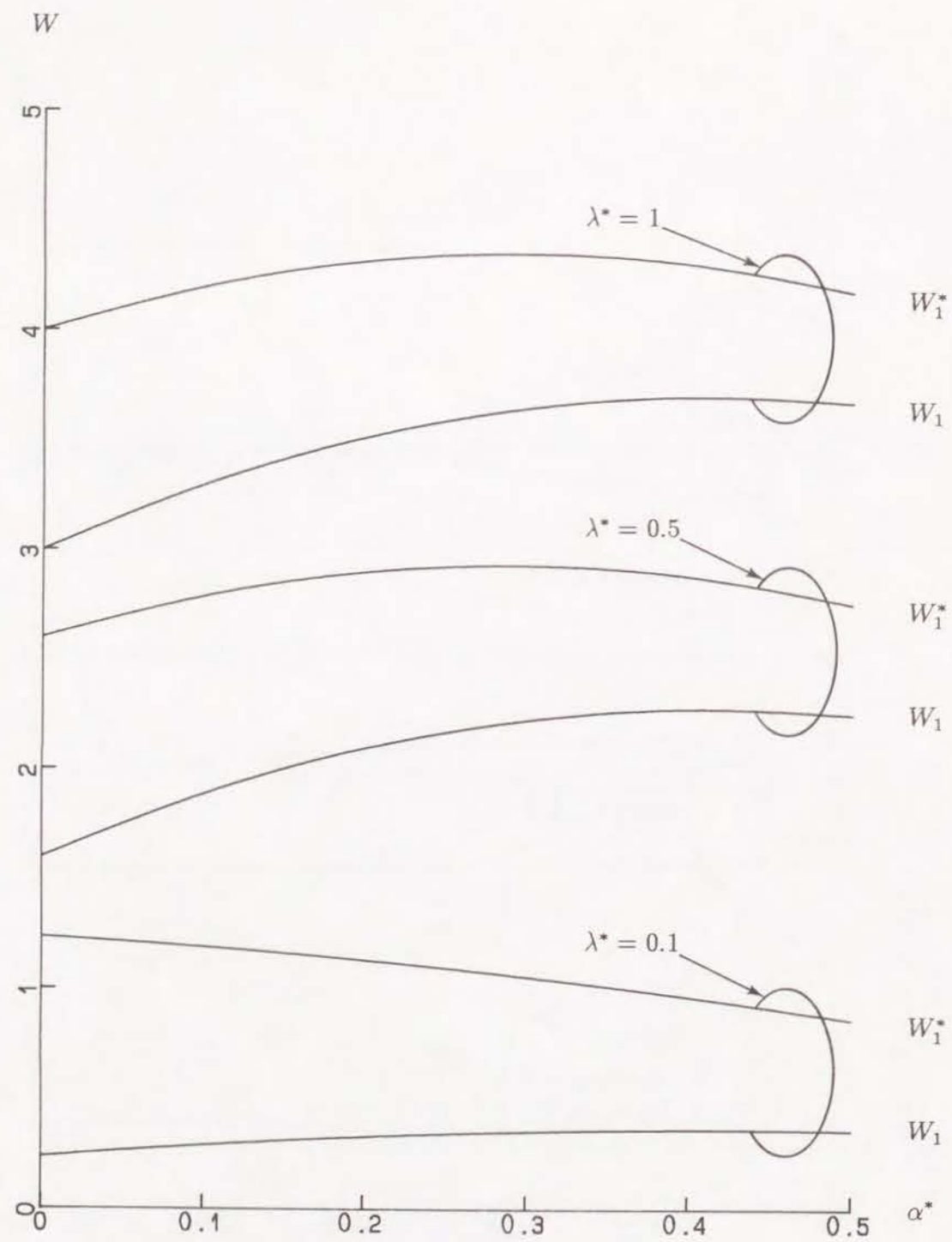


Figure 4.5: Average ordinary and set-up waiting time for  $\mu_1 = 0.4$ ,  $\mu_2 = 0.5$ ,  $\nu_{12} = \nu_{21} = 0.5$ .

