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Barrier option prices under discrete deterministic and stochastic volatility models

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1. Introduction

Listed options are usually simple plane vanilla options. Financial institutions trade complex derivative products called exotic options over the counter and they price the options incorporating the market prices of listed plane vanilla options as best as possible. Path-dependent option such as knockout-barrier option is one of the famous Exotic options and the lattice model is often used for the convenient estimation.

Black-Scholes model (BS model) [1] adopts geometric Brownian motion (volatility of equity return is constant) as the equity model. However, the equity model of the constant volatility is not enough to replicate the actual options market prices. To represent the volatility more flexibly, deterministic volatility models (DVM) (Dupire [4], Derman and Kani [3] and so on) and stochastic volatility models (SVM) (Heston[6], Fouque at al. [5] among others) have been proposed. The former models are easily represented in the lattice framework, while the latter models are not (Britten-Jones and Neuberger [2]). Due to the reason, the empirical comparison of these volatility models in the lattice framework is not discussed from the view of the exotic derivative pricing incorporating the market prices of listed options as best as possible.

Therefore, in this research, first, we discuss the calibration of the lattice between DVMs and SVMs. Second, we evaluate exotic options based on the implied lattice and discuss the magnitude of the price difference among the models.

2. Deterministic volatility models (DVM) and their binomial lattices

In general, the deterministic volatility model (DVM) is given by equation (1).

$$dS_t = rS_t dt + \sigma(S_t, t) S_t d\hat{W}, \tag{1}$$

where S_t , r , $\sigma(\cdot)$ and $d\hat{W}$ are underlying asset, risk-free interest rate, local volatility that consists of the underlying asset and time, and Brownian motion under risk-neutral measure. DVM is specified by local volatility function and five kinds of the models listed in Table1.

Table1. The five kinds of DVMs.

DVM	*LocalVolatility*		
1P	$\sigma(S_t,t)=a$	2P	$\sigma(S_t,t)=aS_t^{\,b}$
3P	$\sigma(S_t,t)=c+a\{1-\tanh[b(S_t-S_0)/S_0]\}$		
5P	$\sigma(S_t,t)=c+a\{1-\tanh[b(S_t-S_0)/S_0]\}+d\{1-\text{sech}[e(S_t-S_0)/S_0]\}$	7P	$\sigma(S_t,t)=a+b\tanh[c\ln(S_t/S_0)-d]+e\{1-\text{sech}[f\ln(S_t/S_0)-g]\}$

Below we explain each of 2, 3, 5 and 7 parameter models (Refer to Mawaribuchi, Miyazaki and Okamoto [9] regarding the features of functions tanh(x) and sech(x)). 2 parameter model has two parameters such as (a,b). It is known that the model represents skewness of the risk-neutral distribution. Nevertheless its flexibility is not quite big. 3 parameter model has three parameters (a,b,c) and its local volatility contains function tanh(x). It is able to represent skewness of the risk-neutral distribution more flexibly than 2 parameter model. 5 parameter model has five parameters (a,b,c,d,e) and its local volatility is extension of 3 parameter model by including function sech(x). Function sech(x) is upward convex and useful to represent kurtosis of the risk-neutral distribution. 7 parameter model has seven parameters (a,b,c,d,e,f,g) and its local volatility also contains function tanh(x) and sech(x). 7 parameter model differs from 5 parameter model in that the log-return is adopted and parameters (d,g) are introduced to adjust the levels inside the two functions. Important point is that function tanh(x) can express skewness of the risk-neutral distribution and function sech(x) can express its kurtosis flexibly.

Having in mind that our paper is based on European option, the option valuation formulas are

$$\text{Call Price} = e^{-rT}\int_0^\infty \max(S_T - K,0)f(S_T)dS_T \qquad \text{Put Price} = e^{-rT}\int_0^\infty \max(K - S_T,0)f(S_T)dS_T, \tag{2}$$

where r, S_T, K and $f(S_T)$ are risk-free rate, equity price at the maturity, strike price and probability density function at the maturity, respectively.

