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A force theory describing several two-particle systems from subatomic to biologic

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A quasi-stability concept applied to a momentum conservation law reveals the dynamics underlying the magic number and the shell model, i.e., the reason why uranium 235 mainly leads to the weight ratios between asymmetric 2:3 and symmetric 1:1 in the resultant smaller child atoms and also the reason why larger atoms such as Th have larger ratios close to 2:3. It reveals the reason why the nitrogenous bases in biological base pairs of nucleic acids, biological cells divided, liquid droplets broken, and stars also show frequencies in size ratios between 1:1 and about 2:3 (between the golden and silver ratios). Then, the higher-order of analysis also clarifies the other ratios over 2:3 for atoms and biological amino acids.

1. Introduction

A neutron impacting uranium 235 produces smaller child atoms that often have an asymmetric weight ratio of about 2:3 between the golden and silver ratios. In contrast, varying the impact speed of neutrons results in a nearly symmetric division of uranium 235. [1] The liquid droplet model proposed by Bohr [2] and recent works based on energy conservation [3] have not revealed much about the fusion of symmetry of 1:1 and asymmetry of around 2:3, i.e., the further dynamics underlying the magic number and the shell model.

Then, purines and pyrimidines in biological base-pairs, biological cells after divisions, and stars in the cosmos also have a fusion of symmetry of 1:1 and asymmetry of around 2:3. [4-7] A model we developed previously based on fluid dynamics has revealed the reason why a neutron impacting uranium 235, the nitrogenous bases in biological base pairs of nucleic acids, cells, and stars often lead to the bimodal feature of symmetry of 1:1 and asymmetry of around 2:3 on the size ratios.[4-7] The fusion of symmetry and asymmetry appears in various systems from atom to star, because each system commonly stems from a "breakup of flexible particles deformed". However, the model was analyzed by the first order of accuracy. Thus, the present paper shows some further possibilities based on a higher-order of analysis.

2. Model [4-7]

Here, we define a parcel as a flexible spheroid having two long and short radii of $a(t)$ and $b(t)$ dependent on time t, for the aggregation of neutrons and protons in each child atom resulting from the fission of a uranium 235 atom, a nitrogenous base in biological base-pair of nucleic acid hydrated with a lot of water molecules, biological cell, and star at breakup in the cosmos. The parcel becomes a sphere of the radius $r_d(=[ab^2]^{1/3})$ under an equilibrium condition. The deformation rate $\gamma(t)$ is defined as

$a(t)/b(t)$, while a sphere without deformation corresponds to $\gamma=1$.

Then, we consider the form of two spheroid parcels connected in line at the time of the breakup processes of uranium 235, at the replication stage of biological base-pair, at cell division, and at division of star. We derive a theory for describing the deformation and motions of the two connected spheroid parcels having two radii of r_{d1} and r_{d2} under equilibrium conditions and two deformation rates of $\gamma_k\,[k=1,2]$, while the size ratio of the two parcels is defined by $\varepsilon=r_{d1}/r_{d2}$.

We model the relative motion between the two parcels, nonlinear convections inside the parcels, and the interfacial force at the parcel surface. The interfacial force is evaluated in the form of σ/r^m where m and σ are constants and r is the curvature of parcel surface. Several types of forces such as nuclear force, van der Waals force, surface tension, coulomb force, and gravity can be approximated by varying m. The relation $m=1$ implies the surface tension of liquid. The mean density of the parcel is ρ_L.

Existing experimental data on breakup processes of liquid droplets of water and oil and biological cells show spheroid shapes at the timing of breakup or the later stage of breakup, which is the rate-determining stage. This also leads to a possibility that, for several levels of parcel breakups from subatomic to cosmic ones, the parcels will take approximately spheroid shape at the timing of breakup or the later stage after deformation. Thus, the present model reveals the essential principle (the specific size ratio of about 2:3) underlying the fission process, biological divisions, and stars, whereas multi-dimensional calculations using supercomputers shows the process in detail.

