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Existence and stability of boundary layers to the Euler-Poisson equation arising in plasma physics

Shinya Nishibata, Masashi Ohnawa,
Department of Mathematical and Computing Sciences,
Tokyo Institute of Technology, Tokyo 152-8552, Japan

Masahiro Suzuki,
Research Institute of Nonlinear Partial Differential Equations,
Waseda University, Tokyo 169-8555, Japan

1 Introduction

In the present paper, we study the stability and the asymptotic behavior of a boundary layer, called a sheath, arising in plasma physics. We begin with a brief explaination of the formation process of the sheath. We consider the situation in which a wall is put in a plasma and is negatively charged. Though plasma is quasi-neutral, the potential gradient attracts positive ions to the wall while repelling most of the electrons. The potential gradient due to this biased distribution of charged particles near the wall remains only within a close neiborhood of the wall, so that the wall is insulated from the rest of the space. Hence the boundary layer is formed. Its typical thickness is given by the Debye distance λ_D. In most cases, the characteristic length scale L, determined by the quantities such as mean free path, the ionization length or the geometry of the system, is much greater than the Debye distance. Hence the sheath is considered to be locally planar and collision free. The research of the sheath was initiated in the 1920s, and in the early pioneer work by Tonks and Langmuir [11], the basic features of the transition between the plasma and the sheath are addressed. They defined the sheath edge as the location where the difference between the solution of the governing equations with the quasi-neutrality and that of governing equation including the Poisson equation becomes apparent. In other words, the sheath edge is the location where quasi-neutrality breaks down. Later, Bohm [4] studied the stationary problem (2.6) with $K = 0$ and obtained the necessary condition for the formation of the sheath. This necessary condition, called the Bohm criterion, is that the mean velocity of the incoming flow of positive ions to the sheath from the inner region is greater than a certain physical constant.

The sheath is intensively studied in plasma physics since the process of sheath formation is important for such industrial applications as material processing, fusion and discharges([6]). In spite of its importance, the definition of the sheath has been left ambiguous because of the difficulty in the consistent understanding of plasma and sheath since different governing equations are used for them. Refer reviews and new insights concerning the sheath to [9] or [1]. Accordingly no mathematical study has been done to validate the Bohm criterion for a long time. Recently Ambroso, Méhats and P.-A. Raviart in [3] showed the existence of the monotone stationary solution to (2.1) under (2.13) over one-dimensional bounded domain. Later Ambroso in [2] numerically showed the solution to (2.1) approaches the stationary solution as time tends to infinity in the same setting as [3]. Suzuki in [10] interpreted the sheath to be a monotone stationary solution to the system of Euler-Poisson equation (2.1) for one-dimensional half space and derived that the Bohm criterion together with the physically natural boundary condition on the electric potential is sufficient for the unique existence of a monotone stationary solution. In [8], asymptotic stability of the stationary solution is proved under (2.13) and also under (2.14). Consequently the Bohm criterion is well justified from the mathematical point of view. The objective of the present paper is to summarize the results obtained in these two papers.

2 Main results

The isothermal flow of positive ions is governed by the Euler-Poisson equations:

$$\rho_t + \mathrm{div}(\rho \mathrm{u}) = 0, \tag{2.1a}$$

$$(\rho u)_t + \mathrm{div}\,(\rho u \otimes u) + K\nabla\rho = \rho\nabla\phi \tag{2.1b}$$

$$\Delta\phi = \rho - e^{-\phi}, \tag{2.1c}$$

where unknown functions ρ, u and ϕ stand for the density and the velocity of positive ions and the electrostatic potential, respectively. Non-negative constant K corresponds to the temperature of ions. Note that ϕ is so scaled that it has opposite sign to the electrostatic potential in physics.

The first equation describes the conservation of mass, and the second one is the equation of momeutum in which the pressure gradient and electrostatic potential gradient as well as the convection effect are taken into account. The third equation is called the Poisson equation, which describes the relation between the potential, the ion density and the electron density. It is obtained by combining the Poisson equation and the Boltzmann relation in which the electron density is given by $\rho_e = e^{-\phi}$.