To construct the binomial lattice for each DVM from Table1, we adopt Li algorism (Li [8]) that proposes setting both up and down transition probabilities at 50%. Hoshika and Miyazaki [7] noticed that the robustness of the Li algorism is higher compared to that of Derman and Kani [3]. S_t^i denotes the underlying asset price at time t, which falls on the i-th node from the top.

In Li algorithm, the asset price dynamics between two consecutive time periods (time interval is Δt) are given in equation (3).

$$\begin{aligned} S_t^1 &= S_{t-1}^1\left[1 + r\Delta t + \sigma(S_{t-1}^1,t)\sqrt{\Delta t}\right], \\ S_t^{i+1} &= S_{t-1}^i\left[1 + r\Delta t - \sigma(S_{t-1}^i,t)\sqrt{\Delta t}\right], \\ S_t^{i+1} &= \frac{1}{2}\left\{S_{t-1}^i\left[1 + r\Delta t - \sigma(S_{t-1}^i,t)\sqrt{\Delta t}\right] + S_{t-1}^{i+1}\left[1 + r\Delta t + \sigma(S_{t-1}^{i+1},t)\sqrt{\Delta t}\right]\right\}. \ (i \neq 0,t) \end{aligned} \tag{3}$$

The first and the second equations generate the top and the bottom stock paths in the lattice and the third equation describes all the paths inside the lattice.

3. Stochastic volatility models (SVM) and their lattices

3.1 Compound binomial lattice for the simple stochastic volatility model

The simple stochastic volatility model

$$dS = rSdt + \sigma S d\hat{W}_1, \tag{4}$$

$$d\sigma = -\kappa(\sigma - \alpha)dt + \gamma d\hat{W}_2, \tag{5}$$

where $S, r, \sigma, \alpha, \kappa, \gamma$ are equity price, risk-free interest rate, volatility, mean of the volatility, mean-reversion parameter and volatility of the volatility, in order. $d\hat{W}_1$ and $d\hat{W}_2$ are independent Brownian motion under the risk-neutral measure. In discrete version of the SVM, we represent the mean-reversion process in the finite-space Markov framework. To this end, we describe the discrete mean-reversion process z_t with the finite-space Markov chain ($z_t = -J,\ldots,0,1,\ldots,J$). Transition probability of the mean-reversion process z_t from $z_t = j$ to $z_{t+h} = k$ is given by equation (6).

$$p_{k,j} = \left\{\frac{1}{2}\kappa h(J - j) \ \ if \ k = j+1; \quad 1 - \kappa hJ \ \ if \ k = j; \quad \frac{1}{2}\kappa h(J + j) \ \ if \ k = j-1; \quad 0 \ \ if \ \text{otherwise}\right\} \tag{6}$$

Utilizing the mean-reverting process z_t, we introduce volatility as equation (7).

$$\sigma_t(z_t) = \alpha + \delta z_t, \tag{7}$$

where $\delta = \gamma/\sqrt{\kappa J}$. σ_t satisfies mean-reverting process (5).

In the modeling, the volatility σ_t moves around the state-space $(\alpha - \delta J,\ldots,\alpha,\ldots,\alpha + \delta J)$.

Role of the parameters of the mean-reverting process (5) and settings in the compound binomial lattice

(1) We set the number of volatility states and the time step as J and h, respectively.

(2) The level of the volatility $\sigma_t(z_t)$ is given by $\alpha + \delta j$ when $z_t = j$.

(3) For the estimated parameter κ, γ once we take enough large J, δ could be as small as we would like to and the mean-reversion parameter α is represented as $\alpha = \delta j_\alpha$ with positive

integer j_α. However, to make the volatility level positive, we need the condition $j_\alpha \geq J+1$.