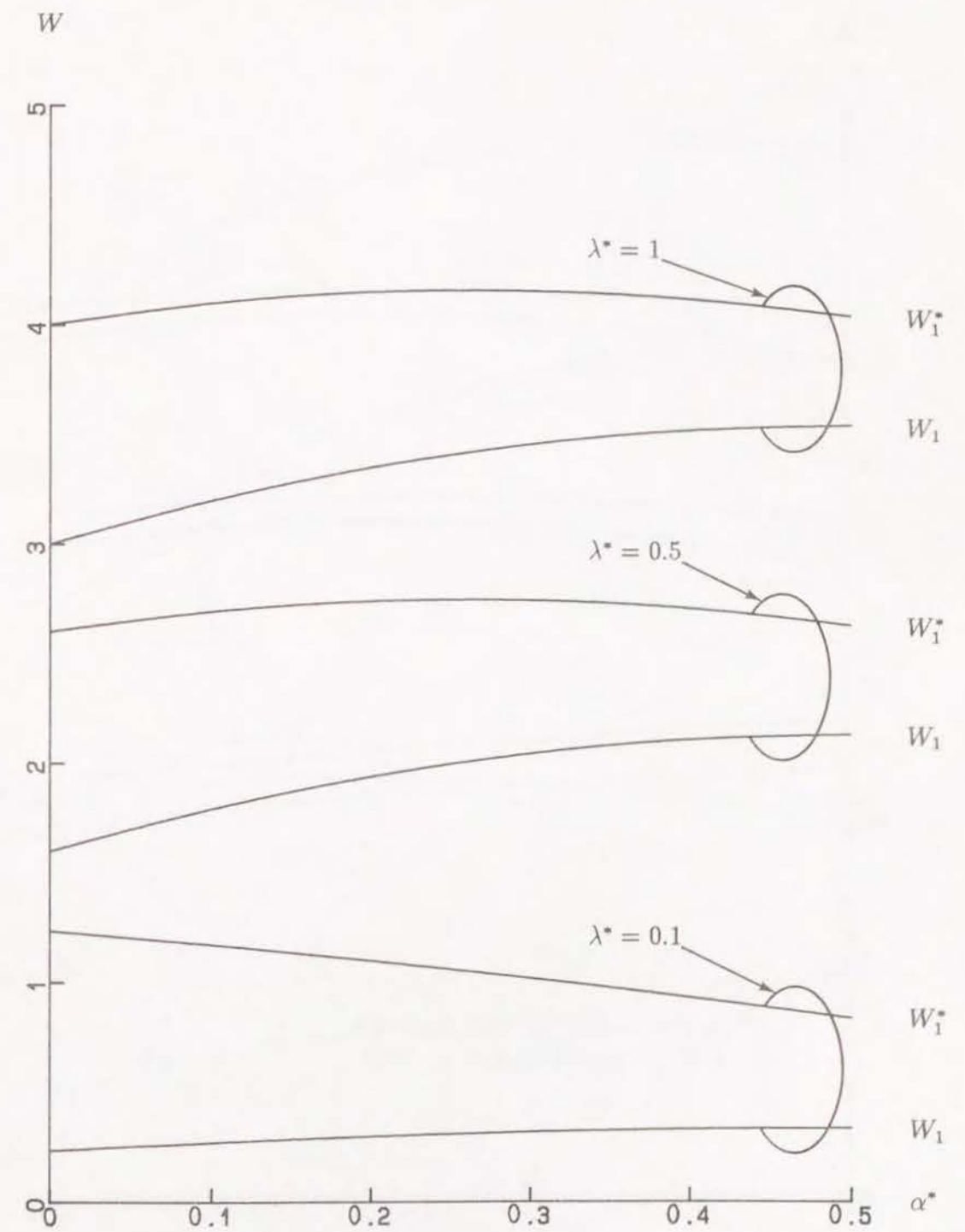


Figure 4.6: Average ordinary and set-up waiting time for  $\mu_1 = \mu_2 = 0.5$ ,  $\nu_{12} = 0.1$ ,  $\nu_{21} = 0.5$ .