We also assume that the convection flow inside parcel is irrotational, i.e., potential one [10]. There are random collisions of water molecules and electrons with the parcel such as biological molecules and cells. It is stressed that these random collisions from the outer region induce potential flow inside the flexible continuum particle, i.e., irrotational flow, because the fluctuations of impulsive starts and stops generate potential flow [10]. This potential flow is also applicable, because fluctuations entering the parcels such as bases, cells, and atoms will be close to those of thermal fluctuations, which are less dissipative. (Fluctuation dissipation theorem) It is well-known that the potential flow assumption is also applicable for liquid droplets. [4] The potential assumption will be no problems also for stars, because of large size and high speed.

Moreover, we must consider that a parcel is not often a continuum, because the number of nucleons and water molecules inside the parcels for atom and nitrogenous base will be less than the order of 1000. The scale for averaging, i.e., the minimum scale representing the phenomenon, will be smaller than that in continuum mechanics. Thus, this small averaging window leads to a weak indeterminacy of physical quantities such as deformation rate and density because of discontinuity of nucleon or molecule.

The system of two flexible parcels in one-dimensional connection is modeled with the above assumptions and also purely mathematical transformation.

Here, we derive the relation between the dimensionless deformation rate $\gamma_k(\equiv a_k/b_k\,[k=1,2])$ of each parcel dependent on dimensionless time $\bar{t}_k=\sqrt{\dfrac{8\sigma}{\rho_L r_{dk}^{\,2+m}}}\,t\quad[k=1,2]$ and the size ratio of the two parcels of $\varepsilon=r_{d1}/r_{d2}$.

The stochastic governing equation ($\gamma - \varepsilon$ equation) having indeterminacy can be described as

$$\frac{d^2}{d\bar{t}_i^{\,2}}\gamma_i = \{[(-\varepsilon - \varepsilon^4 + \frac{2}{3}\varepsilon E_{0j}\gamma_j^{-1/3})B_{0i} + \frac{2}{9}\varepsilon^{2+m}E_{0i}\varepsilon\gamma_i^{-4/3}](\frac{d}{d\bar{t}_i}\gamma_i)^2$$
$$+[\frac{2}{3}\varepsilon^{2+m}E_{0i}\gamma_j^{-1/3}B_{0j} - \frac{2}{9}\varepsilon^{2+m}E_{0i}\gamma_j^{-4/3}](\frac{d}{d\bar{t}_j}\gamma_j)^2$$
$$+(-\varepsilon - \varepsilon^4 + \frac{2}{3}\varepsilon E_{0j}\gamma_j^{-1/3})C_{0i}\,\gamma_i^{\frac{5}{3}-\frac{2}{3}m} + \frac{2}{3}\varepsilon^{2+m}E_{0i}\gamma_j^{-1/3}C_{0j}\,\gamma_j^{\frac{5}{3}-\frac{2}{3}m}\}/Det$$
$$+\delta_{st} \qquad (1)$$

with [for $i = 1, 2.\ j = 1, 2.\ i \neq j$]

$$Det = -\varepsilon - \varepsilon^4 + \frac{2}{3}\varepsilon^4 E_{0i}\gamma_i^{-1/3} + \frac{2}{3}\varepsilon\ E_{0j}\gamma_j^{-1/3},\ B_{0k} = \frac{1}{3\gamma_k}\frac{\gamma_k^{\,2}-2}{\gamma_k^{\,2}-1/2},$$
$$C_{0k} = \frac{3}{8}\frac{2\gamma_k^{\,2m} - 1/\gamma_k^{\,m} - \gamma_k^{\,m}}{\gamma_k^{\,2}-1/2},\ \text{and}\ E_{0k} = 3\frac{\gamma_k^{\,7/3}}{\gamma_k^{\,2}-1/2}\ [\text{for}\ k = 1, 2]$$

where the parameter δ_{st} denotes random fluctuation.

The long derivation of Eq. (1) is in Ref. 5 confirmed by the referees, although only the stochastic term δ_{st} is not in Ref. 5. It is also stressed that this system is not the simple two-body problem of rigid body, because of flexible nonlinear deformations of the parcels.