For convenience, we explain the lattice in case of $j_\alpha = J+1$. We try to construct the compound binomial lattice of $y_t = \ln(S_t/S_0)$. Solving equation (4), we have

$$S_t = S_0 \exp\left\{\left(r - \frac{1}{2}\sigma_t(z_t)^2\right)t + \sigma_t(z_t)\hat{W}_1(t)\right\}, \tag{8}$$

and we know

$$y_t = \left(r - \frac{1}{2}\sigma_t(z_t)^2\right)t + \sigma_t(z_t)\hat{W}_1(t). \tag{9}$$

We call the lattice as the compound binomial lattice in following two reasons. First, when y_t moves to state y_{t+h} with small time interval h, the move consists of $2J+1$ kinds of cases covering binomial tree ($y_{t+h} = y_t \pm \sigma_t(z_t)\sqrt{h}$) with the volatility ($\sigma_t(z_t)$). Second, the probability that the move from state y_{t+h} to state y_{t+2h} is decided by the volatility level of the lattice the move from state y_t to state y_{t+h} and the probability transition matrix (equation(6)).

The y_t is specified by equation (9). The discrete version of the dynamics becomes

$$\Delta y_t \mid z_t = \begin{cases} \sigma_t(z_t)\sqrt{h} & : probability \ \dfrac{1}{2} + \dfrac{\left(r - \sigma_t(z_t)^2/2\right)\sqrt{h}}{2\sigma_t(z_t)} \\ -\sigma_t(z_t)\sqrt{h} & : probability \ \dfrac{1}{2} - \dfrac{\left(r - \sigma_t(z_t)^2/2\right)\sqrt{h}}{2\sigma_t(z_t)} \end{cases}. \tag{10}$$

3.2 Our discrete stochastic volatility models examined in empirical analyses

In empirical analyses, we set the number of the volatility state to be three ($J = 1$) and propose our models below. First, the step sizes of the high, middle and low volatilities in section 3.1 are ($\delta, 2\delta, 3\delta$) (Fig.1). In our empirical analysis, we introduce two models with different kinds of volatility levels. The high, middle and low volatility levels are ($\delta, 3\delta, 5\delta$) and ($\delta, 5\delta, 9\delta$) respectively. Second, the model in section 3.1 assumes that the equity and the volatility processes are independent. Heston [6] and also Fouque, et al. [5] introduced that the factor in the SVM to generate the skewness of the risk-neutral distribution is the correlation between the two processes. Thus, we propose the discrete SVM with two kinds of transition probability matrixes. One is used after the upward move of the equity price and the other is used in the opposite case. In addition, with respect to the transition probability of the volatility, we extend the model by relaxing the constraint of the OU process (Table2). In more detail, we propose the symmetrical probability transition matrix and the free one. Corresponding to the combination of three kinds of volatility levels and probability transition matrixes, we propose total nine kinds of discrete SVMs listed in Table3.

In empirical analyses, we also examine another SVM based on asymptotic expansion (refer to Fouque, et al. [5]).

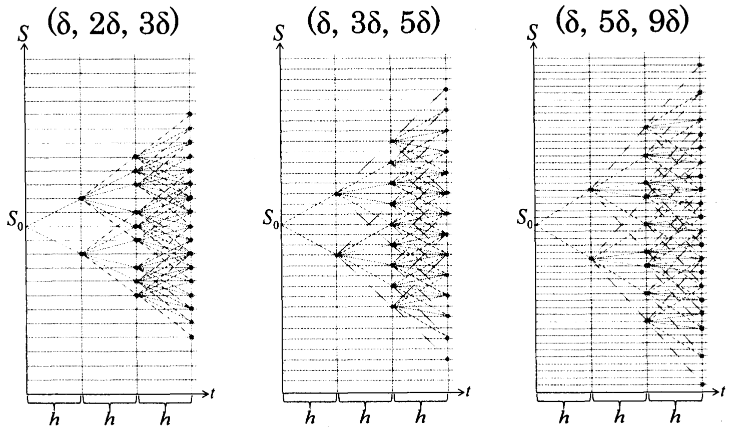

Fig.1 The conceptual figure in case of J=1 (δ, 2δ, 3δ) (δ, 3δ, 5δ) (δ, 5δ, 9δ).

