

# Knots and Links in Physical Systems <sup>1</sup>

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**Abstract:** Few physical systems with topologies more complicated than simple gaussian linking have been explored in detail. Here we focus on examples with higher topologies in non-relativistic quantum mechanics and in QCD.

## 1 Generalized Aharonov-Bohm and Josephson effects

Topology has played an important role in physics in recent years, but unlike in biology, most work done so far has involved the simplest nontrivial topology—gaussian linking. Here we discuss more complex topologies and their implications for physical systems.

Let us start by recalling two prominent examples of the use of gaussian linking in non-relativistic quantum mechanics, the magnetic Aharonov-Bohm effect [1] and the Josephson effect [2], see also Ref. [3]. We can generalize both these systems to higher order linking and/or knotting [4].

The magnetic Aharonov-Bohm effect, see Fig. 1, results when a charged particle travels around a closed path in a region of vanishing magnetic field but nonvanishing vector potential. The wave function of the particle is affected by the vector potential and a vector potential dependent interference pattern proportional to magnetic flux occurs at a detection screen. The conclusion one draws is that the vector potential is more fundamental than the magnetic field. The definitive experiments were done here in Japan [5].

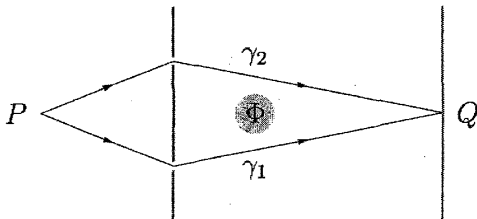


Figure 1: A plane projection of the standard magnetic Aharonov-Bohm effect apparatus.

Shown in Fig. 2 is a schematic of a Borromean ring arrangement to detect the second order phase  $\phi_{12}$ , where  $C_1$  and  $C_2$  are magnetic solenoids carrying flux  $\Phi_1$  and  $\Phi_2$ , and  $C'_3$  and  $C''_3$  correspond to two topologically distinct semi-classical paths and are parts of the closed path  $C_3$  for the wave function of a charged particle starting from the source and ending at the screen [6]. To prevent gaussian linking of the wave function with the solenoids there is a rectangular

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plate in the plane of  $C_1$ . We predict interference proportional to  $(e/\hbar c)^2 \Phi_1 \Phi_2$ . For the case of knotting see Ref. [7].

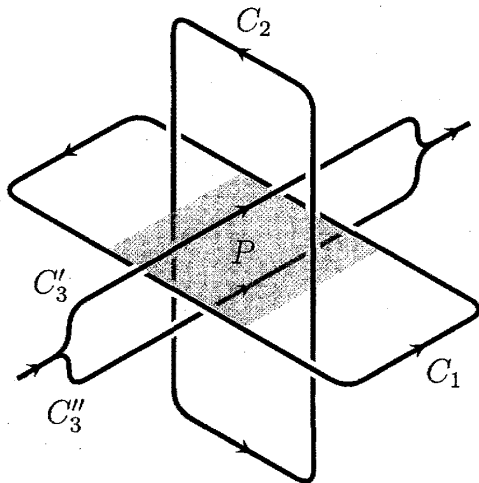


Figure 2: Schematic of a Borromean ring arrangement to detect the second order phase.

A similar generalization of the Josephson experiment to higher order linking, is possible [8], see Fig. 3. There one predicts the maximum current flowing through the superconductor from  $P$  to  $Q$  to be

$$J_{\max} = J_0 \left| \cos \left( \frac{\pi \Phi_1 \Phi_2}{\Phi_0} \right) \right|, \quad (1)$$

where  $\Phi_0$  is the fluxoid.

## 2 Tightly Knotted QCD Flux Tubes as Glueballs

The study of bosonic exchange particles in hadronic physics have a long illustrious history going back to the work of Yukawa [9]. The quark model is sufficient to describe most of the spectrum of hadronic bound states, but after filling the multiplets, a number of states remain and it has been suggested that at least one of these states is a glueball—states with no valance quarks. Two of us have suggested that the glueball spectrum of QCD is a result of tight knots and links

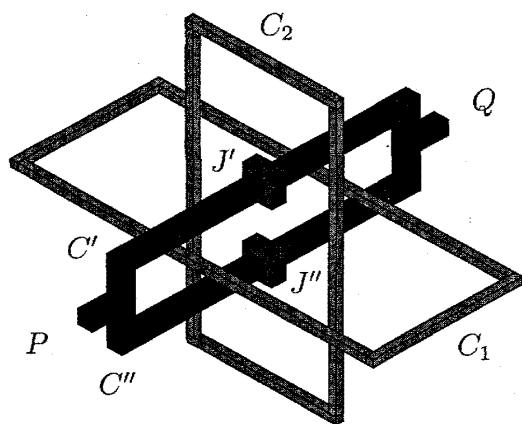


Figure 3: A schematic of a generalized magnetic Josephson effect apparatus.  $J'$  and  $J''$  are Josephson junctions.  $C_1$  and  $C_2$  are solenoids.