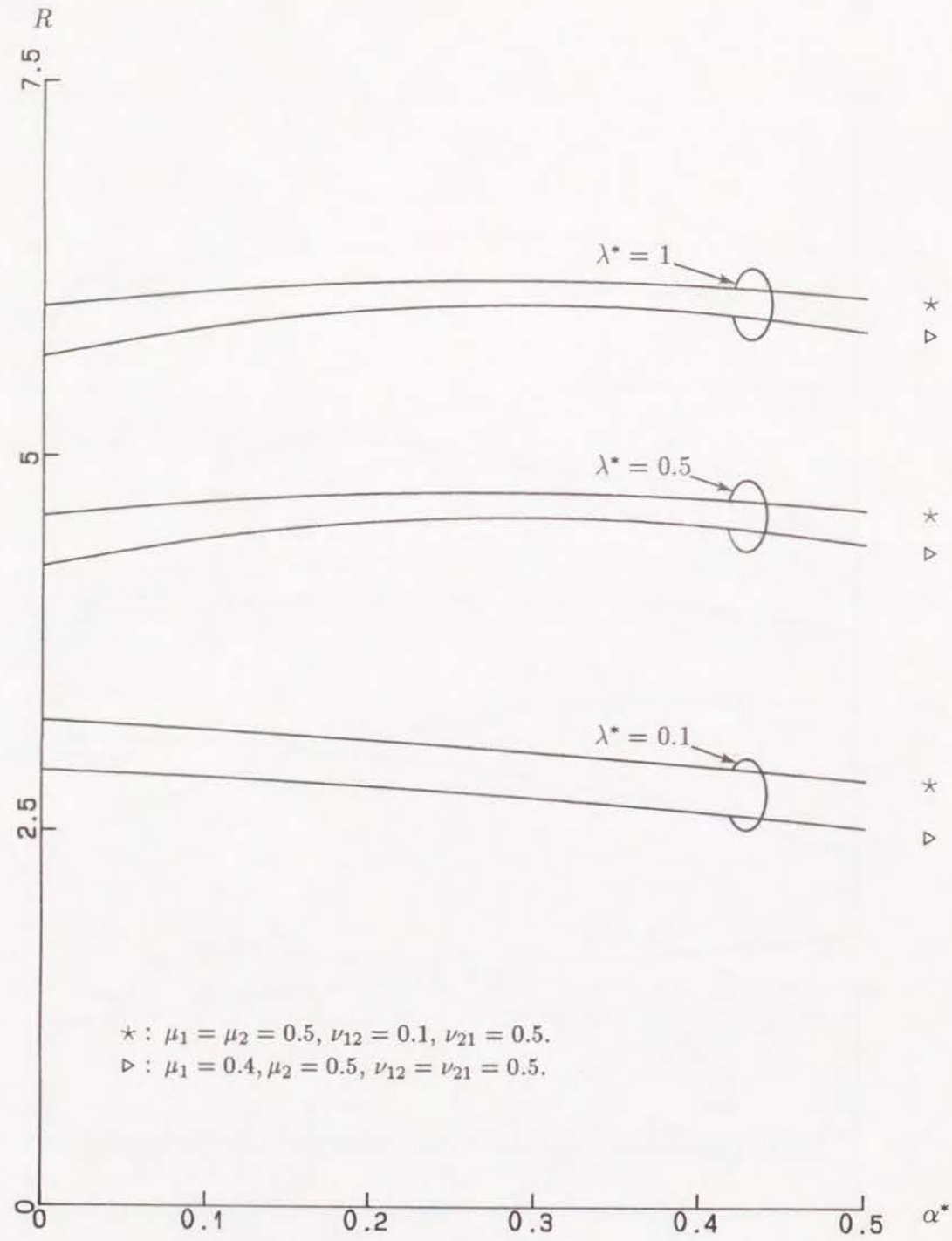


Figure 4.7: Average response time for asymmetric times.

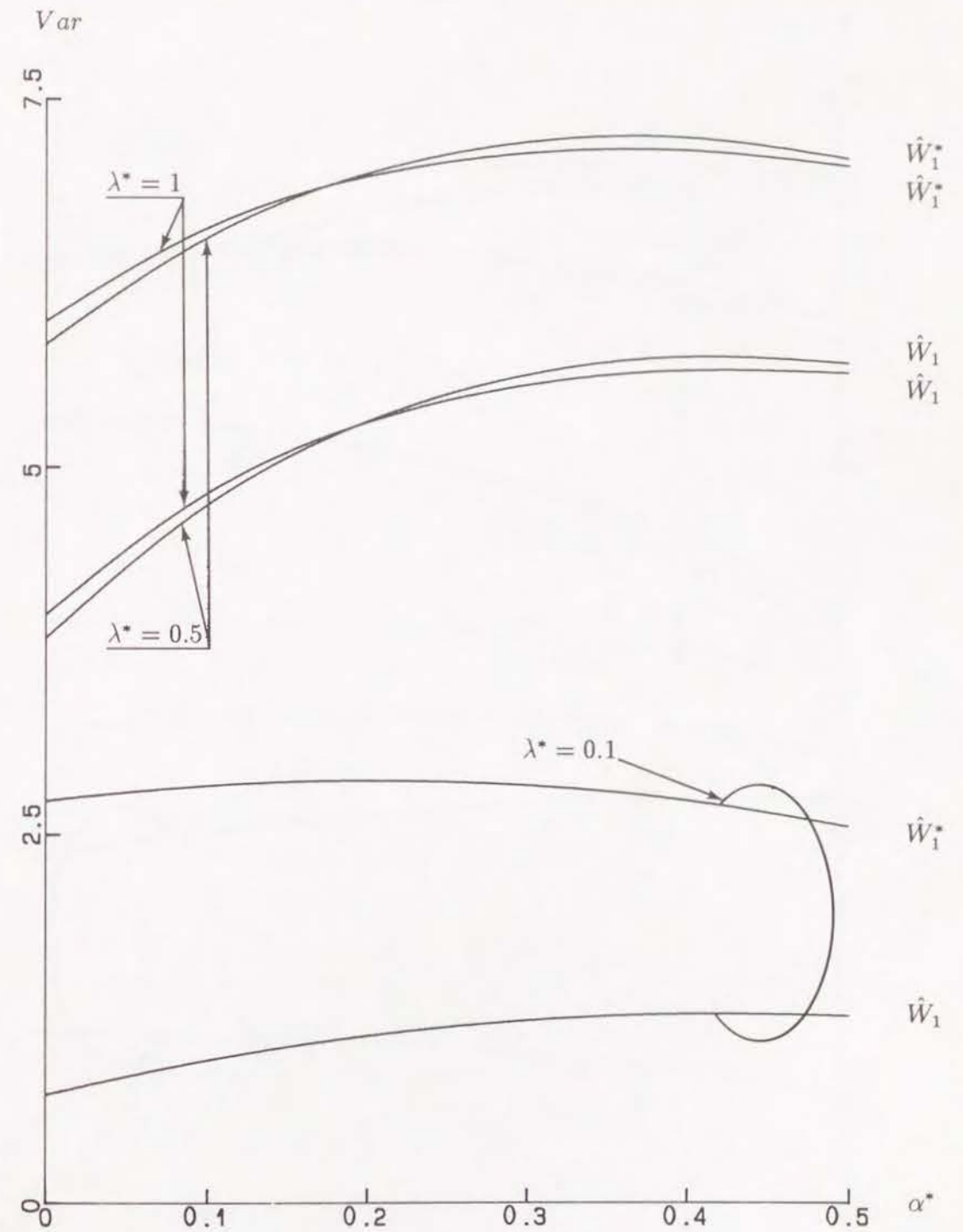


Figure 4.8: Variance of ordinary and set-up waiting time for  $\mu_1 = 0.4, \mu_2 = 0.5, \nu_{12} = \nu_{21} = 0.5.$