Table2. Probability transition matrix.

"OU" type

$k \backslash J$	Low Vol	Middle Vol	High Vol
Low Vol	$1-\kappa h$; $1-2a$	$\kappa h/2$; a	0
Middle Vol	κh; $2a$	$1-\kappa h$; $1-2a$	κh; $2a$
High Vol	0	$\kappa h/2$; a	$1-\kappa h$; $1-2a$

"Symmetrical" type

$k \backslash J$	Low Vol	Middle Vol	High Vol
Low Vol	a	c	$1-(a+b)$
Middle Vol	b	$1-2c$	b
High Vol	$1-(a+b)$	c	a

"Free" type

$k \backslash J$	Low Vol	Middle Vol	High Vol
Low Vol	a	c	e
Middle Vol	b	d	f
High Vol	$1-(a+b)$	$1-(c+d)$	$1-(e+f)$

Table3. The nine kinds of discrete SVMs.

		Transition probability Matrix		
		OU	Symmetrical	free
Vol. State	$(\delta, 2\delta, 3\delta)$	①	④	⑦
	$(\delta, 3\delta, 5\delta)$	②	⑤	⑧
	$(\delta, 5\delta, 9\delta)$	③	⑥	⑨

4. Empirical Analyses

4.1 Objects and Methods of empirical analyses

(Analysis1)

Compare how closely the model prices fit the market prices between DVMs and SVMs.

(Analysis2)

Analysis on the model risk (the deviation from the benchmark model) in the pricing of barrier options based on the implied tree. In this study, we focus the result of this analysis.

The methods of (Analysis1) and (Analysis2) are following (Method1) and (Method2) respectively.

(Method1)

We calibrate the parameters of each model so as to minimize the objective functions (equation (11)) and identify which model well replicates the cross-sectional option market prices. As the options used in the calibration, we adopt total 6 kinds of out-of-the-money options such as OTM1 (the strike price is closest to the current equity price), OTM2 (secondly closest), OTM3 (thirdly closest) call and put options. Objective function is

$$Min \sum_{i=1}^{6} (P_i' - P_i)^2 / 6, \tag{11}$$

where P and P' are option market price and model price, respectively. i indicates type of option.

Here, we describe empirical procedure shortly in the asymptotic expansion approach. Fouque, et al. [5] estimate parameters of volatility by the regression model with the information of the implied volatility surface. For the data used in the regression analysis, we first recover the 6 implied volatilities corresponding to the 6 option market prices and utilize spline-function to derive the implied volatilities corresponding to variety of strike prices. We estimate the parameters of volatility with the derived implied volatilities. To compare the calibration between the lattice framework and the asymptotic expansion approach in the same standard, we compute 6 model prices P' of the asymptotic expansion approach and derive the objective function value.

(Method2)

In the analysis of the model risk we examine up-knockout-barrier-option (hereafter, we call just barrier option) as an example of exotic derivatives. We set the strike price of the barrier option to be that of the CallOTM1 option and compute, model by model, prices of the barrier options that have the barrier prices up to 4000 above the strike price. We capture the model risk by the absolute deviation of the barrier option price by each model from that by the benchmark, which most precisely replicates the options market prices. To the goal, for each model, we take average of the absolute differences in all the contract months of the analysis period. With the product specification of the barrier option, we firstly know that the absolute deviation approaches to 0, when the barrier price becomes closer to the strike price and the barrier option price itself becomes closer to 0. We secondly know that the absolute deviation of barrier option approaches to that (attained in (Method1)) of the European option when the barrier option price itself is close to the European option price with the small barrier hitting probability in the case of the highly set barrier price level. We also statistically test the difference in the sample average of the absolute deviations.

