数理解析研究所講究録
第 1761 巻 2011 年 74-79

An inverse boundary element model to estimate the in situ acoustic impedance on the surfaces of arbitrary-shape interior spaces

Gabriel Pablo Nava*
NTT Communication Science Labs.
Hikaridai 2-4, Seika-cho, Soraku-gun, Kyoto. 619-0237, JAPAN.

Yosuke Yasuda
Graduate School of Frontier Sciences, University of Tokyo.

Yoichi Sato, Shinichi Sakamoto
Institute of Industrial Science, University of Tokyo.

Abstract

Measurements of the acoustic characteristics of materials are often needed to make accurate predictions of sound fields by numerical simulations. From the existent in situ measurement techniques, those based on inverse boundary models have the advantage to deal with surfaces of arbitrary shape. However, a drawback with such approaches is their high sensitivity to noise due to a rank deficient linear model, and although regularization steps have been applied to improve their robustness, their application in practical situations is still limited. The inverse boundary model presented in this paper takes a different approach to deal with such ill-conditioned model by exploiting knowledge of the surface geometry which allows us to formulate an iterative optimization procedure with an improved robustness to noise. This formulation further allows the estimation of the acoustic impedance of not only a single surface but all the modeled surfaces in an interior space. Moreover, in contrast to previous papers of similar methods that have reported numerical simulations, this paper shows results from preliminary experiments in an acoustically controlled environment.

1 Introduction

The traditional measurement techniques with a pair of microphones [1, 2] provide important information of the acoustic absorption/impedance coefficients of the materials in situ, but on the other hand, their application is limited to a few practical situations that meet strict geometrical conditions. Most of such constraints have been overcome with the use of sound pressure and particle velocity sensors recently introduced into the market [3]. Another branch of methods able to deal with vibro-acoustic objects of arbitrary shape are those based on inverse boundary formulations. In this kind of approach, the unknown acoustic parameters at the surface (boundary) are estimated from measurements of the sound field under influence. To achieve that, the wave propagation (linear) model has to be solved in its inverse form. Indeed, pioneering attempts to estimate boundary values in such fashion date back to the appearance of the near-field acoustic holography (NAH) [4], but it was until more recent years that estimation of the acoustic impedance on the surface of materials was coped by acoustical inverse methods [5, 6]. Nevertheless, a significant pitfall of these methods is their high sensitivity to noise due to the rank deficiency of the inverse model. And although engineers have suggested regularization techniques

*Corresponding author. E-mail: pablo@cslab.kecl.ntt.co.jp

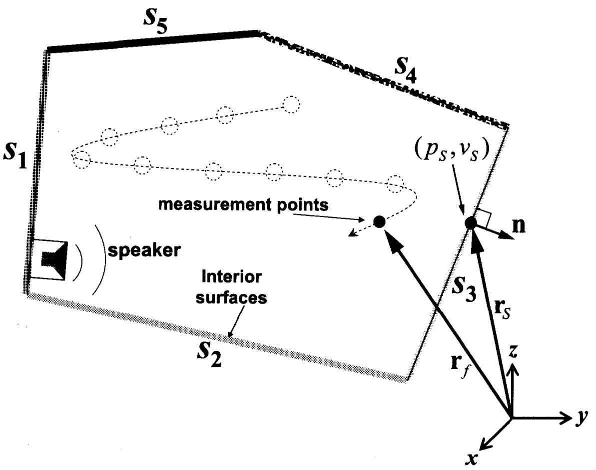


Figure 1: Diagram of the inverse estimation of the acoustic impedance on the interior surfaces of a room.

(e.g. Tikhonov regularization) and measurement procedures [7] to deal with such problem, a practical implementation for in situ acoustic impedance measurement had remained unrealized. In contrast, this paper reports experimental attempts to estimate the acoustic impedance on the (configurable) surfaces of a real (experimental) room, by the use of a readily introduced system [8]. Its theoretical framework is based on the inverse boundary element method formulated as an iterative optimization algorithm which takes as input the geometrical model of the room, the position and strength of the sound source, and samples of the sound pressure measured at arbitrary points in the interior field. Further assumptions on the segmentation of the test surfaces allows simplification of the model leading to a reduction of dimensionality and improvement of its robustness to noise.