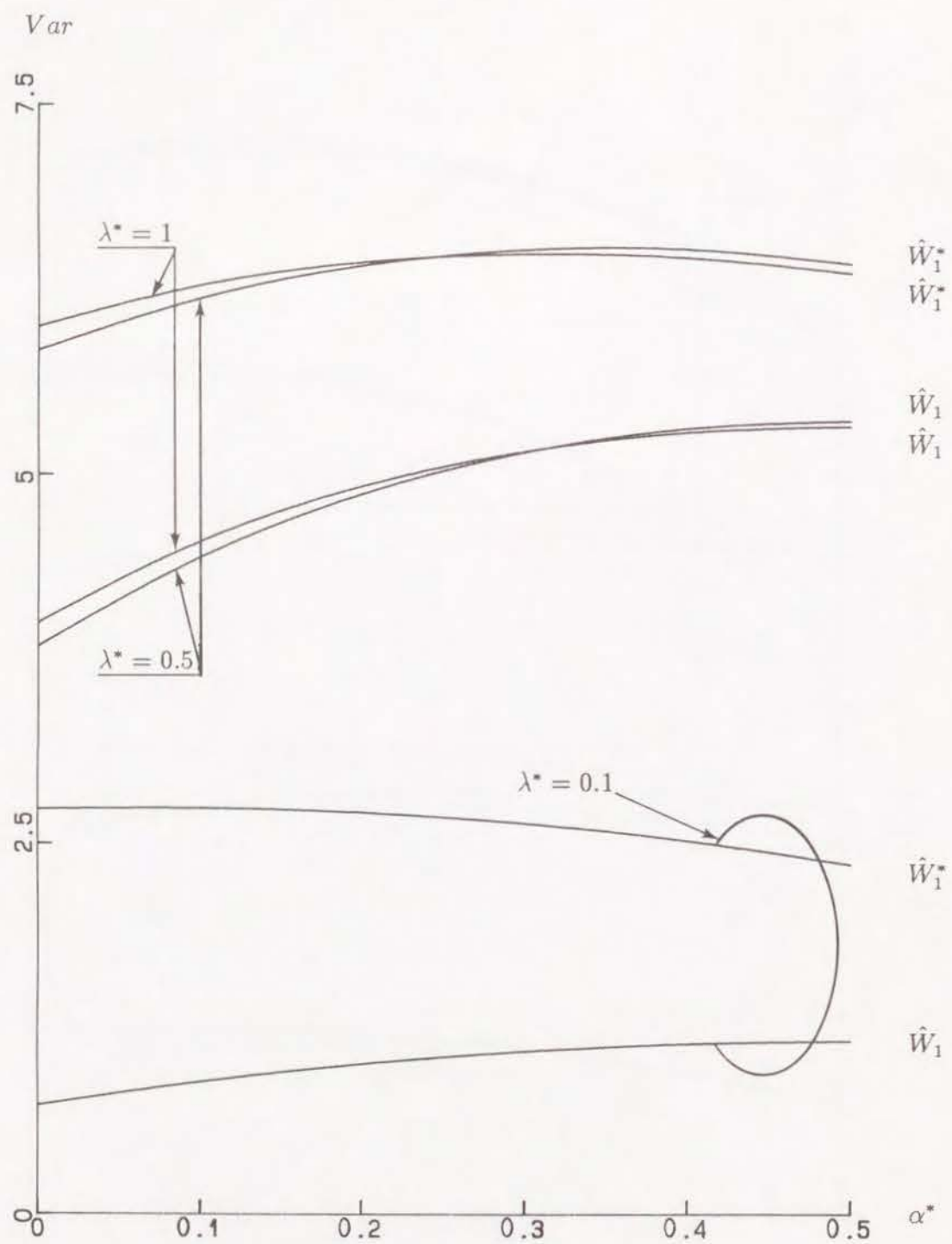


Figure 4.9: Variance of ordinary and set-up waiting time for  $\mu_1 = \mu_2 = 0.5$ ,  $\nu_{12} = 0.1$ ,  $\nu_{21} = 0.5$ .