4.2 Data and settings

For the calibration of the models, we use the 6 kinds of options described in (Method1). The options are monthly contracts and their remaining maturities are 15 business days. The data period covers from June 2003 to April 2008 and total 59 monthly results in the period are attained. The number of the lattice up to the maturity is set to be 30.

4.3 Results and implications of empirical analyses

Results and implications of (Analysis1)

As the results of (Analysis1), average and standard deviation of the objective function values for each model in both periods before and after occurrence of financial crisis (sub-prime problem) are shown in Table 4.

Firstly, we focus on the average of the objective function value for each DVM. The precision of the calibration is dramatically improved in accordance with the extension of the model from 1 parameter to 7 parameter. 7 parameter model almost perfectly replicates the cross-sectional option market prices. Function tanh(x) and sech(x) are important to express skewness and kurtosis of the risk-neutral distribution flexibly.

Secondly, we compare the average of the objective function value between the Heston model in the asymptotic expansion approach and the models in the lattice framework such as OU. The averages of the objective function values for OU3, OU5 and OU9 are 76.81, 64.53 and 56.07, respectively. The average of the objective function value in the asymptotic expansion approach is 61.72 and close to that of OU5 model. This result indicates that the different valuation approaches provide approximately the same level of the precision of the calibration in average, once the same kinds of models are adopted. On the other hand, there is a big difference between the two approaches in the standard deviation of the objective function value. From the standard deviations, the asymptotic expansion approach is thought to be more robust than the lattice framework.

Thirdly, we compare the average objective function value of one model with those of others among the nine SVMs. One notable point is that for all the three probability transition matrixes such as OU, "symmetric" and "free", the larger the difference among high, middle and low volatility levels, the better the calibration. The other notable point is that the weaker the constraint in the probability transition matrix, the better the fit in any volatility level. The sensitivity of the calibration on the constraint in the probability transition matrix is much larger than that on the difference among high, middle and low volatility levels. These results suggest that the flexibility of the probability transition matrix is important to replicate precisely the cross-sectional options market prices.

Lastly, we compare the precision of the calibration for the DVMs with the one for the SVMs. The OU-type of SVM has the same level of the precision of the calibration as that for 2 parameter DVM. When we relax the constraint on the probability transition matrix from OU to "symmetric", the precision of the calibration for the SVM is improved so as to surpass the one of 2 parameter DVM but still inferior to the one of 3 parameter one. This result indicates that the small relax in the volatility transition probability from OU to "symmetric" is not enough to generate the skewness in the risk-neutral distribution as the function tanh(x). With the further relax in the volatility transition probability from "symmetric" to "free", the precision of the calibration for the discrete SVMs is revamped to be better than the one for 3 parameter DVM but still be inferior to the one for 5 and 7 parameter models. The implication of this result is that once we adopt sech(x) in addition to tanh(x) as the functional form of the local volatility, the precision of the calibration for the DVMs becomes superior to the one for the discrete SVMs both in the average and the standard deviation of the minimized sum of the square errors. Taking the convenience in the lattice construction into consideration, 5 and 7 parameter DVMs seems to be advantageous among all of the valuation models examined in our analysis.

Table4. The Average and standard deviation of objective function value.

Before the financial crisis (2003/06～2007/08)														
Deterministic Volatility Model					Stochastic Volatility Model									
1P	2P	3P	5P	7P	①OU3	②OU5	③OU9	④Symmetrical3	⑤Symmetrical5	⑥Symmetrical9	⑦Free3	⑧Free5	⑨Free9	Asymptotic
Average														
103.48	69.02	21.95	4.21	0.21	76.81	64.43	56.07	55.00	40.67	32.84	12.66	8.36	6.83	61.72
Standard Deviation														
125.51	97.35	29.85	7.48	0.57	105.83	95.47	88.00	84.36	69.30	60.55	29.03	13.92	11.41	68.66
After the financial crisis (2007/09～2008/04)														
Deterministic Volatility Model					Stochastic Volatility Model									
1P	2P	3P	5P	7P	①OU3	②OU5	③OU9	④Symmetrical3	⑤Symmetrical5	⑥Symmetrical9	⑦Free3	⑧Free5	⑨Free9	Asymptotic
Average														
351.59	260.17	28.21	6.92	0.59	293.15	263.10	241.13	245.02	203.98	178.33	44.54	16.55	11.25	68.68
Standard Deviation														
737.10	582.66	37.45	13.02	2.26	651.79	602.93	564.54	576.70	501.80	452.80	130.09	41.00	31.69	80.78

