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ON THE KOTTWITZ-SHELSTAD NORMALIZATION OF TRANSFER FACTORS FOR AUTOMORPHIC INDUCTION FOR GL\(_n\) (JOINT WORK WITH K. HIRAGA)

ATSUSHIICHINO

This note is a report on a joint work with Kaoru Hiraga. Details will appear elsewhere.

Automorphic induction for GL\(_n\) over a p-adic field is an example of endoscopic transfer and its character identity was established by Henniart and Herb [2], up to a constant. We discuss a relation of this constant to the Kottwitz-Shelstad transfer factor [5], in particular, to the normalization using \(\epsilon\)-factors.

Let \(F\) be a non-archimedean local field of characteristic zero. Let \(G = \text{GL}_n(F)\) and \(a\in \text{H}^1(W_F, Z(\hat{G}))\), where \(W_F\) is the Weil group of \(F\) and \(Z(\hat{G})\) is the center of the dual group of \(G\). Let \((H, \mathcal{H}, s, \xi)\) be an endoscopic data for \((G, \text{id}, a)\) (see [5]). Then we have a map

\[
\text{Tran}^G_H : \{(\text{stable}) \text{ invariant distributions on } H\} \rightarrow \{\text{twisted invariant distributions on } G\}
\]

defined as follows.

Let \(\omega\) be the character of \(F^\times\) associated to \(a\). We write \(\omega(g) = \omega(\det g)\) for \(g \in G\). For a (strongly) regular semisimple element \(\gamma \in G\) such that \(G_{\gamma} \subset \text{ker } \omega\) and \(f^G \in C_c^\infty(G)\), put

\[
O^\omega_\gamma(f^G) = \int_{G_\gamma \backslash G} \omega(g) f^G(g^{-1} \gamma g) \, dg,
\]

where \(G_{\gamma}\) is the centralizer of \(\gamma\) in \(G\). Similarly, for a (strongly) \(G\)-regular semisimple element \(\gamma_H \in H\) and \(f^H \in C_c^\infty(H)\), put

\[
O_{\gamma_H}(f^H) = \int_{H_{\gamma_H} \backslash H} f^H(h^{-1} \gamma_H h) \, dh,
\]

where \(H_{\gamma_H}\) is the centralizer of \(\gamma_H\) in \(H\). Here we choose suitable Haar measures on \(G, G_{\gamma}, H, \) and \(H_{\gamma_H}\). By a result of Waldspurger [7], for
each \( f^G \in C_c^\infty(G) \), there exists \( f^H \in C_c^\infty(H) \) such that

\[
O_{\gamma_H}(f^H) = \sum_{\gamma} \Delta(\gamma_H, \gamma) O^\omega_{\gamma}(f^G)
\]

for all \( G \)-regular semisimple elements \( \gamma_H \in H \). Here the sum is taken over a set of representatives for the conjugacy classes of \( \gamma \in G \) whose norm is \( \gamma_H \) and \( \Delta \) is a transfer factor (see [5]). Since \( G \) is quasi-split over \( F \), we can normalize \( \Delta \) using Whittaker data and \( \varepsilon \)-factors as in [5, §5.3]. For an invariant distribution \( D \) on \( H \), we define a twisted invariant distribution \( \text{Tran}_H^G(D) \) by

\[
\text{Tran}_H^G(D)(f^G) = D(f^H)
\]

for \( f^G \in C_c^\infty(G) \).

On the other hand, by a result of Henniart and Herb [2], for each irreducible tempered admissible representation \( \pi_H \) of \( H \), there exist an irreducible tempered admissible representation \( \pi \) of \( G \) and a constant \( c \in \mathbb{C}^\times \) such that \( \pi \otimes \omega \cong \pi \) and

\[
\text{Tran}_H^G(\Theta_{\pi_H}) = c \cdot \Theta_{\pi}^\omega.
\]

Here \( \Theta_{\pi_H}(f^H) = \text{trace}(\pi_H(f^H)) \) for \( f^H \in C_c^\infty(H) \) and \( \Theta_{\pi}^\omega(f^G) = \text{trace}(\pi(f^G) \circ A_\omega) \) for \( f^G \in C_c^\infty(G) \), where \( A_\omega : \pi \otimes \omega \rightarrow \pi \) is an isomorphism as vector spaces. Since \( \pi \) is generic, we can normalize \( A_\omega \) using Whittaker functionals. By a result of Henniart and Lemair [3], the constant \( c \) does not depend on the representations.

Our main result is as follows.

**Theorem 1.** We have

\[ c = 1. \]

**Remark 2.** An analogous result for \( F = \mathbb{R} \) was proved by Henniart [1].

**References**


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