2 Theoretical framework

Consider the arbitrary subspace of Fig. 1, bounded by n surfaces S_n whose acoustic impedance is to be calculated from M measurements of sound pressure p_f taken at aleatory points in the interior sound field which is generated by a source (speaker) vibrating harmonically with a normal particle velocity E and frequency ω. At any point of the surfaces, the normal-incidence acoustic impedance z_S is given by the ratio:

$$z_S = \frac{p_S}{v_S}. \tag{1}$$

Thus, z_S is related to p_f through the surface parameters p_S and v_S (sound pressure and particle velocity, respectively), according to the Kirchhoff-Helmholtz expression:

$$p_f + \int_S \left(p_S \frac{\partial \mathrm{G}(r)}{\partial \mathrm{n}} + j\omega\rho \mathrm{G}(r)\, v_S \right) dS = 0\ , \tag{2}$$

where $G(r) = e^{-j\mathrm{k}r}/4\pi r$ is the Green's function in 3D space, dependent on the distance $r = |\mathbf{r}_S - \mathbf{r}_f|$ and the wave number $\mathrm{k} = \omega/c$. ρ and c are the density and sound speed of the propagation medium, and $j = \sqrt{-1}$.

Therefore, in order to estimate z_S, equation (2) should be solved for p_S and v_S. This can be accomplished numerically by discretizing the surfaces into a mesh of N elements (e.g. triangles). By doing so, it can be shown that applying a Boundary Element Method (BEM) framework [9] yields two matrix equations derived from eq. (2):

$$\begin{aligned} \mathbf{A}_S\, \mathbf{p}_S - \mathbf{B}_S\, \mathbf{v}_S &= 0\ , & (3)\\ \mathbf{A}_f\, \mathbf{p}_S - \mathbf{B}_f\, \mathbf{v}_S &= -\,\mathbf{p}_f\ . & (4) \end{aligned}$$

Assuming constant interpolation between elements, the coefficients of the complex matrices $\mathbf{A}_S$, $\mathbf{B}_S$ and $\mathbf{A}_f$, $\mathbf{B}_f$can be computed from:

$$a_{i,k} \;=\; \int_{S_k} \frac{\partial \mathrm{G}(r)}{\partial \mathrm{n}}\,\mathrm{ds} \quad , \quad b_{i,k} \;=\; -j\omega\rho \int_{S_k} \mathrm{G}(r)\,\mathrm{ds}\;, \tag{5}$$

here, s_k is the surface of the k-th elementm, thus $k = 1, 2, \ldots, N$, and $i = 1, 2, \ldots, N$ for eq. (3) or $i = 1, 2, \ldots, M$ for eq. (4). Furhtermore, taking into account the sound source E, eq. 4 can be expanded to express known and unknown parameters at each side:

$$\mathbf{A}_f\mathbf{p}_S - \mathbf{B}_{f,\mathrm{un}}\mathbf{v}_S = \mathbf{B}_{f,E}\mathbf{v}_E - \mathbf{p}_f \;, \tag{6}$$

were $\mathbf{B}_{f,\mathrm{un}}$ and $\mathbf{B}_{f,E}$ represent the coefficients of the unknown velocities $\mathbf{v}_S$ and of the known sources $\mathbf{v}_E$.

We can proceed now to simplify the model by assuming that the surfaces have been clustered by homogeneity as the geometry of the interior space is known. Moreover, if only the acoustic effect at the point of incidence on the surface (i.e. local impedance) is considered, then for each surface type, the acoustic impedance can be approximated by the following relation:

$$\frac{p_{S_i,1}}{v_{S_i,1}} \approx \frac{p_{S_i,2}}{v_{S_i,2}} \approx \ldots \approx \frac{p_{S_i,m_i}}{v_{S_i,m_i}}\;, \tag{7}$$

that is

$$z_{S_i,1} \approx z_{S_i,2} \approx \ldots \approx z_{S_i,m_i} = Z_i\;, \tag{8}$$

where, m_i is the number of discrete element that belongs to the i-th surface. Since p_S and z_S are interchangeable, the sound pressure $\mathbf{p}_S$ in eq. (6) can be substituted by their impedance equivalent and the matrix $\mathbf{A}_f$ can be rearranged according to eq. (7) as follows:

$$\langle \mathbf{A}_f \cdot \tilde{\mathbf{v}}_S \rangle = \begin{pmatrix} \sum_{k=1}^{m_1} a_{f,(1,k)}\,\tilde{v}_{S,k} & \sum_{k=m_1+1}^{m_2} a_{f,(1,k)}\,\tilde{v}_{S,k} & \cdots & \sum_{k=m_{n-1}+1}^{m_n} a_{f,(1,k)}\,\tilde{v}_{S,k} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ \sum_{k=1}^{m_1} a_{f,(M,k)}\,\tilde{v}_{S,k} & \sum_{k=m_1+1}^{m_2} a_{f,(M,k)}\,\tilde{v}_{S,k} & \cdots & \sum_{k=m_{n-1}+1}^{m_n} a_{f,(M,k)}\,\tilde{v}_{S,k} \end{pmatrix}, \tag{9}$$