We study the initial boundary value problem to (2.1) in the N-dimensional half space $\mathbb{R}^N_+ := \{(x_1, \dots, x_N) \in \mathbb{R}^N |\, x_1 > 0\}$ for $N = 1, 2, 3$. Throughout this paper, the space coordinate is denoted by $x = (x_1, \dots, x_N) = (x_1, x')$, where x_1 and $x' = (x_2, \dots, x_N)$ are the normal and the tangential components, respectively. The initial and the boundary data are prescribed as

$$(\rho, u)(0, x) = (\rho_0, u_0)(x), \quad \inf_{x \in \mathbb{R}^N_+} \rho_0(x) > 0,$$

$$\lim_{x_1 \to \infty} (\rho_0, u_0)(x_1, x') = (\rho_+, u_+, 0, \dots, 0) \in \mathbb{R}^{N+1} \quad \text{for an arbitrary } x' \in \mathbb{R}^{N-1}, \tag{2.2}$$

$$\phi(t, 0, x') = \phi_b \quad \text{for an arbitrary } x' \in \mathbb{R}^{N-1}, \tag{2.3}$$

where $\rho_+ > 0$, u_+ and ϕ_b are constants. We take a reference point of the value of the potential ϕ as $x_1 = \infty$, that is,

$$\lim_{x_1 \to \infty} \phi(t, x_1, x') = 0 \quad \text{for an arbitrary } x' \in \mathbb{R}^{N-1}. \tag{2.4}$$

To construct a classical solution of the Poisson equation (2.1c), the condition

$$\rho_+ = 1 \tag{2.5}$$

is necessary. Owing to conditions (2.4) and (2.5), the quasi-neutrality $\rho = \rho_e$ holds as $x_1 \to \infty$ since $\lim_{x_1 \to \infty} \rho_e = e^{-0} = \rho_+$.

The planar stationary solution $(\tilde{\rho}, \tilde{u}, \tilde{\phi})(x_1)$ is a solution to (2.1) independent of the time variable t and of the tangential coordinates x':

$$(\tilde{\rho}\tilde{u})_{x_1} = 0, \tag{2.6a}$$

$$\left(\tilde{\rho}\tilde{u}^2 + K\tilde{\rho}\right)_{x_1} = \tilde{\rho}\tilde{\phi}_{x_1}, \tag{2.6b}$$

$$\tilde{\phi}_{x_1 x_1} = \tilde{\rho} - e^{-\tilde{\phi}}. \tag{2.6c}$$

We assume conditions (2.2)–(2.5), that is,

$$\inf_{x_1 \in \mathbb{R}_+} \tilde{\rho}(x_1) > 0, \quad \lim_{x_1 \to \infty} (\tilde{\rho}, \tilde{u}, \tilde{\phi})(x_1) = (\rho_+, u_+, 0, \dots, 0, 0), \quad \tilde{\phi}(0) = \phi_b. \tag{2.7}$$

In the discussion of the existence of the stationary solution, the Sagdeev potential

$$V(\phi) := \int_0^{\phi} \left[f^{-1}(\eta) - e^{-\eta}\right] d\eta, \quad f(\rho) := K \log \rho + \frac{u_+^2}{2\rho^2} - \frac{u_+^2}{2} \tag{2.8}$$

plays crucial roles. Here the inverse function f^{-1} is defined by adopting the branch which contains the equilibrium point $(\rho, \phi) = (1, 0)$ (see [10] for details). The unique existence of the monotone stationary solution is obtained in [10], summarized as follows:

Theorem 2.1. ([10]) (i) *Let u_+ be a constant satisfying either $u_+^2 \leq K$ or $K+1 \leq u_+^2$. Then stationary problem* (2.6) *and* (2.7) *has a unique monotone solution $(\tilde{\rho}, \tilde{u}, \tilde{\phi})(x_1)$ verifying*

$$\tilde{\rho}, \tilde{u}, \tilde{\phi} \in C(\overline{\mathbb{R}_+}) \quad \text{and} \quad \tilde{\rho}, \tilde{u}, \tilde{\phi}, \tilde{\phi}_{x_1} \in C^1(\mathbb{R}_+) \tag{2.9}$$