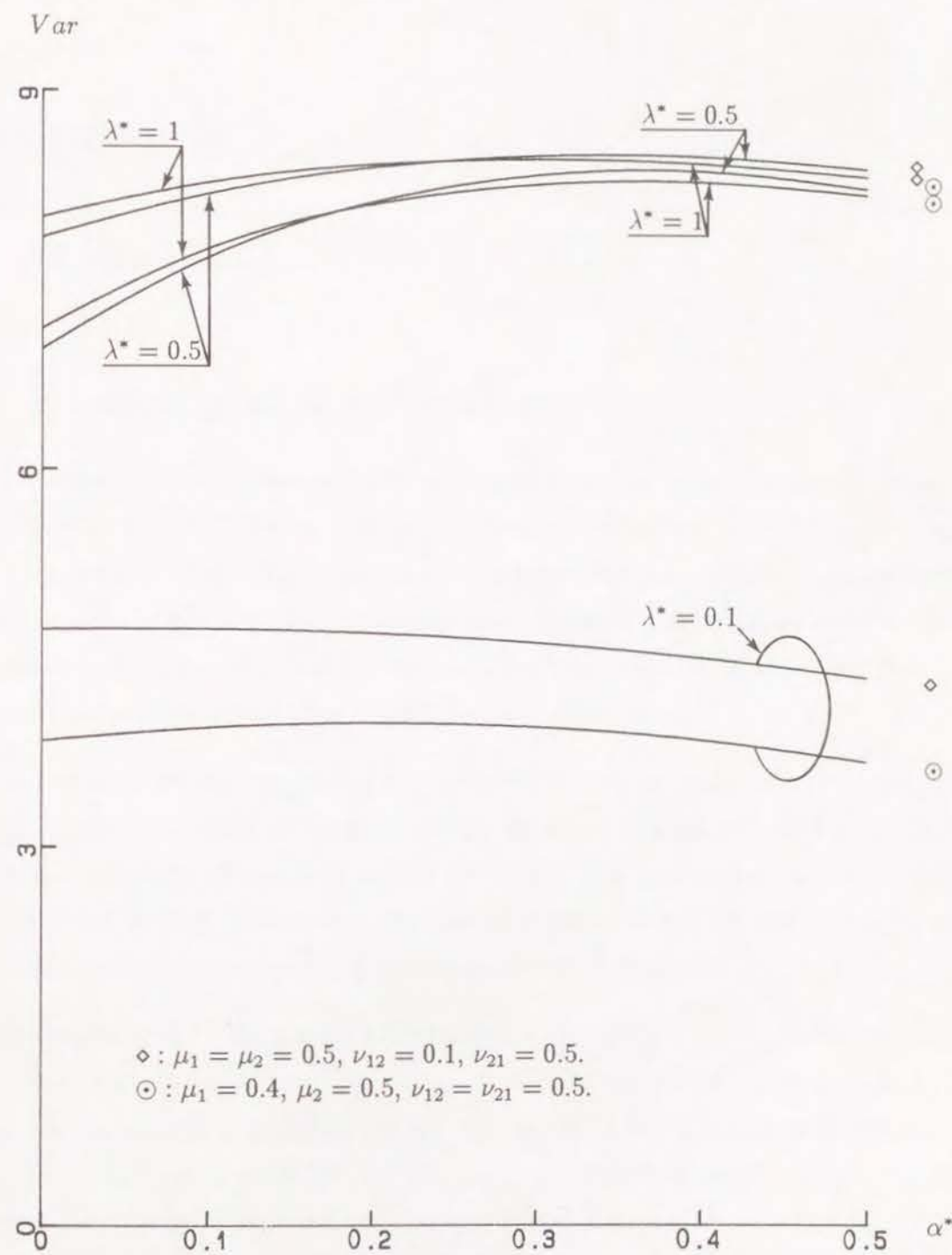


Figure 4.10: Variance of response time for asymmetric times.



## Chapter 5

### Conclusion

#### 5.1 Summary of the Dissertation

In this dissertation, two types of FMC system connected to conveyor material transport system, that is, the FMC system with tandem storage buffers and the FMC system with ATC, were studied to investigate the relation between the conveyor and storage buffers in tandem, and the effects of tool switching time. Especially, an exact analysis on these types of the FMC system by applying the queueing theory is little in the literature. The result summary of this dissertation is described as follows.

1. As for the FMC system having a single station with storage buffers in tandem in the conveyor-serviced production system, the exact analysis was performed on the special case in which queue 1 has one buffer size, and queue 2 has an infinite buffer size. The average performance measures on queue 2, such as the average queue length and the average waiting time, were explicitly derived.
2. In Chapter 3, the FMC system with tandem storage buffers, where the first storage is of finite capacity and the second storage is of infinite capacity was exactly analyzed as the discrete time queueing model. It is noted that this model is the extension of the FMC model provided in Chapter 2. In analytical approach, steady state probability generating functions can be derived through the solution of a set of linear equations which is provided in matrix form for convenient computation. Also the average performance measures for this general case can be numerically obtained. The results are useful in designing the optimal buffer storage capacity at an FMC system.
3. The analytical model of the FMC system equipped with an automatic tool changing device was proposed in Chapter 4. The stationary probability generating function



of waiting time was explicitly derived. The average performance measures such as the average waiting time and average response time can be obtained through the numerical experiment. The numerical results clarified the effects of tool switching time at the FMC system.

## 5.2 Topics for Future Research

As for further study issues on FMS, some researchers proposed the following topics that must be considered [Buza 86a], [Kalk 86], [Kusi 86a].