Results and implications of (Analysis2)

As a benchmark model in (Analysis2), we adopt the deterministic 7 parameter model, which the most precisely replicates the cross-sectional option market prices in (Analysis1).

Firstly, the averages of 59 contract months absolute deviations of the barrier option prices for DVMs (1, 2, 3, 5 parameter models) from that of the benchmark model are shown in Fig.2. In Fig.2, vertical axis indicates the average absolute deviations and horizontal axis shows the barrier price level. From Fig.2, we see that in any barrier price level, the more precise the calibration, the smaller the average absolute deviation. Especially, when the model is extended from 2 parameter model to 3 parameter model, the average absolute deviation is greatly diminished. One of the interesting points in Fig.2 is that the average absolute deviation takes the maximum value in the barrier level from 1000 to 1500 yen. Considering the barrier option price itself is smaller than the price of the European option with the same strike price, the average absolute deviation of the barrier option with the barrier level is quite large relative to the price itself. The result suggests that the difference in the inside of the calibrated lattice between each model and benchmark model is larger than that in the risk-neutral distributions at the maturity of the option between the two.

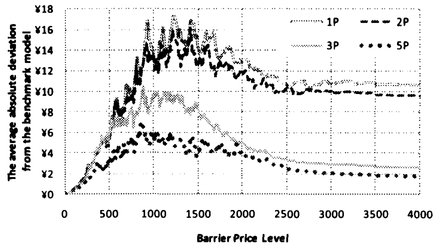

Fig.2 (1P, 2P, 3P, 5P).

Secondly, we examine the model risk in the SVMs. SVMs are specified by the combination of the size in the high, middle, low volatilities and the probability transition matrix as in Table 3. First, for a given probability transition matrix, we examine the volatility size effect to the average absolute deviation in the barrier option price based on the SVM. Fig.3 ~ Fig.5 show the volatility size effect to the average absolute deviation in the barrier option price for the probability transition matrixes such as OU process, symmetrical and free, respectively. From Fig.3 ~ Fig.5, we see that in any probability transition probability matrix, the larger the size of the volatility, the smaller the average absolute deviation. Second, for a given size in the high, middle, low volatilities, we examine the probability transition matrix effect. Fig.6~ Fig.8 show the probability transition matrix effect to the average absolute deviation in the barrier option price for the sizes in the high, middle, low volatilities such as ($\delta,2\delta,3\delta$), ($\delta,3\delta,5\delta$) and ($\delta,5\delta,9\delta$), respectively. The figures tell us that in any volatility size, the more flexible the probability transition matrix, the smaller the average absolute difference. In more detail, the shrink of the average absolute deviation by the replacement of OU process probability transition matrix with Symmetrical one is limited, while the reduction of it by the replacement of Symmetrical probability transition matrix with Free one is quite large.

From the results of the analyses, the effect of the probability transition matrix to the average absolute deviation is much larger than that of the size in the high, middle, low volatilities. The implication from the results is that the flexible probability transition matrix is indispensable to replicate precisely European options and exotic options respectively.

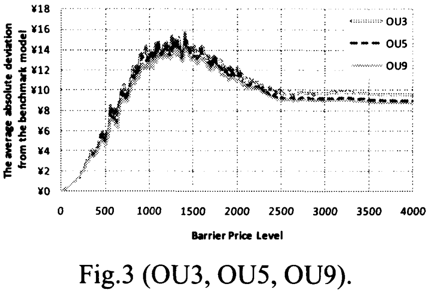

Fig.3 (OU3, OU5, OU9).