if and only if the boundary data ϕ_b satisfies conditions

$$V(\phi_b) \geq 0 \quad \text{and} \quad \begin{cases} \phi_b \geq g(u_+^2) & \text{if} \quad K > 0, \\ \phi_b > g(u_+^2) & \text{if} \quad K = 0, \end{cases} \tag{2.10}$$

where a function g is defined by, for $y > 0$,

$$g(y) := \begin{cases} \dfrac{K}{2}\log y - \dfrac{K}{2}\log K + \dfrac{K}{2} - \dfrac{y}{2} & \text{for} \quad K > 0, \\ -\dfrac{y}{2} & \text{for} \quad K = 0. \end{cases} \tag{2.11}$$

Moreover, if $K+1 < u_+^2$ and $\phi_b \neq g(u_+^2)$, the stationary solution belongs to $C^\infty(\overline{\mathbb{R}_+})$ and verifies

$$|\partial_{x_1}^j(\tilde{\rho} - 1)| + |\partial_{x_1}^j(\tilde{u} - u_+)| + |\partial_{x_1}^j\tilde{\phi}| \leq C|\phi_b|e^{-cx_1} \quad \text{for} \quad j = 0, 1, 2, \cdots, \tag{2.12}$$

where c and C are positive constants.

(ii) *Let u_+ be a constant satisfying $K < u_+^2 < K+1$. If $\phi_b \neq 0$, then stationary problem* (2.6) *and* (2.7) *does not admit any solutions in the function space $C^1(\mathbb{R}_+)$. If $\phi_b = 0$, then a constant state $(\tilde{\rho}, \tilde{u}, \tilde{\phi}) = (1, u_+, 0)$ is the unique solution.*

By this proposition, we see that the condition

$$u_+^2 > K + 1, \quad u_+ < 0 \tag{2.13}$$

together with $|\phi_b| \ll 1$ or

$$u_+^2 = K + 1, \quad u_+ < 0 \tag{2.14}$$

together with $\phi_b \geq 0$ gives a sufficient condition for the unique existence of the monotone stationary solution. The condition

$$u_+^2 \geq K + 1, \quad u_+ < 0 \tag{2.15}$$

is called the Bohm criterion in [9]. Note that some textbooks as [5] drop the equality in (2.15) and its manner is adopted in [10]. In the present paper, considering the physical interest as seen in [1, 9], and our development of mathematical result in Theorem 2.4 under (2.14), we call (2.15) the Bohm criterion and define the sheath by the monotone stationary solution to (2.1) under (2.15).

To study the asymptotic stability of the sheath, we introduce unknown functions $v := \log\rho$, $\tilde{v} := \log\tilde{\rho}$ and the perturbation

$$(\psi,\eta,\sigma)(t,x_1,x') := (v,u,\phi)(t,x_1,x') - (\tilde{v},\tilde{u},\tilde{\phi})(x_1).$$

Subtracting (2.6) from (2.1), we have

$$\psi_t + u\cdot\nabla\psi + \mathrm{div}\eta + \eta_1\tilde{v}_{x_1} = 0, \tag{2.16a}$$
$$\eta_t + u\cdot\nabla\eta + K\nabla\psi - \nabla\sigma + \eta_1\tilde{u}_{x_1} = 0, \tag{2.16b}$$
$$\Delta\sigma = e^{\psi+\tilde{v}} - e^{\tilde{v}} - e^{-(\sigma+\tilde{\phi})} + e^{-\tilde{\phi}}. \tag{2.16c}$$

The initial and the boundary data to (2.16) are obtained from (2.2), (2.3) and (2.7):

$$\begin{gathered}(\psi,\eta)(0,x) = (\psi_0,\eta_0)(x) := (\log\rho_0 - \log\tilde{\rho}, u_0 - \tilde{u}),\\ \lim_{x_1\to\infty}(\psi_0,\eta_0)(x_1,x') = (0,0) \quad \text{for an arbitrary } x' \in \mathbb{R}^{N-1},\end{gathered} \tag{2.17}$$

$$\sigma(t,0,x') = 0 \quad \text{for an arbitrary } x' \in \mathbb{R}^{N-1}. \tag{2.18}$$