to yield the compact matrix form

$$\langle \mathbf{A}_f \cdot \mathbf{v}_S \rangle\, \mathbf{z} - \mathbf{B}_{f,\mathrm{un}}\mathbf{v}_S = \hat{\mathbf{p}}_f \;, \tag{10}$$

where

$$\mathbf{z} = \{Z_1, Z_2, \ldots, Z_n\}^{\mathrm{T}}\;, \quad \hat{\mathbf{p}}_f = \mathbf{B}_{f,E}\mathbf{v}_E - \mathbf{p}_f \;. \tag{11}$$

Hence, the sought acoustic impedances Z_n can be found by solving the optimization problem

$$\begin{gathered} \mathbf{z}_{\min} = \min \|\langle \mathbf{A}_f \cdot \mathbf{v}_S\rangle \mathbf{z} - \mathbf{B}_{f,\mathrm{un}}\mathbf{v}_S - \hat{\mathbf{p}}_f\| \\ \text{s.t.}\;\; \mathrm{Z}_{\min} \le \mathbf{z} \le \mathrm{Z}_{\max} \end{gathered} \tag{12}$$

within the iterative algorithm shown in Algorithm 1. The iteration process is initialized with an initial guess $\mathbf{z}^{(0)} = \{Z_1^{(0)}, Z_2^{(0)}, \ldots, Z_n^{(0)}\}$ and continued until the condition of eq. (16) is satisfied. For each iteration, eq. (14) is solved using a global optimization solver based on Sequential Quadratic Programming implemented on readily available commercial software which allows its straightforward utilization. The matrices $\mathbf{C}$'s are assembled as

$$c_{i,k} = z_{S,k} \cdot a_{i,k} \;, \tag{17}$$

and finally, $\mathbf{p}_G$ and $\mathbf{p}_F$ are the predicted and the measured sound pressures expressed in decibels. The predicted sound pressure is computed at the same points of the actual measurements, therefore, the position of the measurement points is also assumed to be known.

1: Using $\mathbf{z}^{(0)}$, solve for $\mathbf{v}_S^{(\ell)}$ from

$$(\mathbf{C}_S - \mathbf{B}_{S,\mathrm{un}})\mathbf{v}_S = \mathbf{B}_{S,E}\mathbf{v}_E \tag{13}$$

2: **while** condition (16) is **false do**

3: Update $\mathbf{z}$ by

$$\mathbf{z}^{(\ell+1)} = \min \|\langle \mathbf{A}_f \cdot \mathbf{v}_S \rangle \mathbf{z} - \mathbf{B}_{f,\mathrm{un}}\mathbf{v}_S - \hat{\mathbf{p}}_f\| \quad \text{s.t.} \;\; \mathrm{Z}_{\min} \leq \mathbf{z} \leq \mathrm{Z}_{\max} \tag{14}$$

4: Update $\mathbf{v}_S^{(\ell+1)}$ from eq. (13) using $\mathbf{z}^{(\ell+1)}$

5: Compute $\mathbf{p}_g$ by

$$\mathbf{p}_g = \mathbf{B}_{f,E}\mathbf{v}_{S,E} - (\mathbf{C}_f - \mathbf{B}_{f,\mathrm{un}})\mathbf{v}_S^{(\ell+1)} \tag{15}$$

6: Evaluate

$$h(\mathbf{p}_g, \mathbf{p}_f) = \frac{\|\mathbf{p}_g - \mathbf{p}_f\|^2}{\|\mathbf{p}_f\|^2} + \frac{1}{M-1}\frac{\|\mathbf{p}_G - \mathbf{p}_F\|^2}{\|\mathbf{p}_F\|^2} \leq \alpha \tag{16}$$