We then define the deviation from a sphere as y_i, which is equal to $\gamma_i - 1$. Taking the first order of approximation in the Taylor series leads to

$$\frac{d^2 y_i}{d\bar{t}_i^{\,2}} = [-\frac{2}{3}(3-\varepsilon^3-2\varepsilon^{2+m})\left(\frac{dy_i}{d\bar{t}_i}\right)^2 + 3(3-\varepsilon^3)my_i - 4\varepsilon^{1+m}\left(\frac{dy_j}{d\bar{t}_j}\right)^2 + 12\varepsilon^{1+m}my_j]/[3(\varepsilon^3+1)]$$
$$+\delta'_{st}, \qquad (2)$$

where the parameter δ'_{st} denotes random fluctuation.

[The effect of special theory of relativity between mass and energy will not influence for the size ratio of child atoms obtained after the fission process of uranium 235 very much, because the effect will work on both two child atoms at an identical rate. Moreover, only an impact of only one neutron to uranium will not produce fast deformation of uranium at the later stage of fission.]

3. Size (Representative length)

Equation 2 shows that a symmetric ratio of 1.0 (ε=1) makes the first term on the right-hand side of the equation zero, while an asymmetric ratio of $\sqrt[3]{3}$ around 1.5 (ε^3=3) makes the second term zero for each m. (The size ratios of 1.00 and approximately 1.50 can be described by the unified number of the n-th root of n.)

We define a system as being quasi-stable when only one term on the right-hand side of the differential equation system governing the phenomenon is zero. (The quasi-stable principle may also be defined by the condition that either term of volume or surface forces is zero.) The system of two parcels connected in line is relatively quasi-stable because d^2x/dt^2 becomes smaller when the size ratio of connected parcels takes the values of ε=1 or ε^3=3.

These ratios of 1:1 and about 1:1.5 correspond to those of child atoms generated by the breakup of uranium 235. As Eqs. 1 and 2 show a slightly vague solution for the phenomenon, this indeterminacy also implies that size variations of ε are possible in a limited range. This indeterminacy permits the possibility of sizes around 1:1 and also around 1: $\sqrt[3]{3}$, i.e., between 1:1 and about 2:3, although 1:1 and about 2:3 are more.

Next, let us look at biological systems containing water flows. We can classify the five bases of adenines (A), guanines (G), cytosines (C), thymines (T), and uracils (U) into two groups: purines and pyrimidines. Purines, i.e., A and G, have a relatively large size, while pyrimidines, i.e., C, T, and U, are small. Asymmetric base pairs such as the Watson-Crick type of about 1: $\sqrt[3]{3}$ are used in living beings. This grouping specifically refers to the asymmetric size ratio of purines and pyrimidines of around 1.50 in their hydrogen bonds within DNA and RNA, although a symmetric size ratio of 1.00 is often observed in RNA. [8] Symmetric and asymmetric size ratios are also observed at the cell level of microorganisms such as yeast. [4, 5, 6]

The concept of quasi-stability is weaker than neutral stability. [5, 6] The quasi-stability concept is necessary for living beings, because stronger stability cannot bring variations, i.e., adaption for environmental change and evolution. This quasi-stability is also possible for nonliving systems such as atoms, because atoms also vary in a long time. (This is also because atomic and biological systems have dense interaction between nucleons in a closed volume.)

The quasi-stable ratios of 1: $\sqrt[n]{n}$ for n=1 and 3 appear for each m (see Eq. 2). This universality also leads to the possibility that the present model can be applied for several levels of parcels from baryons to stars in the cosmos [9]: specifically, at the level of nuclear force, coulomb force, van der Waals force, surface tension, and force of gravity. [It is emphasized here that a stability analysis based on a mathematical variable transformation for matrix diagonalization is meaningless for revealing the nature of the phenomenon, because natural phenomena are in physical space, not in mathematical space. It is understood because elimination of one term in the Navier-Stokes equation including nonlinear convection, pressure, and viscosity terms essentially changes the solution. The Navier-Stokes equation should be analyzed in physical space for a lot of cases and conditions.]