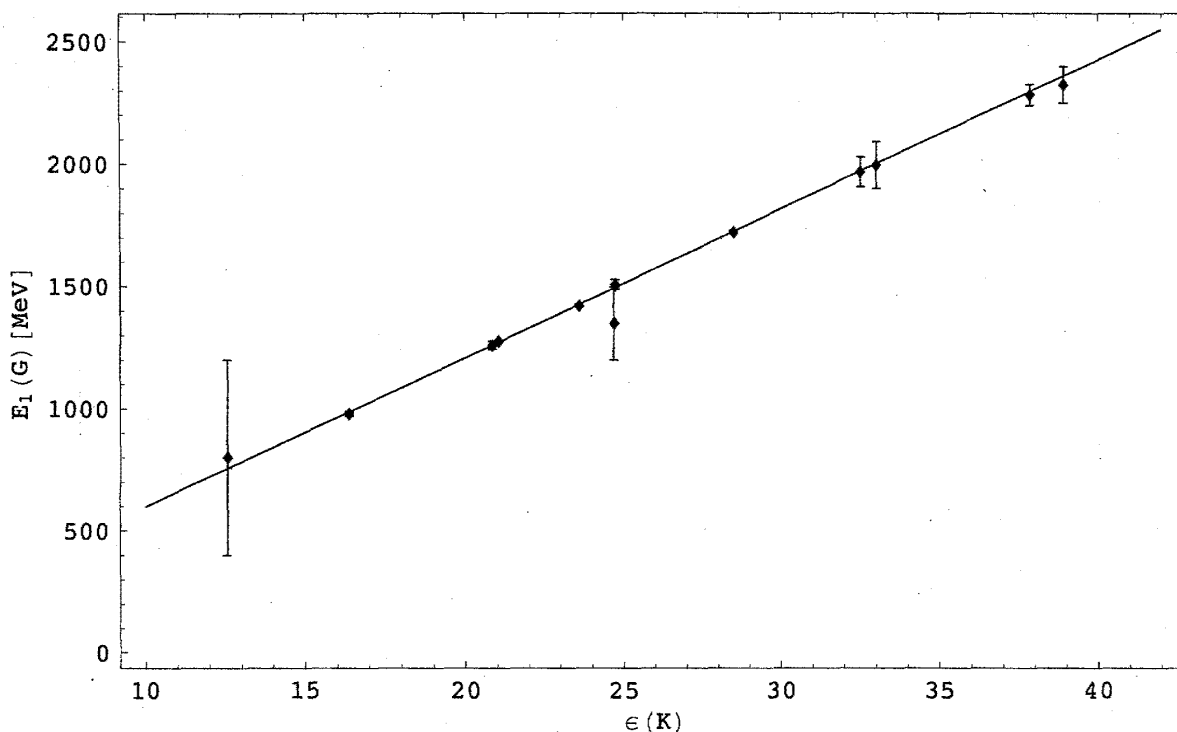


Figure 4: Preliminary fit of  $f_0$  states to the tight knot/link spectrum.

of quantized chromo-electric flux [10]. (The study of tight knots started in biology, see Ref. [11].) This provides an infinite spectrum, up to stability of new hadronic states, and predicts their energies. Identifying knot lengths with particle energies means the glueball spectrum is the same as the tight knot spectrum up to an overall scaling parameter. A preliminary fit matching the most recent knot/link lengths [12] with the presumed glueball states [13] of zero angular momentum, the  $f_0$  states, is shown in Fig. 4, see Ref. [14].

Other systems of tightly knotted fluxes or other types of matter/energy should all generate universal behavior, with spectra all corresponding to the length of knots and links up to a single overall scaling parameter [10].

In the present model, glueball candidates of non-zero angular momentum, called  $f_J$  states, correspond to spinning knots and links. To calculate rotational energies we need the moment of inertia tensor for each tight knot and link. We can get exact results for links with planar components. E.g., in its center of mass frame, the moment of inertia tensor for a Hopf link of uniform density is

$$I_{\text{Hopf}} = \begin{pmatrix} 21 & 0 & 0 \\ 0 & \frac{75}{2} & 0 \\ 0 & 0 & \frac{75}{2} \end{pmatrix} \pi^2 \rho a^5, \quad (2)$$

where one torus is in the  $xz$ -plane and the other is in the  $xz$ -plane. Note this inertia tensor corresponds to a prolate spheroid as do other straight chains of links with an even number of elements. Odd straight chains do not have so much symmetry. Moment of inertia tensors of other knots and links can be calculated by Monte Carlo methods [15]. For further discussion and a detailed analysis of both the zero and non-zero angular momentum  $f_J$  states including calculations of moment of inertia tensor see Ref. [16].

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