1. Models of system operation must be developed to promote efficient integration of an MHS into FMCs to produce a variety of product, and to design the optimal layout of FMS. More components will be incorporated into the FMS to handle more diversified operations such as rotational part processing, inspection, and assembly. Certainly, the development of FMS technological components will generate many new research topics because the FMS will link to other computerized systems to form the CIM system for a fully automated factory.
2. Planning and scheduling methodologies for each type of FMS must be developed to assure the most efficient operation. Particularly, the transient behaviour of system must be analyzed for the operational control of FMS, although it is a very difficult problem about the computation.
3. Models for assigning a tool setting to FMCs must be developed to enhance operating efficiency of FMS. In an FMS with dynamic material routing, it would be essential to model the tool interchanging system. This is a remarkable research topic because interchangeable tool magazines will be widely applied in the future.

In FMS, it is necessary to consider the interactions of the operating factors among all of work stations. Furthermore, the problem of cost concerning to the FMC system must be considered. Especially, the cost associated with storage buffers is a very important factor to design the optimal FMC system. The author is very interested in various models of FMC systems as the fundamental research topics for FMS's and CIM systems. Here, the author suggests some FMC systems and issues which are worth studying as future research topics.

1. As the extensions of model provided in Chapter 2, two models are worthy of exactly analyzing. One model is the case in which the first queue has an infinite buffer size

and the second queue has a finite buffer size. Another model is the case in which the first and the second queues have respectively finite storage buffers. These models are certainly interesting topics for investigating an optimal capacity of storage buffers at an FMC connected with conveyor system.

2. An FMC system with multiple parallel flexible machines having storage buffers in tandem is a challengeable issue as the extension of analytical approaches presented in Chapter 3.
3. An FMC system connected with loop conveyor is a quite invaluable study issue because the loop conveyor provides a cheap common storage buffers. Though its exact analysis seems to be very difficult, this topic is sufficiently worth noting from the analytical point of view.
4. The exact analysis of an FMC system having a tool switching time due to a tool failure is a noticeable topic. From the practical viewpoints, investigating the effect of tool switching times resulting from the cause of tool life, tool wear and tool breakage at an FMC system is quite important to the FMC design as well as FMS's.



## Appendix A

### Derivation of $\hat{\mathbf{G}}(\omega)$

In order to express the entries of  $\hat{\mathbf{G}}(\omega)$  compactly, we introduce the auxiliary functions which satisfy the following recursive equations.

$$e_0(\omega) = \theta(\omega) - \zeta(\omega) \cdot \eta(\omega) \cdot \delta^{-1}(\omega) \quad (\text{A.1})$$

$$e_l(\omega) = \theta(\omega) - \zeta(\omega) \cdot \kappa(\omega) \cdot e_{l-1}^{-1}(\omega) \quad 1 \leq l \leq K-2 \quad (\text{A.2})$$

$$\prod_{l=i}^j e_l(\omega) = \begin{cases} e_i(\omega)e_{i+1}(\omega) \cdots e_j(\omega) & \text{for } j \leq i \\ 1 & \text{for } i > j \end{cases}$$

$$d_{-1}(\omega) = \theta(\omega)$$

$$d_n(\omega) = \theta(\omega) - \zeta(\omega) \cdot \kappa(\omega) \cdot d_{n-1}^{-1}(\omega) \quad 0 \leq n \leq K-3 \quad (\text{A.3})$$

$$\prod_{n=a}^b d_n(\omega) = \begin{cases} d_a(\omega)d_{a+1}(\omega) \cdots d_b(\omega) & \text{for } a \leq b \\ 1 & \text{for } b \leq a-1. \end{cases}$$

The entries of  $\hat{\mathbf{G}}(\omega)$  in (3.15) can be represented as follows

1. if  $i = j$

$$g_{ii} = (-1)^{i+j} \delta(\omega) \left[ \prod_{l=0}^{i-2} e_l(\omega) \right] \zeta^0(\omega) \kappa^0(\omega) \cdot \left\{ \prod_{n=-1}^{K-i-3} d_n(\omega) \cdot \epsilon(\omega) - \prod_{n=-1}^{K-i-4} d_n(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega) \right\} \\ 1 \leq i \leq K-2, \quad 1 \leq j \leq K-2, \quad K \geq 3$$

2. if  $i \neq j$

$$g_{ij} = (-1)^{i+j} \delta(\omega) \left[ \prod_{l=0}^{r-2} e_l(\omega) \right] \zeta^m(\omega) \kappa^p(\omega) \cdot \left\{ \prod_{n=-1}^{K-q-3} d_n(\omega) \cdot \epsilon(\omega) - \prod_{n=-1}^{K-q-4} d_n(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega) \right\} \\ 1 \leq i \leq K-2, \quad 1 \leq j \leq K-2, \quad K \geq 4$$