Let us consider the reason why the energy conservation law and variation principle such as the Bohr model do not explain the fusion of symmetry of 1:1 and asymmetry of around 2:3 in size and density (number density). The first reason is that the momentum conservation in Eq. 1 models all of the relative motion between two parcels, nonlinear convections inside the parcels, interfacial forces at the parcel surfaces, and collisions with smaller molecules, whereas previous models based on energy conservation have tended to eliminate the relative motion between parcels. A second reason is that the quasi-stability principle was proposed instead of the variation principle.

Another important point is that natural systems should be explained in terms of four conservation concepts regarding mass, momentum, moment, and energy, as is seen in the thermo-fluid dynamics. Conservative quantity is not only energy. The present new explanation for two particle systems is necessary, because explanations based on two conservativity principles such as momentum and energy should be done. The following section also shows the importance of mass conservation law.

4. Volume and weight ratios

The weight and size (representative length) ratios of purines and pyrimidines are about 1.5. The asymmetric pair of flexible spheroid parcels of "purines surrounded by water molecules" and "pyrimidines surrounded by water molecules" will also have the size (length) ratio between 1.40-1.50. The weight and size (length) ratios of nitrogenous bases are proportional to the parcel size ratio (parcel length ratio). However, the weight and volume ratios of the two spheroid parcels are 3.0. The difference between the weight ratios of parcels and nitrogenous bases, i.e., about 1.5 for the weight ratio of bases and 3.0 for the weight ratio of parcels, is possible, because the molecular weight ratio of the parcels including water is larger than that of the purine-pyrimidine bases and also because the electric density distribution of Pai electrons will not be in a two-dimensional distribution, while the nitrogenous bases have one-dimensional string (ring).

Emphasis is placed on the fact that the size (length) and weight ratios of child atoms broken up from uranium 235 are also close to 2:3, because atomic cores will be mathematically similar to one-dimensional strings distorted or rings.

What is the difference between the weight ratio of two child atoms separated from uranium 235 and the weight ratio of parcels including child atoms? It will be possible that the weight ratio of parcels is larger than that of protons and neutrons in atom or child atoms split from uranium 235, because of immerse mass due to gluon, quark condensation, and the relativity theory effect. Immerse mass proportional to the parcel weight only appears while deformation and breakup occur. The immerse mass can be understood analogically from the fluid dynamic effect in which impulsive deformation motion is accompanied with added mass [10]. This immerse mass produces a weight ratio of 3.0 for the parcels, although the weight ratio of child atoms is about 1.5. Parcels smaller than cells may be with immerse mass, which may come from lack of observation techniques. The weight ratios of liquid droplets, biological cells, and stars will be equal to those of parcels.

5. Number ratio

It is also well known that several atoms in nature have the number ratios of protons and neutrons between 1:1 and 2:3. Here, let us examine the reason why larger atoms have larger number ratios close to 2:3.

The first and second terms on the right-hand side of Eq. 2 also show that symmetric ($e=1$) and asymmetric ($e^3=3$) parcel divisions are more stable for small disturbances of motion (dx/dt) and parcel deformation x, respectively (Fig. 1).

Let us separate baryon aggregation into two parts: the internal side around the center of the aggregation and the external side close to the surface. External baryons close to the surface move relatively easily, because one part of the baryon is free without any connection to other baryons. However, internal baryons often receive forces from many directions due to the presence of other ones, making it relatively difficult for them to move relative to the origin on the earth. Thus, inner baryons deform relatively easily without any translational motion of the gravity center. As a result, the inner and outer baryons determine whether baryons are asymmetric or symmetric, respectively. [6, 7] An important point is that the concept of inner asymmetry and outer symmetry can be seen.

Larger aggregations of parcels such as Thorium (Th) including baryons more than Helium (He) have more inner baryons, because the surface/volume ratio of the aggregation becomes smaller as size increases. (Larger aggregations have smaller surface, because the volume is proportional to the cubic value of the representative length L, while the surface increases with the square value of L.)