7: $\ell \leftarrow \ell + 1$

8: **end while**

Algorithm 1: Iterative algorithm for the estimation of $\mathbf{z}$

3 Experiments in a room

To demonstrate the performance of the Algorithm 1, experiments in an acoustically isolated room were performed. The objective was to estimate the acoustic impedance of its walls (which are covered by removable sound-absorbent panels), its carpeted floor, and its ceiling (covered with the same absorbent panels) distributed as illustrated in Fig. 2. For that purpose, the real-time measurement setup depicted in Fig. 3 was employed. In brief, the implementation consists of an omnidirectional microphone to which a light has been attached, and a set of four overhead video cameras. Thus, while the microphone records samples of sound pressure in the 3D space, the video system keeps track of the light marker (i.e. the microphone) calculating its 3D position by triangulation. Such real-time implementation has been introduced previously and demonstrated that is able to record massive number of sound pressure measurements while allowing a free displacement of the microphone [8]. For each experiment, a pure tone was output through the sound source (speaker) and $M = 2N$ samples were recorded. Here, the physical parameters to be measured are the complex sound pressure and vibration of the sound source, expressed respectively as:

$$\begin{aligned} p_f &= |\mathrm{P}| \exp^{j(\omega t + \theta_f)} , \\ v_{\mathrm{spk}} &= |\mathrm{V}| \exp^{j(\omega t + \theta_{\mathrm{spk}})} . \end{aligned} \tag{18}$$

The strength of the sound source $|\mathrm{V}|$ is calculated from the laser doppler vibrometer (LDV) which is also taken as reference (i.e. $\theta_{spk} = 0$) to estimate the phase θ_f of the sound pressure p_f. The sound pressure amplitude $|\mathrm{P}|$ is directly observed from the microphone signals. The test frequencies fall in the range 80 Hz to 240 Hz with intervals of 20 Hz.

To estimate the acoustic impedance of the surfaces in the room, the geometric model, the measure data and sound source information was input Algorithm 1. Moreover, the initialization parameters were set to $\mathbf{z}^{(0)} = \mathbf{1}$, i.e. the normalized characteristic acoustic impedance of the media $z = \rho c$, and the bounds of the solution space were constrained to $-Z_{max} \leq \mathrm{Re}\{\mathbf{z}\} \leq Z_{max}$ and $-Z_{\max} \leq \mathrm{Im}\{\mathbf{z}\} \leq Z_{\max}$, where $Z_{\max} = 1000$ (normalized). The stopping parameter was set as $\alpha = 0.01$. Note that these are empirical values that should be further investigated to unify a universally applicable criteria.

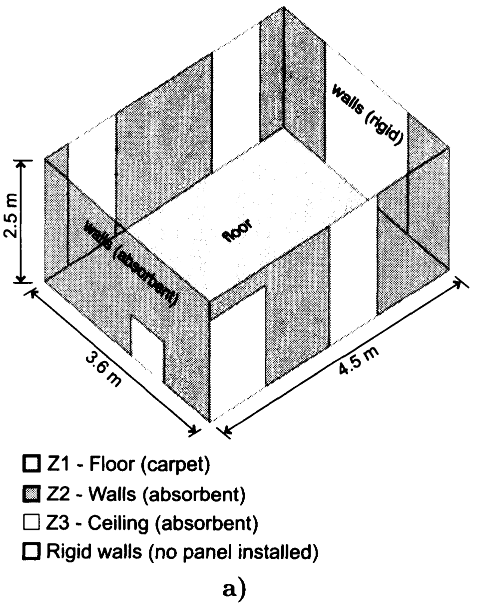


Figure 2: Experimental room.

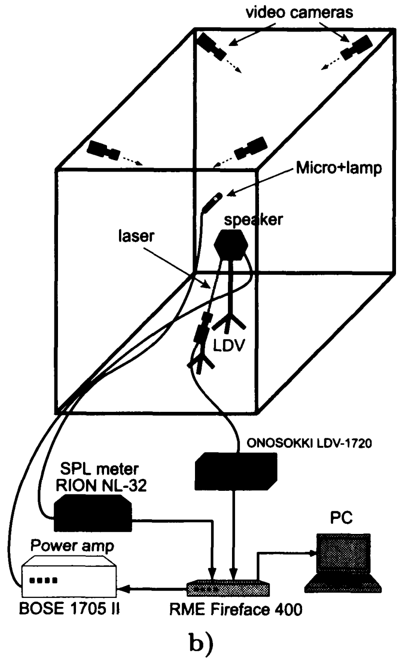


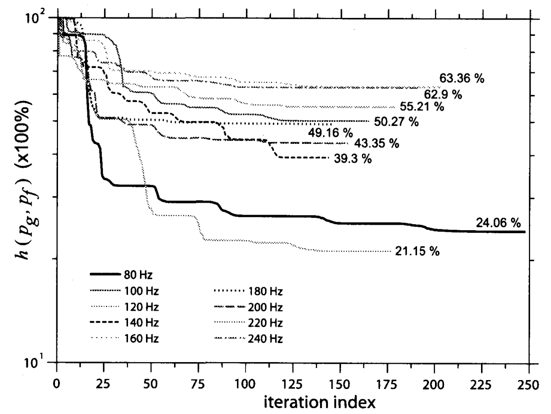
Figure 3: Measurement system.