If the perturbations (ψ,η,σ) and $|\phi_b|$ are sufficiently small, all characteristics in x_1-direction of hyperbolic system (2.16a) and (2.16b) are negative owing to (2.12) and (2.15). Namely,

$$\lambda_1 := \eta_1 + \tilde{u} - \sqrt{K} < 0, \quad \lambda_2 := \eta_1 + \tilde{u} + \sqrt{K} < 0, \quad \lambda_i = \eta_1 + \tilde{u} < 0 \quad \text{for } i = 3,\ldots,N+1. \tag{2.19}$$

Hence no boundary conditions for hyperbolic system (2.16a) and (2.16b) are necessary for the well-posedness of the initial boundary value problem (2.16)–(2.18). Consequently one boundary condition (2.3) is necessary and sufficient.

Linearization of (2.16) around the asymptotic state $(\rho,u,\phi) = (\rho_+,u_+,0,\ldots,0)$ results in

$$\psi_t + u_+\psi_{x_1} + \mathrm{div}\eta = 0, \tag{2.20a}$$
$$\eta_t + u_+\eta_{x_1} + K\nabla\psi - \nabla\sigma = 0, \tag{2.20b}$$
$$\Delta\sigma = \psi + \sigma. \tag{2.20c}$$

Since the spectrums of (2.20) are given by

$$\mu(i\xi) = i\left(-\xi_1 u_+ \pm |\xi|\sqrt{K + \frac{1}{1+|\xi|^2}}\right), \quad -i\xi_1 u_+ \quad \text{for } \xi = (\xi_1,\ldots,\xi_N) \in \mathbb{R}^N, \tag{2.21}$$

the real part of the spectrums are zero. Here $-i\xi_1 u_+$ has a multiplicity of $N-1$. This fact prohibits the application of standard method to our problem. This diffuculty was overcome in [10] by considering the stability problem in a function space with such weight functions as

$$(1+\beta x_1)^\alpha \quad \text{or} \quad e^{\beta x_1}$$

following ideas in [7], which studies an outflow problem for the Navier-Stokes equation.

To derive the asymptotic stabilty of a sheath under (2.13), we introduce new variables $(\Psi, H, \Sigma) := (e^{\beta x_1/2}\psi, e^{\beta x_1/2}\eta, e^{\beta x_1/2}\sigma)$. We then rewrite the system of equations (2.16) with respect to (Ψ, H, Σ) and linearize the result around the asymptotic state $(\rho, u, \phi) = (\rho_+, u_+, 0, \dots, 0, 0)$ to get

$$\Psi_t + u_+\Psi_{x_1} + \mathrm{div} H = \frac{\beta}{2}u_+\Psi, \tag{2.22a}$$

$$H_t^j + u_+H_{x_1}^j + K\Psi_{x_j} - \Sigma_{x_j} = \frac{\beta}{2}u_+H^j - \delta_{j1}\frac{\beta}{2}\Sigma \quad \text{for } j = 1, \dots, N, \tag{2.22b}$$

$$\Delta\Sigma - \beta\Sigma_{x_1} + \left(\frac{\beta^2}{4} - 1\right)\Sigma = \Psi. \tag{2.22c}$$

By a straightforward calculation we see that the spectrums of (2.22) are given by

$$\begin{gathered} \mu(i\xi) = \frac{\beta u_+}{2} + i\left(-\xi_1 u_+ \pm \sqrt{K\zeta - \frac{1}{\zeta} + 1 - K}\right), \quad \frac{\beta u_+}{2} - i\xi_1 u_+ \\ \text{for } \xi = (\xi_1, \dots, \xi_N) \in \mathbb{R}^N, \quad \zeta := 1 + |\xi|^2 - \frac{\beta^2}{4} + i\beta\xi_1, \end{gathered} \tag{2.23}$$

where $\beta u_+/2 - i\xi_1 u_+$ has a multiplicity of $N-1$. The next theorem reassures that condition (2.13) may be sufficient for the stability of the sheath and corroborates the introduction of the weight function.