More inner baryons for larger atoms bring more asymmetric number ratio of protons and neutrons, as larger nucleic acids such as rRNA also have more asymmetric number ratios of purines and pyrimidines close to 2:3. (Fig. 2) This is because of mass conservation law, i.e., because heavier particles such as purines and protons can be generated with less numbers. [6, 7]

A mysterious thing is that the masses of proton and neutron are almost same, while child atoms generated by fission of uranium 235 and nitrogenous bases (pyrimidine and purine) have the different weight ratio around 2:3. Proton is qualitatively a little heavier than neutron. Thus, relatively heavy protons can be with less number in atomic core, because of mass conservation law. We may be able to explain quantitatively this mystery by the fact that the mass ratio of proton and neutron are around 2:3 as the forms such as lambda particle before atomic cores are stabilized, because the up and down quarks have the weight ratio between 1:1 and 1:2. Thus, the later stage of fission process for generating atomic cores is described by the present theory. Then, the final or post-fission stage stabilizes the atomic cores of protons and neutrons.

This explanation is possible by the analogy with biological system, because nitrogenous bases are also connected with heavy molecules such as ribose or deoxyribose, which lead to nucleic acids. The weight ratio of purine- deoxyribose and pyrimidine- deoxyribose is close to 1:1, i.e., the symmetric value.

The present theory also indicates that the number ratios around the center between 1:1 and about 1:1.5, i.e., around 1:1.3, will be relatively unstable. (Fig. 2) This can be understood by the fact that both fluctuations of deformation and velocity enter into the parcels connected with the medium number ratios of about 1:1.3, whereas only velocity fluctuations enter into small parcels such as those of Helium (He) and only deformations occur for large atoms such as Thorium (Th). Actually, atoms, which are a little larger than lead (Pb), are unstable.

Atoms larger over 300 nucleons are unstable. This can be explained by the fact that large deformations enter into the atoms, because larger atoms get larger deformations.

Thus, this theory explains the reason why relative heavy atoms have more asymmetric number ratios close to 2:3 and also why the atoms having the number ratios around 1:1.3 are relatively unstable, whereas the energy conservation law such as those of Bohr clarifies the inevitability that neutron-rich system is more stable than proton-rich one.

The quasi-stability concept applied to momentum conservation law and mass conservation law bring the result compatible with that the Bohr's drop model based on the variation principle for energy conservation law and also gives a further physics underlying the Shell model related to the magic number.

6. Taylor series

Next, we take a higher order of the Taylor series for Eq. 1. Even-numbered terms such as the second and fourth ones show no other quasi-stable size ratios. However, odd-numbered terms result in other quasi-stable ratios. The third term in the Taylor series results in a quasistable ratio of about 3.5, the fifth term in ratios of about 2.5 and 2.1, and the seventh in a ratio of about 1.78. An important point is that the terms of orders higher than the ninth are absolutely unstable, which leads to the existence of the maximum size limit of atoms. (Table 1)

It is also stressed that liquid fuel droplets generated by injectors and child atoms broken up from the fission of uranium 235 also have a threefold variation of sizes at the maximum [1, 4, 5] and also that the molecular weights of the twenty types of amino acids show a threefold variation between 240 of cysteine as the maximum and 75 of glycine as the minimum. The higher-order of analysis clarifies the ratios over 2:3 in several systems. (Table 1)

Free atoms in atmosphere and molecules outside living beings are not with the special ratios such as 1:1, about 2:3, and 3.5, because free atoms and molecules outside cells are sparse, while subatomic particles such as baryons and biological molecules like bases, nucleic acids, amino acids, and proteins are interactional insider closed regions.

7. Conclusion and Outlook

7.1 Spatial structure

Our previous report based on the quasi-stability concept applied to momentum conservation [4-7] revealed the reason why several particles such as biological cells, nitrogenous bases, and liquid droplets have the bimodal size ratios of about 2:3 and 1:1. The present theory can be applied for several levels of parcels from baryons to stars in the cosmos: specifically, at the level of nuclear force, van der Waals force, surface tension, coulomb force, and force of gravity.

The present paper extended with the higher order of analyses also reveals the other ratios over 2:3. The halo structures such as H10 and M32 have the number ratios of neutrons and protons over 2:3. [11] These number ratios similar to those in largest and smallest child atoms broken from uranium 235 and biological molecules such as amino acids will also be explained by the present theory of the higher-order of accuracy.