4 Experimental results

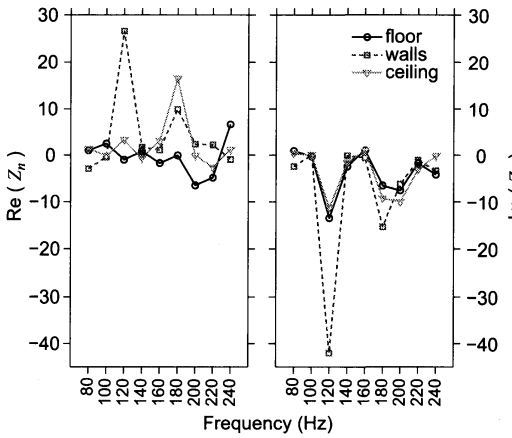
The performance of the iterative Algorithm 1 can be observed in the iteration history shown in Fig. 4. Although care has been put to perform the experiments, none of the analyses achieved the preset criterion $\alpha = 0.01$ (or 1%). The best performance is is shown at 120 Hz where an evaluation 21.5 % was achieved, and 63.3 % at 160 Hz for the worst case. These levels suggest a considerable error introduced during the geometric modeling stage and/or the measurement process itself. Nevertheless, the Algorithm 1 converged to the stable values of acoustic impedance shown in Fig. 5. Note that the real and imaginary parts of the complex acoustic impedance is expressed as normalized values $Z_n = Z/\rho c$. Let us remark that, although a clear tendency can be appreciate it in the results, a definite validation is achieved by comparison with the true values or results obtained by other similar methods. By the moment, the lack of either motives further research on alternative validation methods.

5 Conclusions

An acoustical inverse approach for the estimation of in situ acoustic impedance of arbitrary-shape interiors has been presented. At the core of the method is an iterative optimization algorithm derived from the boundary element method applied to the geometric model of the room. The algorithm takes as input the geometric model, a number of sound pressure measurements, and the position and strength of the sound source. As output, the (normal-incidence) acoustic impedance of all the surfaces are computed at once. A few similar approaches have been reported before, but none had been tested experimentally. However, this paper has introduced an algorithm robust to noise enough to allow for experiments. Also, a real-time measurement implementation that further facilitates experimenting in real rooms was described. Although a definite validation of the system is in an undergoing-stage, the preliminary experiments have shown promising results that motivate further investigation.



a)



b)

Figure 4: Iteration history showing the performance of the Algorithm 1 with the experimental data.

Figure 5: Estimated values of complex acoustic impedance on the surfaces of the experimental room.

References

[1] J. F. Allard, C. Depoiller and P. Guignouard, "Free field surface impedance measurements of sound absorbing materials with surface coatings", *Applied Acoustics*, **26**, 199–207(1989).

[2] A. J. Cramond and C. G. Don, "Reflection of impulses as a method of determining the acoustic impedance", *J. Acoust. Soc. of Am.*, **75**, 328–344(1984).

[3] R. Lanoye, G. Vermeir and W. Lauriks, "Measuring the free field acoustic impedance and absorption of sound absorbing materials with a combined particle velocity-pressure sensor", *J. Acoust. Soc. of Am.*, **119**, 2826–2831(2006).

[4] J. D. Maynard, E. G. Williams and Y. Lee, "Nearfield Acoustic Holography: I. Theory of the generalized holography and the developement of the NAH", *J. Acoust. Soc. Am.*,**78**, 1395–1413(1985).

[5] St. Marburg and H.-J Hardke, "A study on the acoustic boundary admittance. Determination, results and consequences", *Eng. Analaysis with Boundary Elements*, **23**, 737–744(1999).

[6] G. Dutilleux, F. C. Sgarg and U. R. Kristiansen, "Low-frequency assessment of the in situ acoustic absorption of materials in rooms: an inverse problem approach using evolutionary optimization", *Int. J. Numer. Meth. Eng.*, **53**, 2143-2161(2002).

[7] P.A. Nelson and S.H. Yoo, "Estimation of acoustic source strenght by inverse methods: Part I, conditioning of the inverese problem", *J. of Sound and Vibration*, **233**, 643–668(2000).

[8] G. Pablo Nava, Y. Yasuda, Y. Sato and S. Sakamoto, "In situ estimation of acoustic impedance on the surfaces of realistic interiors: An inverse approach", *POMA*, **2**, 015001(2008).

[9] O. von Estorff, "Boundary elements in acoustics – Advances and applications", *WIT Press*, 1–64(2000).