Theorem 2.2. ([8]) *As for the spectrums for* (2.22) *given in* (2.23), *it holds*

$$\sup_{\xi\in\mathbb{R}^N} Re\,(\mu(i\xi)) = \max\{Re\,(\mu(0))\} = \frac{\beta}{2}\left(u_+ + \sqrt{K + \frac{1}{1-\beta^2/4}}\right).$$

Hence under the condition

$$u_+ < 0, \quad u_+^2 > K + \frac{1}{1-\beta^2/4}, \tag{2.24}$$

the linear stability of (2.22) *holds. This condition is fulfilled if and only if* $u_+ < 0$, $u_+^2 > K+1$. *Namely,* (2.13) *is satisfied with the positive weight parameter* β *set suitably small.*

In accordance with the physical observation and the result of the linear stability, in Theorem 2.3 the Bohm criterion is validated by showing the unique existence of global-in-time solution to (2.1) and its stabilty under (2.13). In Theorem 2.4, we show similar results under the degenerate condition of (2.14) corresponding to the limit as $\lambda_D/L \to 0$ (see [1]).

Theorem 2.3. ([8]) *For* $N = 1, 2, 3$, *let* $m = [\frac{N}{2}] + 2$. *Assume that* $K > 0$ *and that the condition* (2.13) *holds.*

(i) *Suppose* $(e^{\lambda x_1/2}\psi_0, e^{\lambda x_1/2}\eta_0)$ *belongs to the Sobolev space* $(H^m(\mathbb{R}^N_+))^{N+1}$ *for some positive constant* λ. *Then there exists a positive constant* δ *such that if* $\beta \in (0, \lambda]$ *and* $\beta + \left(|\phi_b| + \|(e^{\beta x_1/2}\psi_0, e^{\beta x_1/2}\eta_0)\|_{H^m}\right)/\beta \leq \delta$ *are satisfied, the initial boundary value problem* (2.16)–(2.18) *uniquely has a global-in-time solution* (ψ, η, σ) *as* $(e^{\beta x_1/2}\psi, e^{\beta x_1/2}\eta, e^{\beta x_1/2}\sigma) \in \mathfrak{X}_m([0,\infty)) \times \mathfrak{X}_m([0,\infty)) \times \mathfrak{X}^2_m([0,\infty))$. *Moreover, the solution verifies the decay estimate*

$$\|(e^{\beta x_1/2}\psi, e^{\beta x_1/2}\eta)(t)\|^2_{H^m} + \|e^{\beta x_1/2}\sigma(t)\|^2_{H^{m+2}} \leq C\|(e^{\beta x_1/2}\psi_0, e^{\beta x_1/2}\eta_0)\|^2_{H^m}\, e^{-\alpha t} \tag{2.25}$$

for certain positive constants C *and* α *which are independent of the time variable* t.

(ii) *Suppose* $\left((1+\gamma x_1)^{\lambda/2}\psi_0, (1+\gamma x_1)^{\lambda/2}\eta_0\right)$ *belongs to* $(H^m(\mathbb{R}^N_+))^{N+1}$ *for certain constants* λ *and* γ *satisfying* $\lambda \geq 2$ *and* $\gamma > 0$. *Then for an arbitrary* $\alpha \in (0, \lambda]$ *there exist a positive constant* δ *such that if* $\beta \in (0, \gamma]$ *and* $\left(|\phi_b| + \|((1+\beta x_1)^{\lambda/2}\psi_0, (1+\beta x_1)^{\lambda/2}\eta_0)\|_{H^m}\right)/\beta + \beta \leq \delta$ *are satisfied, the initial boundary value problem* (2.16)–(2.18) *uniquely has a global-in-time solution* (ψ, η, σ) *as* $((1+\beta x_1)^{\lambda/2}\psi, (1+\beta x_1)^{\lambda/2}\eta, (1+\beta x_1)^{\lambda/2}\sigma) \in \mathfrak{X}_m([0,\infty)) \times \mathfrak{X}_m([0,\infty)) \times \mathfrak{X}^2_m([0,\infty))$. *Moreover, the solution verifies the decay estimate*

$$\begin{aligned}\|((1+\beta x_1)^{\alpha/2}\psi, (1+\beta x_1)^{\alpha/2}\eta)(t)\|^2_{H^m} + \|(1+\beta x_1)^{\alpha/2}\sigma(t)\|^2_{H^{m+2}}\\ \leq C\|((1+\beta x_1)^{\lambda/2}\psi_0, (1+\beta x_1)^{\lambda/2}\eta_0)\|^2_{H^m}(1+\beta t)^{-(\lambda-\alpha)},\end{aligned} \tag{2.26}$$

where C *is a positive constant determined by* α.