There are mesons with "two" quarks and baryons with "three". Therefore, an analysis based on Eq. 1 may also clarify the ratios around 1:1 and 2:3 in the elemental particles such as quarks. The quasistable size ratio of about 3.5 leads to a possibility of particles with the mass which is 50 times as heavy as the largest atom existing. We will examine, whether or not this may be top quark.

This model may also reveal the quasi-stable size ratios for the sub-particles inside quarks and for the super-cosmos systems larger than the cosmos, although this model is very simple.

7.2 Temporal structure

Let us think about whether or not the quasi-stability concept can also be applied to the evolutionary and morphogenetic processes.

The macroscopic kinetic model describing the populations of the four main groups of species in the evolutionary process shows that only one group of species does not increase at the time of a transition in evolution, while the other groups vary in time. [Figs. 5 and 17 in reference 12] Thus, a quasi-stable condition, i.e., weakly stable situation before transitions such as fission and evolution, can be redefined by the rule that only a part of the equation system, i.e., only one term or one variable, is invalid at the time of a transition.

There is another quasi-stable example, which is the deterministic model explaining the temporal oscillations occurring in the brain system and morphogenetic process. [13, 14, 15] The model of six variables often shows that, when only one variable among six is at a constant level, drastic transitions occur in morphogenetic and economic processes. It is stressed that the model [13, 14, 15] can also explain the neural network pattern inside brain (topological pattern) related to circadian clock, memory, thinking, pyramidal area, emotion, language, visual and auditory senses, which is dominated with a fusion of symmetric and asymmetric connections. (Fig. 3)

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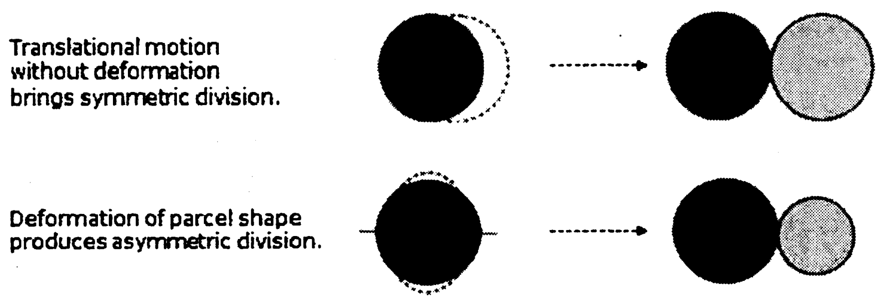

Fig. 1 Asymmetric division due to parcel deformation and symmetric divisions in the absence of deformation.

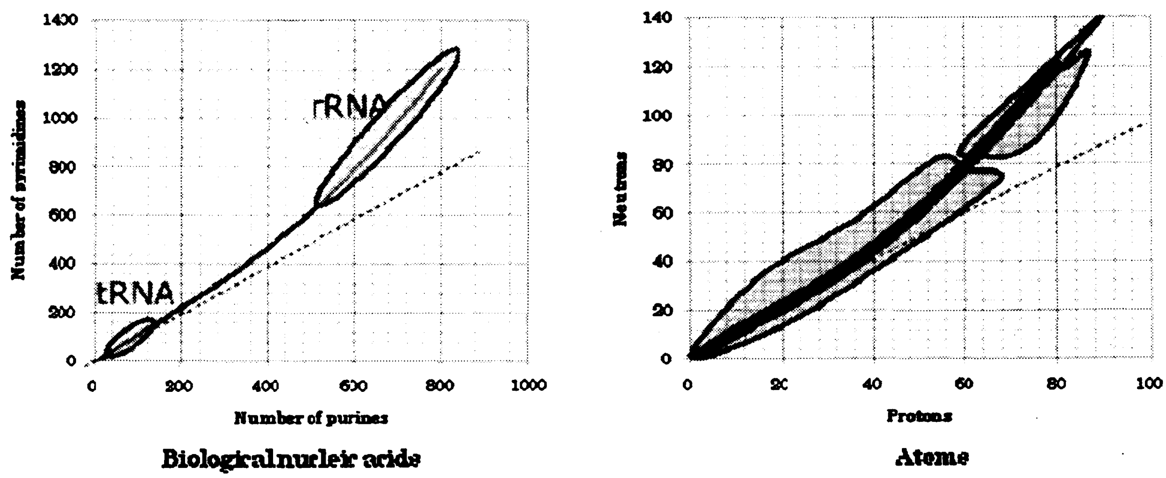

Fig. 2. Density ratio of purines and pyrimidines in RNAs, which is similar to the density ratio of protons and neutrons in stable atom core. (mRNAs are between tRNAs and rRNAs, which are relatively unstable.)