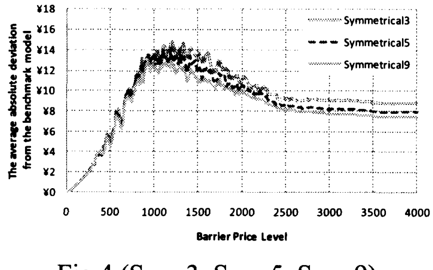

Fig.4 (Sym.3, Sym.5, Sym.9).

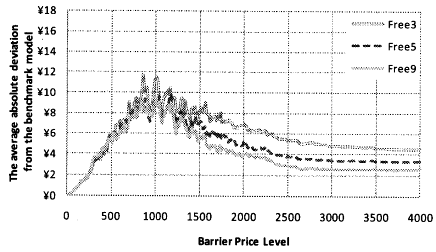

Fig.5 (Free3, Free5, Free9)

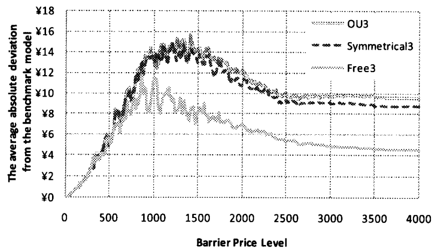

Fig.6 (OU3, Sym.3, Free3).

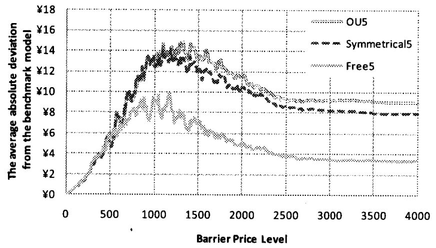

Fig.7 (OU5, Sym.5, Free5).

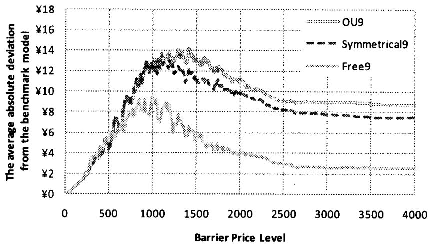

Fig.8 (OU9, Sym.9, Free9).

Thirdly, we compare the average absolute deviations of the DVMs with those of the SVMs. From the DVMs, we adopt 2, 3, 5 parameter models. Considering that the probability transition matrix effect is larger than the volatility size effect to the average absolute deviation, we adopt OU9, Symmetrical9 and Free9 from the SVMs. All in all, we compare the model risks in the 6 kinds of models. First, in (Analysis1), we rank the 6 kinds of models based on the precision of the calibration. From the model of the highest precision in order, the rank is 5parameter, Free9, 3parameter, Symmetrical9, OU9, 2parameter. Second, the averages of the 59 contract month absolute deviations of the barrier option prices for the 6 kinds of models from that of the benchmark model are shown in Fig.9. Fig.9 tells us that the average absolute deviations are ranked as 5parameter, Free9, 3parameter, Symmetrical9, OU9 and 2parameter from the smallest in order. We are able to see that the more precise the calibration, the smaller the average absolute deviation.

It is very interesting that even though the lattice framework is different, the rank of the calibration is almost the same as the rank of the average absolute deviation in the barrier option price.