$$\begin{aligned}
g_{i0} &= (-1)^i \zeta^i(\omega) \left\{ \prod_{n=-1}^{K-i-3} d_n(\omega) \cdot \epsilon(\omega) - \prod_{n=-1}^{K-i-4} d_n(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega) \right\} \\
&\quad 0 \leq i \leq K-2, \quad K \geq 2 \\
g_{0j} &= (-1)^j \eta(\omega) \kappa^{j-1}(\omega) \left\{ \prod_{n=-1}^{K-j-3} d_n(\omega) \cdot \epsilon(\omega) - \prod_{n=-1}^{K-j-4} d_n(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega) \right\} \\
&\quad 1 \leq j \leq K-2, \quad K \geq 3 \\
g_{iK-1} &= (-1)^{i+K-1} \delta(\omega) \left\{ \prod_{l=0}^{i-2} e_l(\omega) \right\} \kappa^{K-i-1}(\omega) \cdot \epsilon(\omega) \\
&\quad 1 \leq i \leq K-1, \quad K \geq 2 \\
g_{iK} &= (-1)^{i+K} \delta(\omega) \left\{ \prod_{l=0}^{i-2} e_l(\omega) \right\} \kappa^{K-i}(\omega) \quad 1 \leq i \leq K, \quad K \geq 1 \\
g_{K-1j} &= (-1)^{K-1+j} \delta(\omega) \left\{ \prod_{l=0}^{j-2} e_l(\omega) \right\} \zeta^{K-j-1}(\omega) \cdot \epsilon(\omega) \\
&\quad 1 \leq j \leq K-2, \quad K \geq 3 \\
g_{Kj} &= (-1)^{K+j} \delta(\omega) \left\{ \prod_{l=0}^{j-2} e_l(\omega) \right\} \zeta^{K-j}(\omega) \quad 1 \leq j \leq K-1, \quad K \geq 2 \\
g_{K-10} &= (-1)^{K-1} \zeta^{K-1}(\omega) \cdot \epsilon(\omega) \quad K \geq 1 \\
g_{0K-1} &= (-1)^{K-1} \eta(\omega) \cdot \kappa^{K-2}(\omega) \cdot \epsilon(\omega) \quad K \geq 2 \\
g_{K0} &= (-1)^K \zeta^K(\omega) \quad K \geq 1 \\
g_{0K} &= (-1)^K \eta(\omega) \cdot \kappa^{K-1}(\omega) \quad K \geq 1.
\end{aligned}$$

Arranging the above iterative equations (A.1), (A.2), (A.3), we can obtain the following equations (A.4), (A.5).

$$e_i(\omega) = \frac{F_i(\omega)}{F_{i-1}(\omega)} \quad 0 \leq i \leq K-2 \quad (\text{A.4})$$

where

$$\begin{aligned}
F_i(\omega) &= 1 & i &= -2 \\
F_i(\omega) &= \delta(\omega) & i &= -1 \\
F_i(\omega) &= F_{i-1}(\omega) \cdot \theta(\omega) - F_{i-2}(\omega) \cdot \zeta(\omega) \cdot \eta(\omega) & i &= 0 \\
F_i(\omega) &= F_{i-1}(\omega) \cdot \theta(\omega) - F_{i-2}(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega) & 1 \leq i &\leq K-2.
\end{aligned}$$

$$d_j(\omega) = \frac{D_j(\omega)}{D_{j-1}(\omega)} \quad 0 \leq j \leq K-3 \quad (\text{A.5})$$

where

$$\begin{aligned}
D_j(\omega) &= 1 & j &= -2 \\
D_j(\omega) &= \theta(\omega) & j &= -1 \\
D_j(\omega) &= D_{j-1}(\omega) \cdot \theta(\omega) - D_{j-2}(\omega) \cdot \zeta(\omega) \cdot \kappa(\omega) & 0 \leq j &\leq K-3.
\end{aligned}$$

And then, calculating the following items in entries of matrix  $\hat{G}(\omega)$ , which can be given by

$$\prod_{l=i}^j e_l(\omega) = \frac{F_j(\omega)}{F_{i-1}(\omega)} \quad \text{for } i \leq j$$

$$\prod_{l=i}^j e_l(\omega) = 1 \quad \text{for } i < j$$

$$\prod_{n=a}^b d_n(\omega) = D_b(\omega) \quad \text{for } a \leq b$$

$$\prod_{n=a}^b d_n(\omega) = 1 \quad \text{for } b \leq a-1.$$

Therefore, substituting these values into the preceding expression of entries in  $\hat{G}(\omega)$ , we can get the expression of entries in  $\hat{G}(\omega)$  shown on pages 51 and 52.



## Appendix B

### Derivation of Eq.(4.22)

In the recursive equations (4.20) and (4.21), let

$$\mathbf{A} = \begin{pmatrix} \alpha^* B^{(1)}(z) & \beta^* S^{(12)}(z) B^{(2)}(z) \\ \alpha^* S^{(21)}(z) B^{(1)}(z) & \beta^* B^{(2)}(z) \end{pmatrix}. \quad (\text{B.1})$$

The eigenvalues of  $\mathbf{A}$  are

$$\lambda_i^*(z) = \frac{\alpha^* B^{(1)}(z) + \beta^* B^{(2)}(z) \pm \sqrt{[\alpha^* B^{(1)}(z) + \beta^* B^{(2)}(z)]^2 - 4\alpha^* \beta^* B^{(1)}(z) B^{(2)}(z) D(z)}}{2} \quad (\text{B.2})$$

