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Periodic points on the boundaries of rotation domains

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Abstract

We are interested in periodic points on the boundaries of rotation domains of rational functions. In this talk, we show that the boundary of an invariant rotation domain contains no periodic points except for the Cremer points when the boundary has an injective neighborhood.

1 Introduction and the result

The dynamics on a periodic Fatou component is well understood, actually there are three possibilities. They are the attracting case, the parabolic case or the irrational rotation case. However, it is difficult to see the dynamics on the boundary of a periodic Fatou component.

It is interesting that the periodic points on the boundary of an immediate attracting or parabolic basin are dense in the boundary [**PrZ**, Theorem A]. So we may ask can the boundaries of rotation domains have periodic points? According to R. Pérez-Marco, the injectivity on a simply connected neighborhood of the closure of a Siegel disk implies that no periodic points on the boundary of the Siegel disk [**PM**, Theorem IV.4.2].

In general, it may be hard to find such a simply connected domain where the function is injective. The following theorem implies that there are still no periodic points except for the Cremer points on the boundary of invariant rotation domains even when the injective neighborhood is not a finitely connected domain.

Theorem 1.1 *Let Ω be an invariant rotation domain of a rational function R, and let U be a neighborhood of $\overline{\Omega}$. If R is injective on U, then the boundary $\partial\Omega$ contains no periodic points except for the Cremer points.*

2 Local surjectivity

We see local surjectivity of a rational function R of degree at least two. The notion of local surjectivity is referred from [**Sch**].

Definition 2.1 Let Ω be a Fatou component, and let $z_0 \in \partial\Omega$. We say R is *locally surjective* for (z_0, Ω), if there exists $\epsilon > 0$ such that $R(N \cap \Omega) = R(N) \cap R(\Omega)$ for any neighborhood $N \subset B_\epsilon(z_0)$ of z_0.

Lemma 2.1 *Let Ω be a Fatou component, and let $z_0 \in \partial\Omega$. Assume that R is locally surjective for (z_0, Ω) and $(R(z_0), R(\Omega))$. Then R^2 is locally surjective for (z_0, Ω).*

The following fact has been pointed out in [**Sch**].

Lemma 2.2 *Let Ω be a Fatou component, and let $z_0 \in \partial\Omega$. Assume that R is not locally surjective for (z_0, Ω). Then there exists a Fatou component $\Omega' \neq \Omega$ such that $z_0 \in \partial\Omega'$ and $R(\Omega') = R(\Omega)$.*

3 The proof of the result

We show Theorem 1.1 by using the following key proposition [**Sch**, Theorem 1].

Proposition 3.1 *Let Ω be an invariant Fatou component, and let $z_0 \in \partial\Omega$ be a weakly repelling fixed point. If R is locally surjective for (z_0, Ω), then z_0 is accessible from Ω by a periodic curve.*

4 Some related topics

We shall give some results on related topics.

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