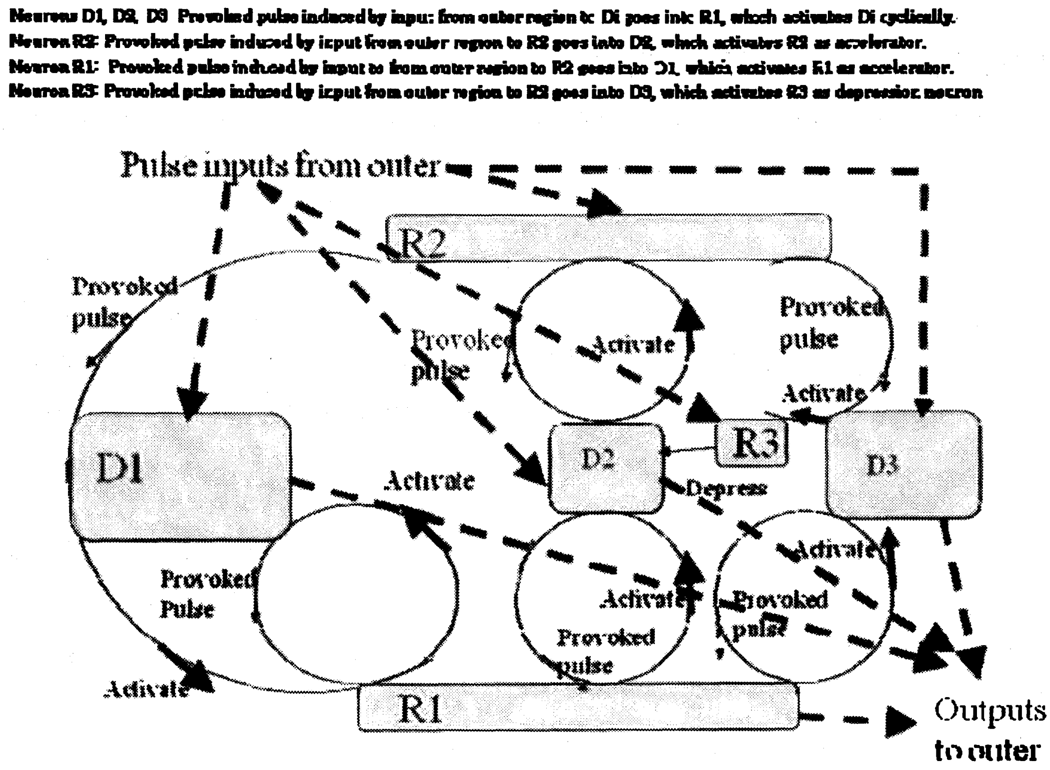

Fig. 3 Fundamental topology of neural network.

Table 1 Quasi-stable ratios observed in atoms and biological systems.

n-th term in Taylor series	Quasi-stable ratio	Atoms	Biological molecules
1	1:1 & about 1.1.5	Main size ratios of child atoms broken from uranium 235. Main number ratios of neutrons and protons in atoms	Nitrogenous bases in DNAs and RNAs (Purines:Pyrimidines).
3	about 1:3.5	Higher limit of size ratios of child atoms broken from uranium. Number ratio of protons and neutrons in He 10	Amino acids
5	about 1:2.5 & about 1:2.1	The intermediate size ratios of child atoms broken from uranium.	rRNA (23S:16S)
7	about 1:1.78	The intermediate size ratios of child atoms broken from uranium.	Ribosome (50S:30S)