$i = 1, 2$

where

$$D(z) = 1 - S^{(12)}(z) S^{(21)}(z).$$

The eigenvectors for  $\lambda_1^*(z)$  and  $\lambda_2^*(z)$  are given as

$$\begin{pmatrix} c_1 \\ d_1 \end{pmatrix} = \begin{pmatrix} \beta^* S^{(12)}(z) B^{(2)}(z) \\ \lambda_1^*(z) - \alpha^* B^{(1)}(z) \end{pmatrix} \quad (\text{B.3})$$

$$\begin{pmatrix} c_2 \\ d_2 \end{pmatrix} = \begin{pmatrix} \lambda_2^*(z) - \beta^* B^{(2)}(z) \\ \alpha^* S^{(21)}(z) B^{(1)}(z) \end{pmatrix}. \quad (\text{B.4})$$

Hence, we can get the matrix  $\mathbf{P}$  consisting of the eigenvectors as follows

$$\mathbf{P} = \begin{pmatrix} \beta^* S^{(12)}(z) B^{(2)}(z) & \lambda_2^*(z) - \beta^* B^{(2)}(z) \\ \lambda_1^*(z) - \alpha^* B^{(1)}(z) & \alpha^* S^{(21)}(z) B^{(1)}(z) \end{pmatrix} \quad (\text{B.5})$$

and the inverse matrix

$$\mathbf{P}^{-1} = \frac{1}{\alpha^* S^{(21)}(z) B^{(1)}(z) \beta^* S^{(12)}(z) B^{(2)}(z) - [\lambda_2^*(z) - \beta^* B^{(2)}(z)][\lambda_1^*(z) - \alpha^* B^{(1)}(z)]} \cdot \begin{pmatrix} \alpha^* S^{(21)}(z) B^{(1)}(z) & \lambda_1^*(z) - \alpha^* B^{(1)}(z) \\ \lambda_2^*(z) - \beta^* B^{(2)}(z) & \beta^* S^{(12)}(z) B^{(2)}(z) \end{pmatrix}. \quad (\text{B.6})$$



Therefore,

$$\begin{aligned}
 \mathbf{A}^n &= \mathbf{P} \begin{pmatrix} \lambda_1^{*n}(z) & 0 \\ 0 & \lambda_2^{*n}(z) \end{pmatrix} \mathbf{P}^{-1} \\
 &= \begin{pmatrix} \beta^* S^{(12)}(z) B^{(2)}(z) & \lambda_2^*(z) - \beta^* B^{(2)}(z) \\ \lambda_1^*(z) - \alpha^* B^{(1)}(z) & \alpha^* S^{(21)}(z) B^{(1)}(z) \end{pmatrix} \begin{pmatrix} \lambda_1^{*n}(z) & 0 \\ 0 & \lambda_2^{*n}(z) \end{pmatrix} \\
 &= \frac{1}{\alpha^* S^{(21)}(z) B^{(1)}(z) \beta^* S^{(12)}(z) B^{(2)}(z) - [\lambda_2^*(z) - \beta^* B^{(2)}(z)][\lambda_1^*(z) - \alpha^* B^{(1)}(z)]} \\
 &\quad \cdot \begin{pmatrix} \alpha^* S^{(21)}(z) B^{(1)}(z) & \lambda_1^*(z) - \alpha^* B^{(1)}(z) \\ \lambda_2^*(z) - \beta^* B^{(2)}(z) & \beta^* S^{(12)}(z) B^{(2)}(z) \end{pmatrix} \\
 &= \frac{1}{\delta(z)} \begin{pmatrix} \epsilon(z) & \zeta(z) \\ \eta(z) & \kappa(z) \end{pmatrix}. \tag{B.7}
 \end{aligned}$$

where

$$\begin{aligned}
 \delta(z) &= \alpha^* S^{(21)}(z) B^{(1)}(z) \beta^* S^{(12)}(z) B^{(2)}(z) - [\lambda_2^*(z) - \beta^* B^{(2)}(z)][\lambda_1^*(z) - \alpha^* B^{(1)}(z)] \\
 \epsilon(z) &= \alpha^* S^{(21)}(z) B^{(1)}(z) \beta^* S^{(12)}(z) B^{(2)}(z) \lambda_1^{*n}(z) \\
 &\quad - \lambda_2^{*n}(z) [\lambda_1^*(z) - \alpha^* B^{(1)}(z)][\lambda_2^*(z) - \beta^* B^{(2)}(z)] \\
 \zeta(z) &= \beta^* S^{(12)}(z) B^{(2)}(z) [\lambda_2^*(z) - \beta^* B^{(2)}(z)][\lambda_2^{*n}(z) - \lambda_1^{*n}(z)] \\
 \eta(z) &= \alpha^* S^{(21)}(z) B^{(1)}(z) [\lambda_1^*(z) - \alpha^* B^{(1)}(z)][\lambda_1^{*n}(z) - \lambda_2^{*n}(z)] \\
 \kappa(z) &= \alpha^* S^{(21)}(z) B^{(1)}(z) \beta^* S^{(12)}(z) B^{(2)}(z) \lambda_2^{*n}(z) \\
 &\quad - \lambda_1^{*n}(z) [\lambda_1^*(z) - \alpha^* B^{(1)}(z)][\lambda_2^*(z) - \beta^* B^{(2)}(z)].
 \end{aligned}$$

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