Theorem 2.3 is an extention of the theorem obtained in [10], in which the spatial dimension is limited only to one and more crucially, strictly stronger conditions than (2.13) is assumed.

Theorem 2.4. ([8]) *For* $N = 1, 2, 3$, *let* $m = [\frac{N}{2}] + 2$. *Assume that* $K > 0$ *and that the condition* (2.14) *holds. Set* $\lambda_0 = 5.5693\ldots$ *be the unique real solution to the equation* $\lambda_0(\lambda_0 - 1)(\lambda_0 - 2) - 12(\lambda_0 + 2) = 0$ *and* $\Gamma := \sqrt{(K+1)/6}$. *Suppose* $\left((1+\gamma x_1)^{\lambda/2}\psi_0, (1+\gamma x_1)^{\lambda/2}\eta_0\right)$ *belongs to* $(H^m(\mathbb{R}^N_+))^{N+1}$ *for certain constants* λ *and* γ *satisfying* $\lambda \in [4, \lambda_0)$ *and* $\gamma > 0$. *Then for arbitrary* α, θ *satisfying* $\alpha \in (0, \lambda]$, $\theta \in (0, 1]$, *there exists a positive constant* δ *such that if* $\phi_b \in (0, \delta]$, $\beta \leq \gamma$, $\beta/(\Gamma\phi_b^{1/2}) \in [\theta, 1]$ *and* $\|((1+\gamma x_1)^{\lambda/2}\psi_0, (1+\gamma x_1)^{\lambda/2}\eta_0)\|_{H^m}/\beta^3 \leq \delta$ *are satisfied, the initial boundary value problem* (2.16)–(2.18) *has a unique global-in-time solution* (ψ, η, σ) *as* $\left((1+\beta x_1)^{\alpha/2}\psi,\ (1+\beta x_1)^{\alpha/2}\eta, (1+\beta x_1)^{\alpha/2}\sigma\right) \in \mathfrak{X}_m([0,\infty)) \times \mathfrak{X}_m([0,\infty)) \times \mathfrak{X}^2_m([0,\infty))$. *Moreover, the solution verifies the decay estimate*

$$\begin{aligned}\|((1+\beta x_1)^{\alpha/2}\psi, (1+\beta x_1)^{\alpha/2}\eta)(t)\|^2_{H^m} + \|(1+\beta x_1)^{\alpha/2}\sigma(t)\|^2_{H^{m+2}}\\ \leq C\|((1+\beta x_1)^{\lambda/2}\psi_0, (1+\beta x_1)^{\lambda/2}\eta_0)\|^2_{H^m}(1+\beta t)^{-(\lambda-\alpha)/3},\end{aligned} \tag{2.27}$$

where C *is a positive constant determined by* α *and* θ.

Notation. For a real number x, $[x]$ denotes a maximum integer which does not exceed x. For a nonnegative integer $l \geq 0$, $H^l(\mathbb{R}^N_+)$ denotes the l-th order Sobolev space in the L^2 sense, equipped with the norm $\|\cdot\|_{H^l}$. We denote by $C^k([0,T];H^l(\mathbb{R}^N_+))$ the space of k-times continuously differentiable functions on the interval $[0,T]$ with values in $H^l(\mathbb{R}^N_+)$. The function space $\mathfrak{X}^j_i$ is defined by

$$\mathfrak{X}^j_i([0,T]) := \bigcap_{k=0}^{i} C^k([0,T];H^{j+i-k}(\mathbb{R}^N_+)),\ \mathfrak{X}_i([0,T]) := \mathfrak{X}^0_i([0,T]) \text{ for } i=0,1,2,3,\ j=0,1,2.$$

Lastly c and C denote generic positive constants.

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