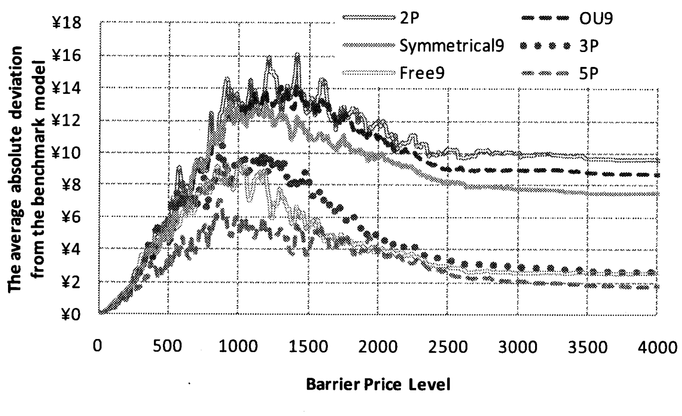

Fig.9 (2P, OU9, Sym.9, 3P, Free9, 5P)

Finally, we statistically examine whether the average absolute deviation of the barrier option for one model is different from that for the other model with some confidence level. In Table5, the positive T-value indicates that the model with the precise calibration provides the smaller average absolute deviation (model risk) of the barrier option price. The statistical results in Table5 support the results of model comparisons observed in the graph.

Table5. Statistical test on the model risk.

***significant at 1% level **significant at 5% level *significant at 10% level

DVM

Barrier price level	1P-2P	1P-3P	1P-5P	2P-3P	2P-5P	3P-5P
¥510～1000	1.41 *	7.44 ***	14.60 ***	6.17 ***	13.96 ***	13.91 ***
¥1010～1500	6.46 ***	31.50 ***	53.31 ***	27.82 ***	53.22 ***	33.08 ***

SVM

O="OU", S="Symmetrical", F="Free"

Barrier price level	O3-S3	S3-F3	O3-F3	O5-S5	S5-F5	O5-F5	O9-S9	S9-F9	O9-F9
¥510～1000	0.30	4.21 ***	4.61 ***	-0.32	5.87 ***	5.65 ***	-0.39	5.33 ***	4.75 ***
¥1010～1500	3.79 ***	29.13 ***	29.44 ***	7.94 ***	34.72 ***	39.49 ***	12.83 ***	26.95 ***	34.85 ***
Barrier price level	O3-O5	O5-O9	O3-O9	S3-S5	S5-S9	S3-S9	F3-F5	F5-F9	F3-F9
¥510～1000	1.23	0.97	2.20 **	0.59	0.91	1.52 *	2.90 ***	0.60	3.31 ***
¥1010～1500	3.09 ***	5.12 ***	7.59 ***	7.40 ***	9.28 ***	16.35 ***	5.46 ***	3.22 ***	7.92 ***

DVM and SVM

Barrier price level	2P-O9	2P-S9	2P-F9	O9-3P	O9-F9
¥510～1000	1.91 **	1.55 *	7.06 ***	3.82 ***	4.75 ***
¥1010～1500	2.82 ***	10.67 ***	30.22 ***	38.03 ***	34.85 ***
Barrier price level	O9-5P	S9-3P	S9-5P	3P-F9	F9-5P
¥510～1000	11.78 ***	4.39 ***	12.48 ***	1.63 *	11.77 ***
¥1010～1500	79.23 ***	25.38 ***	63.45 ***	9.72 ***	12.45 ***

5. Summary and Concluding Remarks

In this research, we discussed how precisely DVM and SVM, which are extension of BS model replicated the cross-sectional options market prices in the lattice framework adopting NIKKEI225 options market data. As the DVMs, we examined five kinds of models from BS model to the model consisting of functions tanh(x) and sech(x) that might generate skewness and kurtosis in risk-neutral density at maturity. Regarding the SVMs, nine kinds of models were examined. They were the discrete SVMs with the probability transition matrix of OU process as Heston model, Symmetrical and Free. From the empirical results, the precision of the calibration for any discrete SVM was not up to those of the DVMs that had both function tanh(x) and sech(x).

Regarding the analysis on the model risk, we defined the model risk as the average absolute deviation of the barrier option price for each model from that by the benchmark model and discussed about it based on the graph and the statistical test of the average absolute deviation. From the results, we found that more precise the calibration, the smaller the average absolute deviation in the barrier option price either in the DVMs or the SVMs.

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