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Kyoto University
ON THE KOTTWITZ-SHELSTAD NORMALIZATION
OF TRANSFER FACTORS FOR AUTOMORPHIC
INDUCTION FOR $GL_n$
(JOINT WORK WITH K. HIRAGA)

ATSUSHI ICHINO

This note is a report on a joint work with Kaoru Hiraga. Details will appear elsewhere.

Automorphic induction for $GL_n$ over a $p$-adic field is an example of endoscopic transfer and its character identity was established by Henniart and Herb [2], up to a constant. We discuss a relation of this constant to the Kottwitz-Shelstad transfer factor [5], in particular, to the normalization using $\varepsilon$-factors.

Let $F$ be a non-archimedean local field of characteristic zero. Let $G = GL_n(F)$ and $a \in H^1(W_F, Z(\hat{G}))$, where $W_F$ is the Weil group of $F$ and $Z(\hat{G})$ is the center of the dual group of $G$. Let $(H, \mathcal{H}, s, \xi)$ be an endoscopic data for $(G, \text{id}, a)$ (see [5]). Then we have a map

$$\text{Tran}_G^H : \{(\text{stable) invariant distributions on } H\} \longrightarrow \{(\text{twisted) invariant distributions on } G\}$$

defined as follows.

Let $\omega$ be the character of $F^\times$ associated to $a$. We write $\omega(g) = \omega(\det g)$ for $g \in G$. For a (strongly) regular semisimple element $\gamma \in G$ such that $G_{\gamma} \subset \ker \omega$ and $f^G \in C_c^\infty(G)$, put

$$O_\gamma^\omega(f^G) = \int_{G_{\gamma} \backslash G} \omega(g) f^G(g^{-1} \gamma g) \, dg,$$

where $G_{\gamma}$ is the centralizer of $\gamma$ in $G$. Similarly, for a (strongly) $G$-regular semisimple element $\gamma_H \in H$ and $f^H \in C_c^\infty(H)$, put

$$O_{\gamma_H}(f^H) = \int_{H_{\gamma_H} \backslash H} f^H(h^{-1} \gamma_H h) \, dh,$$

where $H_{\gamma_H}$ is the centralizer of $\gamma_H$ in $H$. Here we choose suitable Haar measures on $G, G_{\gamma}, H,$ and $H_{\gamma_H}$. By a result of Waldspurger [7], for
each $f^G \in C^\infty_c(G)$, there exists $f^H \in C^\infty_c(H)$ such that
\[ O_{\gamma H}(f^H) = \sum_{\gamma} \Delta(\gamma_H, \gamma) O^\omega_{\gamma}(f^G) \]
for all $G$-regular semisimple elements $\gamma_H \in H$. Here the sum is taken over a set of representatives for the conjugacy classes of $\gamma \in G$ whose norm is $\gamma_H$ and $\Delta$ is a transfer factor (see [5]). Since $G$ is quasi-split over $F$, we can normalize $\Delta$ using Whittaker data and $\varepsilon$-factors as in [5, §5.3]. For an invariant distribution $D$ on $H$, we define a twisted invariant distribution $\text{Tran}^G_H(D)$ by
\[ \text{Tran}^G_H(D)(f^G) = D(f^H) \]
for $f^G \in C^\infty_c(G)$.

On the other hand, by a result of Henniart and Herb [2], for each irreducible tempered admissible representation $\pi_H$ of $H$, there exist an irreducible tempered admissible representation $\pi$ of $G$ and a constant $c \in \mathbb{C}^\times$ such that $\pi \otimes \omega \cong \pi$ and
\[ \text{Tran}^G_H(\Theta_{\pi_H}) = c \cdot \Theta^\omega_{\pi}. \]
Here $\Theta_{\pi_H}(f^H) = \text{trace}(\pi_H(f^H))$ for $f^H \in C^\infty_c(H)$ and $\Theta^\omega_{\pi}(f^G) = \text{trace}(\pi(f^G) \circ A_\omega)$ for $f^G \in C^\infty_c(G)$, where $A_\omega : \pi \otimes \omega \to \pi$ is an isomorphism as vector spaces. Since $\pi$ is generic, we can normalize $A_\omega$ using Whittaker functionals. By a result of Henniart and Lemair [3], the constant $c$ does not depend on the representations.

Our main result is as follows.

**Theorem 1.** We have $c = 1$.

**Remark 2.** An analogous result for $F = \mathbb{R}$ was proved by Henniart [1].

**References**


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