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On weak notion of $p-$dividing

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Abstract

I considered the restricted notions of weak dividing. In this note, I try to define a weak notion of $p-$dividing (thorn-dividing).

1. Preliminaries

We recall some definitions.

**Definition 1** Let $\varphi(x_0,x_1,\cdots, x_{n-1})$ be a formula and $p(x)$ be a type. We denote the type $\{\varphi(x_0,x_1,\cdots, x_{n-1})\} \cup p(x_0) \cup \cdots \cup p(x_{n-1})$ by $[p]^\varphi$.

Let $A \subset B$ and $p(x) \in S(B)$.

$p(x)$ divides over $A$ if there is a formula $\varphi(x, b) \in p(x)$ and an infinite sequence $\{b_i : i < \omega\}$ with $b \equiv b_i(A)$ such that $\{\varphi(x, b_i) : i < \omega\}$ is $k-$inconsistent for some $k < \omega$.

$p(x)$ weakly divides over $A$ if there is a formula $\varphi(\overline{x}) \in L_n(A)$ such that $[p[A]^\varphi$ is consistent, while $[p]^\varphi$ is inconsistent.

We can define weak dividing for formulas.

Let $b \notin A$.

$\psi(x,b)$ weakly divides over $A$ if there is a formula $\varphi(\overline{x}) \in L_n(A)$ and a realization $a$ of $\psi(x,b)$ such that $[tp(a/A)]^\varphi$ is consistent, while $[\psi(x,b)]^\varphi$ is inconsistent.

And we can consider weak forking.

$p(x)$ weakly forks over $A$ if there is a $q(x,y) \in S(A)$ such that $p(x) \cup q(x,y)$ is consistent, and any completion $r(x,y) \in S(B)$ of $p(x) \cup q(x,y)$ weakly divides over $A$.

If we exchange the role between variables and parameters in the definition of weak dividing, we could define weak forking naturally.

In this note, we call such formula "$\varphi(\overline{x})$" in the definition above the witness formula of weak dividing for the sake of convenience.

I introduce an example from [3].
Example 2  Let $T$ be the theory of an equivalence relation with two infinite classes of the language $L = \{\text{a binary relation } E(x, y)\}$. And let $\models \neg E(a, b)$. Then the type $tp(a/b)$ does not divide over $\emptyset$, while $tp(a/b)$ weakly divides over $\emptyset$ by the formula $\neg E(x, y)$.

I tried to divide witness formulas into some classes according to their properties ago. And I told about the next characterization at the RIMS meeting last year.

Definition 3  Let $A \subset B$ and $p(x) \in S(B)$.

$p(x)$ $M-weakly$ $divides$ $over$ $A$ if there is a formula $\varphi(\overline{x}) \in L_n(A)$ and a Morley sequence $I = \{a_i : i < n+1\}$ of $p[A]$ such that $\models \varphi(a_0, a_1, \cdots, a_{n-1})$, while the type $[p]^\varphi$ is inconsistent.

Theorem 4  Let $T$ be simple.

Then $T$ is stable if and only if $M-weak$ $dividing$ $over$ $models$ $is$ $symmetric$.

2. Weak notion of $p-$dividing

In recent years another variant of dividing, "thorn"-dividing has been characterized in rosy theory (see e.g. [4]). I tried to define weak notion of $p-$dividing (thorn-dividing). We recall some definitions first.

Definition 5  Let $A \subset B$ and $p(x) \in S(B)$.

$p(x)$ $strongly$ $divides$ $over$ $A$ if there is a formula $\varphi(x, b) \in p(x)$ such that $b \notin acl(A)$ and $\{\varphi(x, b_i) : b_i \models tp(b/A)\}$ is $k$-inconsistent for some $k < \omega$.

$p(x)$ $p-$divides over $A$ if $p(x)$ strongly divides over $A$ for some parameter $c$.

$p(x)$ $p-$forks over $A$ if there is a formula $\varphi(x, b) \in p(x)$ such that $\varphi(x, b)$ implies a finite disjunction of formulas which $p$-divides over $A$.

Given a formula $\varphi$, a set $\Delta$ of formulas in variables $x, y$, a set of formulas $\Pi$ in variables $y, z$, and a number $k$, we define $p(\varphi, \Delta, \Pi, k)(\text{thorn-rank})$ inductively as follows:

1. $p(\varphi, \Delta, \Pi, k) \geq 0, \infty, \lambda$ for limit ordinal $\lambda$ is defined as usual.
2. $p(\varphi, \Delta, \Pi, k) \geq \alpha + 1$ if and only if there is a $\delta \in \Delta$, some $\pi(y, z) \in \Pi$ and parameters $c$ such that
   a. $p(\varphi \land \delta(x, a), \Delta, \Pi, k) \geq \alpha$ for infinitely many $a \models \pi(y, c)$
   b. $\{\delta(x, a)\}_{a \models \pi(y, c)}$ is $k$-inconsistent.

For a type $p$, we define $p(p, \Delta, \Pi, k) = \min\{p(\varphi, \Delta, \Pi, k) : \varphi \in p\}$.

A theory $T$ is rosy if for any type $p(x)$, any finite sets of formulas $\Delta$ and $\Pi$, and any finite $k$, $p(\varphi, \Delta, \Pi, k)$ is finite.

Remark 6  (1) In rosy theories, $p-$forking satisfies the independence axioms.
(2) If $a \models \varphi(x, b)$ and $\varphi(x, b)$ $p$-divides over $C$ by the set $\{b_i \models \theta(y, d)\}$, then $b \subseteq \text{acl}(Cda) - \text{acl}(Cd)$.

Weak notions of $p$-dividing could be defined in many ways. By the definition, $p$-dividing implies dividing. So we expect that weak $p$-dividing implies weak dividing.

**Definition 7** Let $b \notin A$.

$\psi(x, b)$ weakly $p$-divides over $A$ if there is a formula $\varphi(\overline{x}) = \exists y \wedge \theta(x_i, y)$ $\in L_n(A)$ and a realization $a$ of $\psi(x, b)$ such that $[\text{tp}(a/A)]^\varphi$ is consistent, while $[\psi(x, b)]^\varphi$ is inconsistent.

We define weak $p$-dividing ($p$-forking) for types just like weak dividing (forking).

We can check the next fact easily.

**Fact 8** Let $T$ be rosy. Then $p$-forking implies weak $p$-forking.

3. Weak $p$-dividing and NIP theories

**Definition 9** A formula $\varphi(x, y)$ has the independence property if for every $n < \omega$, there are sequences $a_l$ ($l < n$) such that for every $w \subseteq n$, $\models (\exists x) \left[ \bigwedge_{i < n} \varphi(x, a_i)^{\text{if}(l \in w)} \right]$.

A theory $T$ is NIP if no formula $\varphi(x, y)$ has the independence property.

Weak $p$-dividing is a kind of algebraic extension.

**Lemma 10** ($T$ is any theory.) $A \subseteq B$.

Then $\text{tp}(a/B)$ does not weakly $p$-divide over $A$ if and only if

for any $n < \omega$, any $C$ and any extension $q(x, C, A)$ of $\text{tp}(a/A)$ over $AC$, if $\bigcup_{i < n} q(x_i, C, A)$ is consistent, then $\bigcup_{i < n} q(x_i, Z, A) \cup \bigcup_{i < n} r(x_i, Y, A)$ is consistent where $\text{tp}(a/B) := r(x, B, A)$.

By the lemma above, we can prove the next fact.

**Proposition 11** Let $T$ be NIP and unstable.

Then weak $p$-dividing is not symmetric.
References